

Solution for exercise 1.3.11 in Pitman

$$P(A \cup B \cup C) = P(A \cup (B \cup C))$$

now applying inclusion-exclusion

$$P(A \cup (B \cup C)) = P(A) + P(B \cup C) - P(A \cap (B \cup C)) = P(A) + P(B \cup C) - P((A \cap B) \cup (A \cap C))$$

once again we apply inclusion-exclusion (the second and the third time) to get

$$\begin{aligned} P(A \cup (B \cup C)) &= P(A) + P(B) + P(C) - P(B \cap C) - (P(A \cap B) + P(A \cap C) - P((A \cap B) \cap (A \cap C))) \\ &= P(A) + P(B) + P(C) - P(B \cap C) - P(A \cap B) - P(A \cap C) + P(A \cap B \cap C) \end{aligned}$$