IMM - DTU

02405 Probability
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BFN/bfn

Solution for exercise 1.5.1 in Pitman

We introduce the events

O The odd box is picked

B A black marble is picked

We have $P(O) = \frac{1}{2}$ and $P(B|O) = \frac{1}{4}, P(B|O^c) = \frac{2}{6} = \frac{1}{3}$.

Question a) The events O and O^c is a partition (page 20, see also page 40). We apply the rule of averaged conditional probabilities (box at top of page 41, summary page 73) to get

$$P(B) = P(B|O)P(O) + P(B|O^c)P(O^c) = \frac{1}{4} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{2}{6} = \frac{7}{24}$$

Question b) The probability in question is $P(O^c|B^c)$,which is a standard setting for the application of Bayes rule (interchange of conditioning) page 49 or summary page 73. We get

$$P(O^c|W) = \frac{P(W|O^c)P(O^c)}{P(W)} = \frac{\frac{4}{6} \cdot \frac{1}{2}}{\frac{17}{24}} = \frac{8}{17}$$

Remark: We could have written the denominator in full as on page 49. However the denominator is simply the probability of the event which we condition upon on the left side of Bayes rule; that is the event A in the genereal form page 49, or in this case the event B.

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