

Solution for exercise 6.1.3 in Pitman

Question a) Assuming that the total number of families is n we can deduce that we have $i \cdot P(T = i) \cdot n$ tickets from families with i children, giving a total of $0 \cdot 0.1 \cdot n + 1 \cdot 0.2 \cdot n + 2 \cdot 0.4 \cdot n + 3 \cdot 0.2 \cdot n + 4 \cdot 0.1 \cdot n = 2n$ tickets, $3 \cdot 0.2 \cdot n$ of those from families with 3 children. Using equally likely outcomes (section 1.1) we get $P(U = 3) = 0.3$.

Question b) The probability in question is $P(U = 3, G = 2)$, we find this probability sequentially like in example 1. $P(U = 3, G = 2) = P(U = 3)P(G = 2|U = 3) = 0.3 \cdot \binom{3}{2} 2^{-3} = \frac{9}{80}$

Question c) $P(T = 3, G = 2) = P(T = 3)P(G = 2|T = 3) = 0.2 \cdot \binom{3}{2} 2^{-3} = \frac{3}{40}$