

Solution for exercise 5.2.6 in Pitman

Question a) Draw a small figure showing the area of integration. Using page 349 we get

$$\begin{aligned} P(Y > 2X) &= \int_0^{\frac{1}{2}} \int_{2x}^1 90(y-x)^8 dy dx = \int_0^{\frac{1}{2}} [10(y-x)^9]_{y=2x}^{y=1} dx = \int_0^{\frac{1}{2}} (10(1-x)^9 - 10x^9) dx \\ &= [-(x-1)^{10} - x^{10}]_{x=0}^{x=\frac{1}{2}} = 1 - 2 \left(\frac{1}{2}\right)^{10} \end{aligned}$$

Question b) The marginal density of X is given by (using page 349)

$$f_X(x) = \int_x^1 90(y-x)^8 dy = 10(1-x)^9$$

with CDF

$$F_X(x) = \int_0^x 10(1-u)^9 du = 1 - (1-x)^{10}$$

The marginal density of Y is given by (using page 349)

$$f_Y(y) = \int_0^y 90(y-x)^8 dy = 10y^9$$

with CDF

$$F_Y(y) = \int_0^y 10u^9 du = y^{10}$$

Question c) Maximum and minimum (see exercise 4.6.3). Also note that the marginal distributions are those of max and min from page 316/317.