Solution for exercise 3.3.2 in Pitman

The random variable Y is binominally distributed. The mean $E(Y) = 3\frac{1}{2} = \frac{3}{2}$, the variance is $V(Y) = 3\frac{1}{2}\frac{1}{2} = \frac{3}{4}$. The Mean of Y^2 is

$$E(Y^2) = \sum_{i=0}^{3} i^2 \binom{3}{i} \left(\frac{1}{2}\right)^i \left(\frac{1}{2}\right)^{3-i} = 1\frac{3}{8} + 4\frac{3}{8} + 9\frac{1}{8} = 3$$

Alternatively we could have used the computational formula for the variance

$$V(Y) + (E(Y))^2 = \frac{3}{4} + \frac{9}{4} = 3$$

$$P(Y = 0) = \left(\frac{1}{2}\right)^3 \qquad P(Y = 1) = 3\left(\frac{1}{2}\right)^3 \qquad P(Y = 2) = 3\left(\frac{1}{2}\right)^3 \qquad P(Y = 3) = \left(\frac{1}{2}\right)^3$$

The variance of Y^2 can be found by

$$V(Y^2) = E((Y^2)^2) - (E(Y^2))^2$$

We need to calculate $E((Y^2)^2) = E(Y^4)$

$$E(Y^4) = \frac{3 + 16 \cdot 3 + 81 \cdot 1}{8} = \frac{132}{8} = \frac{33}{2}$$

Finally

$$V(Y^2) = \frac{33}{2} - 9 = \frac{15}{2}$$