IMM - DTU

02405 Probability
2004-4-17
BFN/bfn

Solution for exercise 6.4.1 in Pitman

Question a) From the definition of conditional proability we have $P(A|B) = \frac{A \cap B}{P(B)}$. Now from inclusion-exclusion e.g. page 22 we have $P(A \cap B) = P(A) + P(B) - P(A \cup B)$. Thus

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A) + P(B) - P(A \cup B)}{P(B)} = \frac{1}{2}$$

Question b) Since $P(A)P(B) = 0.12 < 0.2 = P(A \cap B)$ we conclude that A and B are positive depdendent (page 431).

Question c) Using $B = (A \cap B) \cup (A^c \cap B)$ we find $P(A^c \cap B) = P(B) - P(A \cap B) = 0.2$

Question d) We find for the Bernoulli distribution which is the binomial distribution with $n = 1$ (e.g. page 479) $\sigma_X = \sqrt{0.3 \cdot 0.7}$ and $\sigma_Y = \sqrt{0.4 \cdot 0.6}$. Further $E(XY) = P(I_A \cdot I_B = 1) = P(A \cap B) = 0.2$. Using $Cov(X, Y) = E(XY) - E(X)E(Y)$ page 430 and the correlation definition page 432 we get

$$Corr(X, Y) = \frac{0.2 - 0.12}{\sqrt{0.21 \cdot 0.24}} = 0.356$$