

### Solution for exercise 6.2.1 in Pitman

We have the joint distribution (distribution table) (see e.g. exercise 3.1.4)

$X \setminus Y$	1	2	3	4	5	6
1	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{18}$	$\frac{1}{18}$	$\frac{1}{18}$	$\frac{1}{18}$
2	0	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{18}$	$\frac{1}{18}$	$\frac{1}{18}$
3	0	0	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{18}$	$\frac{1}{18}$
4	0	0	0	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{18}$
5	0	0	0	0	$\frac{1}{36}$	$\frac{1}{18}$
6	0	0	0	0	0	$\frac{1}{36}$

We find the conditional distributions to be

$Y :$	1	2	3	4	5	6
$P(X = 1 Y = y)$	1	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{11}$
$P(X = 2 Y = y)$	0	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{11}$
$P(X = 3 Y = y)$	0	0	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{11}$
$P(X = 4 Y = y)$	0	0	0	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{11}$
$P(X = 5 Y = y)$	0	0	0	0	$\frac{1}{9}$	$\frac{1}{11}$
$P(X = 6 Y = y)$	0	0	0	0	0	$\frac{1}{11}$

and

$x$	$P(Y = 1 X = x)$	$P(Y = 2 X = x)$	$P(Y = 3 X = x)$	$P(Y = 4 X = x)$	$P(Y = 5 X = x)$	$P(Y = 6 X = x)$
1	$\frac{1}{11}$	0	0	$\frac{2}{11}$	$\frac{2}{11}$	$\frac{2}{11}$
2	0	$\frac{1}{9}$	0	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
3	0	0	$\frac{1}{7}$	0	$\frac{1}{7}$	$\frac{1}{7}$
4	0	0	0	$\frac{1}{5}$	0	$\frac{1}{5}$
5	0	0	0	0	$\frac{1}{3}$	$\frac{1}{3}$
6	0	0	0	0	0	$\frac{1}{3}$

where we have used the short  $P(Y = y|x)$  for  $P(Y = y|X = x)$ .

Such that

$Y :$	1	2	3	4	5	6
$E(X Y = y)$	1	$\frac{4}{3}$	$\frac{9}{5}$	$\frac{16}{7}$	$\frac{25}{9}$	$\frac{36}{11}$

and

$x$	$E(Y X = x)$
1	$\frac{41}{11}$
2	$\frac{38}{9}$
3	$\frac{33}{7}$
4	$\frac{26}{5}$
5	$\frac{17}{3}$
6	6