IMM - DTU

02405 Probability
2004-4-12
BFN/bfn

Solution for exercise 5.4.1 in Pitman

Question a) The joint density of (X_1, X_2) is

$$f(x_1, x_2) = \begin{cases} \frac{1}{2} & 0 \le x_1 \le 1, 0 \le x_2 \le 2 \\ 0 & \text{otherwise} \end{cases}$$

We find

$$P(X_1 + X_2 \le 2) = \int_{x_1+x_2 \le 2} \frac{1}{2} \mathrm{d}x_1 \mathrm{d}x_2$$

or use straightforward area considerations, to get $P(X_1 + X_2 \le 2) = \frac{3}{4}$

Question b) We use the boxed expression page 372 to get

$$f_{x_1+x_2}(z) = \begin{cases} \int_0^z 1 \cdot \frac{1}{2} \mathrm{d}x_1 & 0 \le z \le 1 \\ \int_0^1 1 \cdot \frac{1}{2} \mathrm{d}x_1 & 1 \le z \le 2 \\ \int_{z-2}^1 1 \cdot \frac{1}{2} \mathrm{d}x_1 & 2 \le z \le 3 \end{cases}$$

thus

$$f_{x_1+x_2}(z) = \begin{cases} \frac{z}{2} & 0 \le z \le 1 \\ \frac{1}{2} & 1 \le z \le 2 \\ \frac{3-z}{2} & 2 \le z \le 3 \end{cases}$$

Question c)

$$F_{x_1+x_2}(z) = \int_0^z f_{x_1+x_2}(u) \mathrm{d}u = \begin{cases} \frac{z^2}{4} & 0 \le z \le 1 \\ \frac{2z-1}{4} & 1 \le z \le 2 \\ \frac{6z-z^2-5}{4} & 2 \le z \le 3 \end{cases}$$