Written examination: December 18 2019 *Course no.* : 02405

Name of course: Probability theory *Duration* : 4 hours

Aids allowed: All

The questions have been answered by:

________________________ ________________________ ________

(name) (signature) (table no.)

There are 30 exercises with a total of 30 questions. The numbering of the exercises are given as 1,2,..., 30 in the text. Every single question is also numbered and given as question 1,2,...,30 in the text. The answers to the test must be uploaded through campusnet, using the file "answers.txt"or an equivalent file. In this file your student ID should be on the first line, the number of the question and your answer should be on the following lines, with one line for each question. The table below can potentially be handed in as a supplement to the electronic hand-in. In case of disagreement the electronic version will be used.

Question	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
answer															

Question	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
answer															

The options for each question are numbered from 1 to 6. If a wrong number has been given, it can be corrected by "blackening"out the wrong answer and writing the correct number below. In case of doubt about a correction, the question will be considered unanswered.

Draft and intermediate calculations will **not** be taken into account. Only the numbers written in the table above will be scored.

5 points are obtained for a correct answer, and -1 point for a wrong answer. Questions left unanswered or with "6" (for 'do not know') are given 0 points. The number of points needed for a sufficient exam will be determined in connection with the examination of the papers.

Notice, the idea behind the exercises is that there is one and only one correct number for every single question. All the given number options may not necessarily make sense. The last page of the exam set is page no 16; flip to that page to be sure it is there.

The notation $\log(\cdot)$ is used for the natural logarithm, i.e. logarithms with base e, while Φ denotes the cumulative distribution function for a standardised normally distributed variable.

Exercise 1

A specific geographical area has a population of 1 million, 5% belong to a certain ethnic group. A group of 100 people is selected at random for a survey.

Question 1

The probability that there are at least 7 persons from the ethnic group in the sample is

1 ☐ $1-\Phi\left(\frac{2}{\sqrt{19}}\right)$

2 ☐ $1-\Phi\left(\frac{3}{\sqrt{18}}\right)$

3 ☐ $\frac{1}{12}$

4 ☐ $1-\sum_{i=0}^{6}\binom{100}{i}(0.05)^i(1-0.05)^{100-i}$

5 ☐ $1-\sum_{i=0}^{6}\frac{e^{-5}5^i}{i!}$

6 ☐ Do not know

Exercise 2

Consider a polar coordinate system with a bombing target at the origin. A bomber throws a bomb hitting a distance R from the origin with the density $f_R(r) = re^{-\frac{1}{2}r^2}$. The probability that the target will be destroyed is e^{-r^2} if the bomb hits at a distance r from the origin.

Question 2

What is the probability that the bombing target will be destroyed?

1 ☐ $\frac{1}{4}$

2 ☐ $\frac{1}{3}$

3 ☐ $\frac{1}{2}$

4 ☐ $\frac{2}{3}$

5 ☐ $\frac{3}{4}$

6 ☐ Do not know

Continue at side 3

Exercise 3

A swimming club has 18 swimmers. Two of those are backstroke specialists, 7 are crawl specialists, 5 are butterfly specialist, and 4 are medley specialists. Four swimmers are selected randomly for a relay event.

Question 3

The probability that there are exactly two butterfly specialist among the four is

1 ☐ $\binom{4}{2}\left(\frac{5}{18}\right)^2\left(\frac{13}{18}\right)^2$

2 ☐ $\frac{5}{18}\frac{4}{17}$

3 ☐ $2 \cdot \frac{5}{18}\frac{4}{17}$

4 ☐ $\dfrac{\binom{5}{2}\binom{13}{2}}{\binom{18}{4}}$

5 ☐ $\frac{5}{18}\frac{4}{18}$

6 ☐ Do not know

Exercise 4

A material emits particles, such that the time between particle emissions can be described by independent exponentially distributed random variables with a mean of 2 minutes.

Question 4

The probability that the third particle is emitted exactly between the third and fourth minute after the start of registration is

1 ☐ $\left(e^{-\frac{3}{2}} - e^{-2}\right)^3$

2 ☐ $\frac{29}{8}e^{-\frac{3}{2}} - 5e^{-2}$

3 ☐ $\frac{4}{3}e^{-2} - \frac{9}{16}e^{-\frac{3}{2}}$

4 ☐ $\Phi(2)^3 - \Phi(1)^3$

5 ☐ $(\Phi(2) - \Phi(1))^3$

6 ☐ Do not know

Continue at side 4

Exercise 5

Let X_1 and X_2 be two independent exponential random variables. Then $X = \min(X_1, X_2)$ and $Y = \max(X_1, X_2)$ with joint density $f(x, y) = 2\lambda^2 e^{-\lambda(x+y)}$.

Question 5

The density $f_Z(z)$ of $Z = \frac{X}{Y}$ is

1 ☐ $\frac{2}{(z+1)^2}$, $0 \leq z \leq 1$

2 ☐ $\lambda e^{-\lambda z}$, $0 \leq z$

3 ☐ $\frac{1}{\sqrt{2\pi}} \frac{1}{\lambda} \exp\left(-\frac{1}{2}\left(\frac{z-\frac{1}{\lambda}}{\sqrt{\frac{1}{\lambda}}}\right)^2\right)$, $0 \leq z \leq 1$

4 ☐ 1 , $0 \leq z \leq 1$

5 ☐ $\frac{1}{(z+1)^2}$, $0 \leq z$

6 ☐ Do not know

Exercise 6

For the events A and B is it known that $\mathsf{P}(A \cap B) = \mathsf{P}(A)\mathsf{P}(B)$.

Question 6

Which of the following statements is correct

1 ☐ $\mathsf{P}(A \cup B) = \mathsf{P}(A) + \mathsf{P}(B)$.

2 ☐ $\mathsf{P}(A|B) = \mathsf{P}(A \cap B)$

3 ☐ $\mathsf{P}(A|B) = \mathsf{P}(A)$

4 ☐ $\mathsf{P}(A|B) = \mathsf{P}(B|A)$

5 ☐ None of the above are correct.

6 ☐ Do not know

Continue at side 5

Exercise 7

The discrete uniformly distributed random variable X takes one of the values $\{-2,-1,1,2\}$. A new random variable is formed by $Y = X^2$.

Question 7

The covariance $\mathsf{Cov}(X,Y)$ between X and Y is

1 ☐ -1

2 ☐ 0

3 ☐ $\frac{1}{2}$

4 ☐ 1

5 ☐ The covariance can not be determined based on the information given

6 ☐ Do not know

Exercise 8

It is known, that 20% of the airline passengers on a specific route brings carry on luggage with a weight substantially higher than allowed. It is assumed that each passenger has excessive carry on luggage independently of and with the same probability as all other passengers.

Question 8

On a departure with 400 passengers, give a good judgement of the probability that at most 60 passengers have excessive carry on luggage.

1 ☐ $\begin{pmatrix} 400 \\ 60 \end{pmatrix} \left(\frac{1}{5}\right)^{60} \left(\frac{4}{5}\right)^{340}$

2 ☐ $\sum_{i=61}^{400} \frac{80^i}{i!} e^{-80}$

3 ☐ $\sum_{i=0}^{60} \frac{\begin{pmatrix} 80 \\ i \end{pmatrix}\begin{pmatrix} 320 \\ 100-i \end{pmatrix}}{\begin{pmatrix} 400 \\ 100 \end{pmatrix}}$

4 ☐ $\Phi(2.2) - \Phi(-1.96)$

5 ☐ $\Phi(-2.4)$

6 ☐ Do not know

Continue at side 6

Exercise 9

The pair (X, Y) is standard bivariate normally distributed with correlation $\rho = \frac{3}{5}$.

Question 9

The probability $\mathsf{P}\left(\frac{1}{2}X < Y < 2X\right)$ is

1 ☐ $\frac{2\pi - \text{Arctan}(\frac{7}{4})}{2\pi}$

2 ☐ $\frac{1}{6}$

3 ☐ $\frac{\text{Arctan}(\frac{15}{8})}{2\pi}$

4 ☐ $\frac{\text{Arctan}(\frac{7}{4}) + \text{Arctan}(\frac{1}{8})}{2\pi}$

5 ☐ $\frac{\text{Arctan}(\frac{7}{8}) + \text{Arctan}(\frac{1}{8})}{\pi}$

6 ☐ Do not know

Exercise 10

You have $X_i \sim \exp(\lambda), i = 1, 2, 3$ independent. One defines $Y = X_1 + X_2 + X_3$.

Question 10

The joint density $f(x, y)$ of X_1 and Y is

1 ☐ $f(x, y) = \lambda e^{-\lambda x} \lambda (\lambda y) e^{-\lambda y} = \lambda^3 y e^{-\lambda(x+y)}$

2 ☐ $f(x, y) = \lambda e^{-\lambda x} \lambda \frac{(\lambda y)^2}{2} e^{-\lambda y} = \lambda^4 \frac{y^2}{2} e^{-\lambda(x+y)}$

3 ☐ $f(x, y) = \lambda e^{-\lambda x} \lambda (\lambda (y - x)) e^{-\lambda(y-x)} = \lambda^3 (y - x) e^{-\lambda y}$

4 ☐ $f(x, y) = \lambda e^{-\lambda x} \lambda \frac{(\lambda(y-x))^2}{2} e^{-\lambda y} = \lambda^4 \frac{(y-x)^2}{2} e^{-\lambda y}$

5 ☐ $f(x, y) = \lambda \frac{(\lambda y)^2}{2} e^{-\lambda y} \cdot \frac{1}{3} = \lambda^3 \frac{y^2}{6} e^{-\lambda y}$

6 ☐ Do not know

Continue at side 7

Exercise 11

It is assumed that cardiac arrest patients arrive at an emergency department independently of each other with an average frequency of 24 per day. As a first approximation one assumes that this frequency is constant during the day.

Question 11

The probability that at most 2 cardiac arrest patients arrive during a 4 hour period is

1 ☐ $\frac{\binom{12}{2}^2}{\binom{24}{4}}$

2 ☐ $\left(\frac{5}{6}\right)^2 + 2 \cdot \frac{5}{6}\frac{1}{6}$

3 ☐ $\sum_{i=0}^{2} \binom{4}{i} \frac{1}{16}$

4 ☐ $\Phi(-1)$

5 ☐ $\left(1 + 4 + \frac{4^2}{2!}\right) e^{-4}$

6 ☐ Do not know

Exercise 12

Let X and Y be two independent random variables, with $\mathsf{E}(X) = \mu_X, \mathsf{E}(Y) = \mu_Y, \mathsf{Var}(X) = \sigma_X^2$ and $\mathsf{Var}(Y) = \sigma_Y^2$.

Question 12

You find

1 ☐ $\mathsf{Var}(XY) = \sigma_X^2\sigma_Y^2 + \mu_X^2\sigma_Y^2 + \mu_Y^2\sigma_X^2 - \mu_X^2\mu_Y^2$

2 ☐ $\mathsf{Var}(XY) = \sigma_X^2\sigma_Y^2$

3 ☐ $\mathsf{Var}(XY) = \sigma_X^2\sigma_Y^2 - \mu_X^2\mu_Y^2$

4 ☐ $\mathsf{Var}(XY) = \sigma_X^2\sigma_Y^2 + \mu_X^2\sigma_Y^2 + \mu_Y^2\sigma_X^2$

5 ☐ $\mathsf{Var}(XY) = \sigma_X^2\sigma_Y^2 + \mu_X^2\mu_Y^2$

6 ☐ Do not know

Continue at side 8

Exercise 13

Given the density $f(x,y) = 6(x-y)$ for the joint distribution of maximum (X) and minimum (Y) of 3 independent $uniform(0,1)$ distributed random variables.

Question 13

The probability for observing a maximum between 0.9 and 0.91 along with a minimum between 0.1 and 0.11 is found (possibly approximately) to

1 ☐ $6 \cdot 0.8 \cdot 0.01^2$

2 ☐ $\int_0^{0.1} \int_0^{0.9} 6(x-y)\mathrm{d}x\mathrm{d}y$

3 ☐ 0.48

4 ☐ $6(0.91 - 0.10) - 6(0.9 - 0.11)$

5 ☐ $6(0.9 - 0.1)$

6 ☐ Do not know

Exercise 14

A random variable X is uniformly distributed on the interval from $-\frac{1}{2}$ to $\frac{1}{2}$. A new random variable is formed as $Y = X^2$ with density $f_Y(y)$.

Question 14

The density $f_Y(y)$ is

1 ☐ $f_Y(y) = \frac{1}{2\sqrt{|y|}}$ for $y \in \left[0; \frac{1}{4}\right]$ og 0 ellers.

2 ☐ $f_Y(y) = \frac{1}{\sqrt{|y|}}$ for $y \in \left[0; \frac{1}{4}\right]$ og 0 ellers.

3 ☐ $f_Y(y) = \frac{1}{2\sqrt{|y|}}$ for $y \in \left[-\frac{1}{4}; \frac{1}{4}\right]$ og 0 ellers.

4 ☐ $f_Y(y) = \frac{1}{\sqrt{|y|}}$ for $y \in \left[-\frac{1}{4}; \frac{1}{4}\right]$ og 0 ellers.

5 ☐ $f_Y(y) = 4$ for $y \in \left[0; \frac{1}{4}\right]$

6 ☐ Do not know

Continue at side 9

Exercise 15

The pair of random variables (X,Y) follows a bivariate normal distribution with $X \sim normal(1,4), Y \sim normal(2,9)$ and correlation coefficient $\rho = -\frac{1}{4}$.

Question 15

The probability $\mathsf{P}(X - Y \leq 0)$ is

1 ☐ $\Phi\left(\frac{1}{\sqrt{13}}\right)$

2 ☐ $\Phi\left(\frac{1}{3}\right)$

3 ☐ $\Phi\left(\frac{1}{\sqrt{12}}\right)$

4 ☐ $\Phi\left(\frac{1}{4}\right)$

5 ☐ $\Phi\left(\frac{1}{\sqrt{2}}\right)$

6 ☐ Do not know

Exercise 16

For the events A, B and C, $\mathsf{P}(A)$, $\mathsf{P}(B)$, $\mathsf{P}(C)$, $\mathsf{P}(A \cap B)$, $\mathsf{P}(A \cap C)$, $\mathsf{P}(A \cap B \cap C)$ and $\mathsf{P}(A \cup B \cup C)$ are known. One wants to calculate $\mathsf{P}(B \cap C)$.

Question 16

The probability $\mathsf{P}(B \cap C)$ is determined by

1 ☐ $\mathsf{P}(B \cap C) = \mathsf{P}(A) + \mathsf{P}(B) + \mathsf{P}(C) - \mathsf{P}(A \cap B) - \mathsf{P}(A \cup B \cup C)$

2 ☐ $\mathsf{P}(B \cap C) = \mathsf{P}(A) + \mathsf{P}(B) + \mathsf{P}(C) - \mathsf{P}(A \cap B) + \mathsf{P}(A \cap B \cap C) - \mathsf{P}(A \cup B \cup C)$

3 ☐ $\mathsf{P}(B \cap C) = \mathsf{P}(A) + \mathsf{P}(B) + \mathsf{P}(C) - \mathsf{P}(A \cap B) - \mathsf{P}(A \cap C) + \mathsf{P}(A \cap B \cap C) - \mathsf{P}(A \cup B \cup C)$

4 ☐ $\mathsf{P}(B \cap C) = \mathsf{P}(A \cup B \cup C) - \mathsf{P}(A) - \mathsf{P}(B) - \mathsf{P}(C) + \mathsf{P}(A \cap B) - \mathsf{P}(A \cap B \cap C)$

5 ☐ $\mathsf{P}(B \cap C) = \mathsf{P}(B) + \mathsf{P}(C) - \mathsf{P}(A \cup B \cup C)$

6 ☐ Do not know

Continue at side 10

Exercise 17

The number of defects N in some material can be described by a Poisson distribution with mean γ. In another model F_i is the event that there are i defects in the material. One wants to express the probability for the event $H = (F_0 \cup F_1 \cup F_2)$ using N.

Question 17

$P(H)$ can be expressed alternatively as

1 ☐ $\mathsf{P}(H) = \mathsf{P}(N \leq 2)$

2 ☐ $\mathsf{P}(H) = \mathsf{P}(N_0) + \mathsf{P}(N_1) + \mathsf{P}(N_2)$

3 ☐ $\mathsf{P}(H) = \mathsf{P}(F_i \leq 2)$

4 ☐ $\mathsf{P}(H) = \mathsf{P}(N = 0, N = 1, N = 2)$

5 ☐ It is not possible to express $\mathsf{P}(H)$ using N with the information given.

6 ☐ Do not know

Exercise 18

A certain fish species spawn a number of eggs that can be described by a Poisson distribution with mean μ. Each egg gives rise to a female fish larvae with probability p and gives rise to a male fish larvae with probability $1 - p$.

Question 18

The expected number of eggs from fish of this species, that gives rise to a female fish larvae is

1 ☐ $\frac{\mu}{p}$

2 ☐ μp

3 ☐ $\mu(1 - p)$

4 ☐ $\mu(p + p^2)$

5 ☐ $\mu p(1 - p)$

6 ☐ Do not know

Continue at side 11

Exercise 19

In a physical experiment, electrons are emitted at high velocity towards a screen. Here the screen can be considered to be of infinite size. The electrons are emitted towards a specific point on the screen. Due to randomness the electrons do not hit this point precisely. The hitting point of an electron on the screen can be described by the coordinates (X, Y), where X and Y are independent standard normal random variables. It is of special interest to know whether the electrons hit between one and two length units from the target (the origin of the coordinate system).

Question 19

The probability that an electron hits between 1 and 2 length units from the target is

1 ☐ $\Phi(2) - \Phi(1)$
2 ☐ $2\left(\Phi(2) - \Phi(1)\right)$
3 ☐ $\exp\left(-\frac{1}{2}\right) - \exp\left(-2\right)$
4 ☐ $1 - \sum_{r=0}^{1} \frac{x^r}{r!} e^{-x}$
5 ☐ e^{-2}
6 ☐ Do not know

Exercise 20

A communication system for railways has a critical time limit for a handshaking process - the so-called round trip time. Mean and standard deviation for this time is known to be 12ms and 6ms respectively. One wants to determine a threshold value for the round trip time, which is exceeded with a probability of at most $\frac{1}{9}$.

Question 20

The limit in question is determined to be

1 ☐ 30ms
2 ☐ 66ms
3 ☐ 120ms
4 ☐ $\left(12 + 6\Phi^{-1}\left(\frac{17}{18}\right)\right)$ms
5 ☐ $\left(12 + 6\Phi^{-1}\left(\frac{8}{9}\right)\right)$ms
6 ☐ Do not know

where Φ^{-1} is the inverse function of the standard normal distribution function.

Continue at side 12

Exercise 21

The life time of an electronic component can with reasonable accuracy be described by an *exponential*(λ) random variable. The component has reached the age t.

Question 21

With the given information, determine the probability that the component will fail in the interval $[t, t + \mathrm{d}t]$. The answer can possibly be given only approximatively.

1 ☐ $\lambda \mathrm{d}t$

2 ☐ $\lambda e^{-\lambda t} \mathrm{d}t$

3 ☐ $e^{-\lambda t} - e^{-\lambda(t+\mathrm{d}t)}$

4 ☐ $\frac{\lambda e^{-\lambda t}}{1-e^{-\lambda t}} \mathrm{d}t$

5 ☐ $\lambda e^{-\lambda t}$

6 ☐ Do not know

Exercise 22

Let X be a positive random variable, such that $\mathsf{P}(X \leq x) = 1 - e^{-\lambda x}$ and define $Y = \frac{1}{X}$.

Question 22

The probability $\mathsf{P}(Y \leq y)$ is

1 ☐ $\mathsf{P}(Y \leq y) = 1 - e^{-\frac{\lambda}{y}}$

2 ☐ $\mathsf{P}(Y \leq y) = 1 - e^{-\frac{y}{\lambda}}$

3 ☐ $\mathsf{P}(Y \leq y) = e^{-\frac{y}{\lambda}}$

4 ☐ $\mathsf{P}(Y \leq y) = e^{-\frac{1}{\lambda y}}$

5 ☐ $\mathsf{P}(Y \leq y) = e^{-\frac{\lambda}{y}}$

6 ☐ Do not know

Continue at side 13

Exercise 23

Consider a game of dice using six normal six-sided dice. The aim is to get as many sixes as possible. You can throw at most two times, as all sixes obtained in the first throw is taken aside and the second throw is performed with only the dice not showing six on the first throw.

Question 23

Assume that you got v sixes in the first throw, then the probability of getting a total of w sixes is found to be

1 ☐ $\begin{pmatrix} 6 \\ w \end{pmatrix} \left(\frac{1}{6}\right) \left(\frac{5}{6}\right)$

2 ☐ $\frac{1}{6-v}$

3 ☐ $\begin{pmatrix} 6-v \\ w-v \end{pmatrix} \left(\frac{1}{6}\right)^{w-v} \left(\frac{5}{6}\right)^{6+v-w}$

4 ☐ $\frac{1}{6}$

5 ☐ $\begin{pmatrix} 6-v \\ w-v \end{pmatrix} \left(\frac{1}{6}\right)^{w-v} \left(\frac{5}{6}\right)^{6-w}$

6 ☐ Do not know

Exercise 24

A point is chosen at random in a circular disk. A straight line is drawn through the point and the center of the circle.

Question 24

The probability that the absolute value of the slope of the line is less than 1 is

1 ☐ $\frac{\text{Arctan}\left(\frac{2}{5}\right)}{\pi}$

2 ☐ $\frac{1}{\pi}$

3 ☐ $\frac{1}{4}$

4 ☐ $\frac{1}{3}$

5 ☐ $\frac{1}{2}$

6 ☐ Do not know

Continue at side 14

Exercise 25

The cause of an air-crash can roughly be ascribed to a. human error, b. mechanical failure, c. act of terrorism. The probability of a human error at a departure is $\frac{1}{50}$, while the probability, that a mechanical failure occurs is $\frac{1}{500}$, and finally, the probability that an airline departure is exposed to an act of terrorism is $\frac{1}{500000}$. If a human error occurs, then the probability that an air-crash occurs $\frac{1}{10000}$, while the probability, that an air-crash occurs at the occurrence of a mechanical failure is $\frac{1}{1000}$, and finally the probability that an air-crash occurs in the event of an act of terrorism is $\frac{2}{3}$. You can assume that at most one of the three potential causes can occur, and, that an air-crash can always be ascribed to one of these three causes.

Question 25

Knowing that an air-crash has occurred, what is the probability that the crash was caused by an act of terrorism

1 ☐ $\frac{1}{4}$

2 ☐ $\frac{2}{3}$

3 ☐ $\frac{1}{3} \cdot 10^{-6}$

4 ☐ $\frac{20000}{20033}$

5 ☐ $\frac{1}{3}$

6 ☐ Do not know

Exercise 26

You have 5 independent *exponential*(μ) distributed variables.

Question 26

The density $g(x)$ of the second largest of the 5 variables is

1 ☐ $g(x) = \mu \frac{(\mu x)^4}{4!} e^{-\mu x}$

2 ☐ $g(x) = \frac{\mu}{4} e^{-\frac{\mu}{4} x}$

3 ☐ $g(x) = \frac{5!}{3!} x^3 (1 - x)$

4 ☐ $g(x) = 20\mu e^{-4\mu x} - 20\mu e^{-4\mu x}$

5 ☐ $g(x) = 20\mu e^{-2\mu x} - 60\mu e^{-3\mu x} + 60\mu e^{-4\mu x} - 20\mu e^{-5\mu x}$

6 ☐ Do not know

Continue at side 15

Exercise 27

The exponential distribution relates to the geometric distribution like the gamma distribution relates to?

Question 27

1 ☐ The binomial distribution

2 ☐ The hyper-geometric distribution

3 ☐ The uniform distribution

4 ☐ The negative binomial distribution

5 ☐ The Poisson distribution

6 ☐ Do not know

Exercise 28

Regarding the two random variables X and Y it is known, that $\mathsf{E}(X) = 4$, $\mathsf{E}(Y) = 7$, $\mathsf{SD}(X) = 10$, $\mathsf{SD}(Y) = 5$ and $\mathsf{Cov}(X, Y) = 0,5$. In addition one defines $U = 3X - 2Y + 1$.

Question 28

$\mathsf{E}(U)$ is calculated to be

1 ☐ -2

2 ☐ -1

3 ☐ 26

4 ☐ 26.5

5 ☐ 27

6 ☐ Do not know

Continue at side 16

Exercise 29

While fishing after sandeels one obtains a certain by-catch of herring. The catch of each species by a random trawl haul can be described with reasonable accuracy by a normal distribution with mean 10 ton (sandeels) and 1 ton (herring) and standard deviation 2 ton (sandeels) and 0.3 ton (herring). The joint distribution can be assumed to be bivariate normal with correlation coefficient 0.7. In a trawl haul one obtained 14 ton of sandeels.

Question 29

The expected by-catch of herring is

1 ☐ 1 ton

2 ☐ 1.24 ton

3 ☐ 1.42 ton

4 ☐ 1.49 ton

5 ☐ 1.6 ton

6 ☐ Do not know

Exercise 30

Define the two continuous random variables X and Y with joint density
$f(x, y) = K(y - x)(1 - y), 0 < x < y < 1$, where K is a normalizing constant.

Question 30

In the range of Y, the marginal density $f_Y(y)$ of Y is

1 ☐ $f_Y(y) = \frac{K}{2}(1 - y)^3$

2 ☐ $f_Y(y) = \frac{K}{2}y(1 - y)^2$

3 ☐ $f_Y(y) = \frac{K}{2}y^2(1 - y)$

4 ☐ $f_Y(y) = \frac{K}{4}(1 - y)$

5 ☐ $f_Y(y) = 1$

6 ☐ Do not know

End of exam