IMM - DTU

02405 Probability
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BFN/bfn

Solution for exercise 1.5.9 in Pitman

Denote the event that a shape of type i is picked by T_i, the event that it lands flat by F and the event that the number rolled is six by S. We have $P(T_i) = \frac{1}{3}, i = 1, 2, 3$, $P(F|T_1) = \frac{1}{3}, P(F|T_2) = \frac{1}{2}$, and $P(F|T_3) = \frac{2}{3}$ $P(S|F) = \frac{1}{2}$, and $P(S|F^c) = 0$.

Question a) We first note that the six events $T_i \cap F$ and $T_i \cap F^c$ $(i = 1, 2, 3)$ is a partition of the outcome space. Now using The Rule of Averaged Conditional Probabilities (The Law of Total Probability) page 41

$$P(S) = P(S|T_1 \cap F)P(T_1 \cap F) + P(S|T_2 \cap F)P(T_2 \cap F) + P(S|T_3 \cap F)P(T_3 \cap F) + P(S|T_1 \cap F^c)P(T_1 \cap F^c) +$$

The last three terms are zero. We apply The Multiplication Rule for the probabilities $P(T_i \cap F)$ leading to

$$P(S) = P(S|T_1 \cap F)P(F|T_1)P(T_1) + P(S|T_2 \cap F)P(F|T_2)P(T_2) + P(S|T_3 \cap F)P(F|T_3)P(T_3)$$

a special case of The Multiplication Rule for n Events page 56. Inserting numbers

$$P(S) = \frac{1}{2}\frac{1}{3}\frac{1}{3} + \frac{1}{2}\frac{1}{2}\frac{1}{3} + \frac{1}{2}\frac{2}{3}\frac{1}{3} = \frac{1}{4}$$

Question b) The probability in question is $P(T_1|S)$. Applying Bayes' rule page 49

$$P(T_1|S) = \frac{P(S|T_1)P(T_1)}{P(S)} = \frac{\frac{1}{6}\frac{1}{3}}{\frac{1}{4}} = \frac{2}{9}$$