

Solution for exercise 2.2.1 in Pitman

All questions are answered by applying The Normal Approximation to the Binomial Distribution page 99 (131). We have $\mu = n \cdot p = 400 \cdot \frac{1}{2} = 200$, $\sigma = \sqrt{npq} = \sqrt{400 \cdot \frac{1}{2} \cdot \frac{1}{2}} = 10$. The questions differ only in the choice of a and b in the formula.

Question a) $a = 190, b = 210$

$$\begin{aligned} P(190 \text{ to } 210 \text{ successes}) &= \Phi\left(\frac{210.5 - 200}{10}\right) - \Phi\left(\frac{189.5 - 200}{10}\right) \\ &= \Phi(1.05) - \Phi(-1.05) = 0.8531 - (1 - 0.8531) = 0.7062 \end{aligned}$$

Question b) $a = 210, b = 220$

$$\begin{aligned} P(210 \text{ to } 220 \text{ successes}) &= \Phi\left(\frac{220.5 - 200}{10}\right) - \Phi\left(\frac{209.5 - 200}{10}\right) \\ &= \Phi(2.05) - \Phi(0.95) = 0.9798 - 0.8289 = 0.1509 \end{aligned}$$

Question c) $a = 200, b = 200$

$$\begin{aligned} P(200 \text{ successes}) &= \Phi\left(\frac{200.5 - 200}{10}\right) - \Phi\left(\frac{199.5 - 200}{10}\right) \\ &= \Phi(0.05) - \Phi(-0.05) = 0.5199 - (1 - 0.5199) = 0.0398 \end{aligned}$$

Question d) $a = 210, b = 210$

$$\begin{aligned} P(210 \text{ successes}) &= \Phi\left(\frac{210.5 - 200}{10}\right) - \Phi\left(\frac{209.5 - 200}{10}\right) \\ &= \Phi(1.05) - \Phi(0.95) = 0.8531 - 0.8289 = 0.0242 \end{aligned}$$