IMM - DTU

02405 Probability
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BFN/bfn

Solution for exercise 5.4.2 in Pitman

Question a) Consider the joint distribution on the unit square. The area of the triangle $x+y>1, x<1, y<1$ is $\frac{1}{8}$, thus $F_{S_2}(1.5) = 1-\frac{1}{8} = \frac{7}{8}$. Alternatively one could use the boxed result page 372 with $S_2 = X_1+X_2$, X_i uniforum. We find

$$f_{S_2}(x) = \begin{cases} \int_0^x 1 \mathrm{d}x_1 & 0 \le x \le 1 \\ \int_{x-1}^1 1 \mathrm{d}x_1 & 1 \le x \le 2 \end{cases} = \begin{cases} x & 0 \le x \le 1 \\ 2-x & 1 \le x \le 2 \end{cases}$$

leading to

$$F_{S_2}(x) = \begin{cases} \frac{x^2}{2} & 0 \le z \le 1 \\ 2z - \frac{z^2}{2} - 1 & 1 \le z \le 2 \end{cases}$$

and $F_{S_2}(1.5) = 0.875$.

Question b) This is a) in example 3 page 379. $P(S_3 \le 1.5) = 0.5$.

Question c) Now using the results of example 3 we get

$$P(S_3 \le 1.1) = \int_0^1 \frac{t^2}{2} \mathrm{d}t + \int_1^{1.1} \left(-t^2 + 3t - \frac{3}{2}\right) \mathrm{d}t = \frac{1}{6} + \left[-\frac{t^3}{3} + \frac{3t^2}{2} - \frac{3t}{2}\right]_{t=1}^{t=1.1} = 0.2213$$

Question d) Using the standard approximation $P(x < S_3 < x + \mathrm{d}x) \simeq f_{S_3}(x)\mathrm{d}x$ we find $P(1 \le S_3 \le 1.001) \simeq \frac{1}{2} \cdot 0.001 = 0.0005$.