Written exam: December 15. 2023 *Course no* : 02405

Course name: Probability Theory *Duration* : 4 hours

Aids allowed: All

The questions have been answered by:

(name)	(signature)	(table no)

In the text there are 30 exercises, called exercise 1,2. . . , 30, with a total of 30 questions. The individual questions are also numbered and indicated as questions 1,2,. . . ,30 in the text. The answers must be uploaded via the DE Digital Exam, using the file ”answers.txt”. In the file, the study number is stated on the first line, the question number and answer are stated on the following lines using one line for each question.

The answer options for each question are numbered from 1 to 6.

5 points are given for a correct answer and -1 for an incorrect answer. Unanswered questions or a 6 (equivalent to ”don't know”) give 0 points. The number of points required for a set to be considered satisfactorily answered is finally determined when the sets are censored.

Please note that the idea behind the exercises is that there is one and only one correct answer to the individual questions. Furthermore, it is not given that all the listed alternative answer options are meaningful. The last page of the set is page 17.

In the text $\log(\cdot)$ is used for natural logarithms, i.e. logarithms with base e, while Φ denotes the distribution function of a standardized normally distributed variable.

Exercise 1

A company has an image processing system to analyze the quality of coffee beans, as the beans have substantial color variations. In this exercise, we will concentrate on the detection of whether the beans have been attacked by a beetle species. If the beans have been attacked by beetles, they will be discolored with probability 0.8, while the probability that a non attacked bean is discolored is 0.01. The probability that a bean has been attacked by beetles is 0.001.

Question 1

The probability that a discolored bean has been subject to beetle attack is

1 ☐ 0.80

2 ☐ 0.0080

3 ☐ 0.0074

4 ☐ 0.074

5 ☐ 0.080

6 ☐ Don't know

Exercise 2

A car dealer assumes that 45% of customers who visit the store will buy an electric car, 15% will buy a petrol car and 40% will not buy anything.

Question 2

If 5 customers visit his store on a given day, what is the probability that he will sell exactly 2 electric cars and 1 petrol car on that day?

1 ☐ $(0.45)^2 + (0.15) + (0.4)^2$

2 ☐ $(0.45)^2(0.15)(0.4)^2$

3 ☐ $\frac{5!}{2!2!}(0.45)^2(0.15)(0.4)^2$

4 ☐ $\frac{\binom{3}{2}\binom{3}{1}}{\binom{5}{2}}$

5 ☐ 0.75

6 ☐ Don't know

Continue at Page 3

Exercise 3

The maximum daily wave height at a dyke can be described by a distribution with mean value 2 meters and variance 1 meter^2.

Question 3

The best upper bound for the probability that a dyke with a height of 6 meters will be flooded on the given day is

1 ☐ $1 - \Phi\left(\frac{6-2}{1}\right)$

2 ☐ $\Phi\left(\frac{6-2}{1}\right)$

3 ☐ $\frac{1}{4}$

4 ☐ $\frac{1}{9}$

5 ☐ $\frac{1}{16}$

6 ☐ Don't know

Exercise 4

For two different shares, the change in the share price over a calendar year is considered. The simultaneous change in the two share prices can be described as a standardised bivariate normal distribution with correlation coefficient $\frac{1}{2}$.

Question 4

The probability that both changes are positive and that neither change is more than twice as large as the other is given by

1 ☐ $1 - \Phi(1)$

2 ☐ $\frac{\text{Acrtan}(3)}{2\pi}$

3 ☐ $\frac{\text{Arctan}\left(\frac{\sqrt{3}}{2}\right)}{2\pi}$

4 ☐ $\frac{1}{4}$

5 ☐ $\frac{1}{6}$

6 ☐ Don't know

Continue at Page 4

Exercise 5

Let X_1 and X_2 be independent identically distributed random variables with distribution function $P(X_i \leq x) = F(x), i = 1{,}2$, and define $X_{(1)} = \min_i X_i$, $X_{(2)} = \max_i X_i$.

Question 5

For $x \leq y$ the joint distribution function $F^*(x,y) = P(X_{(1)} \leq x, X_{(2)} \leq y)$ is

1 ☐ $F^*(x,y) = F(y)^2 - (F(y) - F(x))^2$

2 ☐ $F^*(x,y) = (F(y) - F(x))^2$

3 ☐ $F^*(x,y) = \int_{-\infty}^{x} \int_{-\infty}^{y} \frac{1}{2\pi} \frac{1}{\sigma^2} e^{-\frac{1}{2} \frac{(u-\mu)^2 + (v-\mu)^2}{\sigma^2}} \mathrm{d}u\mathrm{d}v$

4 ☐ $F^*(x,y) = F(y)^2 - F(x)^2$

5 ☐ $F^*(x,y) = F(y)^2 - (F(y) - F(x))$

6 ☐ Don't know

Where μ and σ^2 denote the mean value and the variance in the distribution of X_i.

Exercise 6

The Eastern Quoll is a now threatened Australian predatory species. It has previously been estimated that the occurrence of these in an area near Adelaide was 3 per km^2 . It is assumed that these are distributed randomly and independently of each other over the area.

Question 6

One has found 1 Eastern Quoll in an area of 2 km^2. Under the above assumptions, what is the probability of finding at most 1 Eastern Quoll in an area of 2 km^2?

1 ☐ $\Phi\left(\frac{1-6}{\sqrt{6}}\right)$

2 ☐ $\left(\frac{1}{3}\right)^2 + 2 \cdot \frac{1}{3}\frac{2}{3}$

3 ☐ $7e^{-6}$

4 ☐ $\frac{1}{6}$

5 ☐ 0

6 ☐ Don't know

Continue at Page 5

Exercise 7

A non-negative continuous random variable X has survival function $G(x)$.

Question 7

We know about the properties of G(x),

1 ☐ that $G(x) \geq 0$ and $\sum_{x=0}^{\infty} G(x) = 1$

2 ☐ that $0 \leq G(x) \leq 1$ and $G(x)$ is not increasing

3 ☐ that $0 \leq G(x) \leq 1$ and $G(x)$ is not decreasing

4 ☐ that $G(x) \geq 0$ and $\int_0^{\infty} G(x)\mathrm{d}x = 1$

5 ☐ only, that $G(x) \geq 0$

6 ☐ Don't know

Exercise 8

The time between emissions of particles from a radioactive source is assumed to be exponentially distributed with mean value 0.2s. The number of emitted particles is counted from time t_0.

Question 8

Determine, possibly approximately, the probability that particle number 10 000 will be emitted before 33 minutes have passed from t_0.

1 ☐ $1 - \Phi(1.00)$

2 ☐ $\sum_{i=33}^{\infty} \frac{\left(10000 \cdot \frac{0,2}{60}\right)^i}{i!} e^{-10000 \cdot t \frac{0.2}{60}}$

3 ☐ $1 - e^{-\frac{33 \cdot 60}{2000}}$

4 ☐ $\frac{1}{2000} - e^{-\frac{33 \cdot 60}{2000}}$

5 ☐ $1 - \frac{200}{33 \cdot 60}$

6 ☐ Don't know

Continue at Page 6

Exercise 9

The probability of sunshine on a winter day is 0.45, while the probability of sunshine and frost is 0.35.

Question 9

The probability of sunshine and plus degrees on a winter day is

1 ☐ 0.45 + 0.35 - 0.65

2 ☐ 0.55 - 0.35

3 ☐ 0.65 - 0.45

4 ☐ $0.45 \cdot 0.35$

5 ☐ 0.45 - 0.35

6 ☐ Don't know

Exercise 10

An engineer has been hired by the FDA to test the validity of a pharmaceutical company's claim that only 5% of their pills have too low an active ingredient concentration. The engineer randomly selects 20 pills and finds that three of them have too little of the active substance.

Question 10

What is the probability that three or more pills out of 20 are too low in active ingredient if the pharmaceutical company's claim is true?

1 ☐ $\sum_{i=3}^{\infty} \frac{1}{i!} e^{-1}$

2 ☐ $\sum_{i=0}^{3} \binom{20}{i} 0.05^i 0.95^{20-i}$

3 ☐ $\sum_{i=0}^{3} \frac{1}{i!} e^{-1}$

4 ☐ $\sum_{i=3}^{20} \binom{20}{i} 0.05^i 0.95^{20-i}$

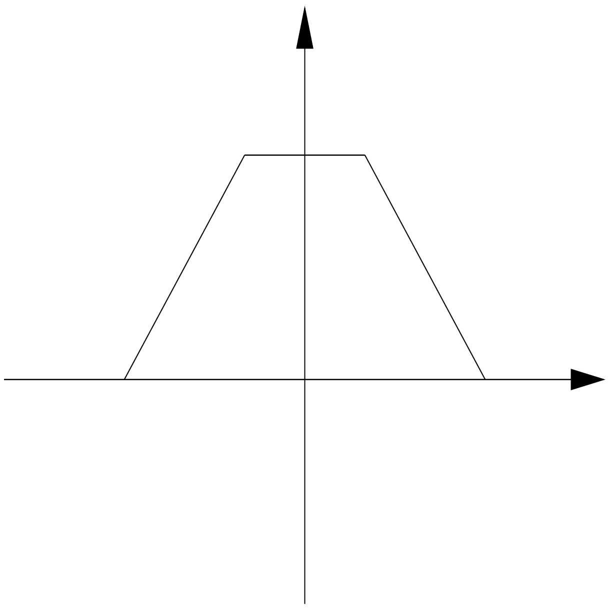
5 ☐ $1 - \Phi(\frac{3.5-1}{\sqrt{0.95}})$

6 ☐ Don't know

Continue at Page 7

Exercise 11

A point is randomly selected in the area shown in the figure. The area can be analytically described as the area bounded by the lines $y = 3$, $y = 0$, $y = \frac{3}{2}(x+3)$ and $y = -\frac{3}{2}(x-3)$. The first coordinate of the point is denoted by the random variable X.



Question 11

The probability $P(X \geq 1)$ is

1 ☐ $\frac{1}{3}$

2 ☐ $\frac{1}{4}$

3 ☐ $\frac{1}{6}$

4 ☐ $\frac{1}{8}$

5 ☐ $\frac{\sqrt{2}}{12}$

6 ☐ Don't know

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Exercise 12

Calls to a call center are assumed to occur completely randomly. On average, there are λ calls per minute.

Question 12

With which distribution can the time between two calls be described (calculated in minutes)?

1 ☐ $uniform(0,1)$

2 ☐ $normal(1/\lambda,1)$

3 ☐ $normal(\lambda,1)$

4 ☐ $exponential(1/\lambda)$

5 ☐ $exponential(\lambda)$

6 ☐ Don't know

Exercise 13

One has the density function $f(x,y) = 6(x - y)$ for the joint distribution of maximum (X) and minimum (Y) of 3 independent $uniform(0,1)$ distributed random variables.

Question 13

The conditional expectation $\mathsf{E}(Y|X = x)$ of Y for $X = x$ is

1 ☐ $\mathsf{E}(Y|X = x) = \frac{x}{2}$

2 ☐ $\mathsf{E}(Y|X = x) = \frac{x}{3}$

3 ☐ $\mathsf{E}(Y|X = x) = \frac{x}{4}$

4 ☐ $\mathsf{E}(Y|X = x) = x^3$

5 ☐ $\mathsf{E}(Y|X = x) = \frac{1}{4}$

6 ☐ Don't know

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Exercise 14

The random variable X has density $f_x(x) = 2x$ on the interval $[0;1]$. We consider another random variable $Y = X^2$ with density $f_Y(y)$

Question 14

In the range of Y we have for $f_Y(y)$

1 ☐ $f_Y(y) = \frac{3}{2}\sqrt{y}$

2 ☐ $f_Y(y) = \frac{5}{2}y\sqrt{y}$

3 ☐ $f_Y(y) = 1$

4 ☐ $f_Y(y) = \frac{1}{2\sqrt{y}}$

5 ☐ $f_Y(y) = 2y$

6 ☐ Don't know

Exercise 15

A system administrator needs to access a database, but it is overloaded. Everyone who tries to enter the site has an equal probability of being let through, and there is a $\frac{1}{20}$ probability of entering on each attempt.

Question 15

What is the probability that the system administrator gets through exactly on the third try?

1 ☐ $\frac{e^{-0.95}0.95^3}{3!}$

2 ☐ $\frac{e^{-0.05}0.05^3}{3!}$

3 ☐ $\frac{1}{20^3}$

4 ☐ $\frac{19^2}{20^2}$

5 ☐ $\frac{19^2}{20^3}$

6 ☐ Don't know

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Exercise 16

The probability that a heavy rainfall event will occur at a given location on a given day in July is $\frac{1}{9}$. At a nearby sewage treatment plant, it is estimated that the probability of overflow of the plant given a heavy rainfall event is $\frac{1}{4}$, while the probability of overflow without a heavy rainfall event is negligible.

Question 16

The probability that an overflow will be experienced at the sewage treatment plant on a given day in July is

1 ☐ $\frac{1}{9}$

2 ☐ $\frac{5}{36}$

3 ☐ $\frac{1}{36}$

4 ☐ $\frac{13}{36}$

5 ☐ $\frac{1}{49}$

6 ☐ Don't know

Exercise 17

The joint density for X og Y is given by

$$f(x,y) = \frac{6}{7}\left(x^2 + \frac{xy}{2}\right), \quad 0 < x < 1, \quad 0 < y < 2.$$

Question 17

The probability $\mathsf{P}(Y > \frac{1}{2} | X < \frac{1}{2})$ is

1 ☐ $\int_0^2 \int_0^1 \frac{6}{7}\left(x^2 + \frac{xy}{2}\right) dxdy$

2 ☐ 0.8625

3 ☐ $\int_{1/2}^{\infty} \int_{-\infty}^{1/2} \frac{6}{7}\left(x^2 + \frac{xy}{2}\right) dxdy$

4 ☐ $\frac{6}{7}$

5 ☐ $\int_{1/2}^{2} \int_0^{1/2} \frac{6}{7}\left(x^2 + \frac{xy}{2}\right) dxdy$

6 ☐ Don't know

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Exercise 18

On a regular dartboard, there is a red circular field in the center called the *inner bull* with a diameter of 0.5 *inches*. Outside is a green ring-shaped field called the *outer bull* with an outer diameter of 1.25 *inches*. Phil throws an arrow at the center of the disc. The coordinates (X,Y), measured from the center, to the place he hits can be described by two independent normally distributed random variables both with mean value 0 *inches* and variance 1 *inch*^{2}.

Question 18

What is the probability that Phil hits the outer bull?

1 ☐ $e^{-\frac{1}{32}} - e^{-\frac{25}{128}}$

2 ☐ $1 - e^{-\frac{25}{128}}$

3 ☐ $\left(2\Phi(\frac{5}{4} - 1)\right)^2 - \left(2\Phi(\frac{1}{2} - 1)\right)^2$

4 ☐ $\left(2\Phi(\frac{5}{8} - 1)\right)^2 - \left(2\Phi(\frac{1}{4} - 1)\right)^2$

5 ☐ $\Phi\left(\frac{5}{4}\right)^2 - \Phi\left(\frac{1}{2}\right)^2$

6 ☐ Don't know

Exercise 19

Let the joint density of the random variables X and Y be given by

$$f(x,y) = \begin{cases} 2, & \text{for } 0 < x < y < 1 \\ 0, & \text{ellers} \end{cases}$$

Question 19

The covariance $\mathsf{Cov}(X,Y)$ between X and Y is determined to be

1 ☐ 0

2 ☐ $\frac{5}{36}$

3 ☐ $\frac{1}{36}$

4 ☐ $\frac{1}{4}$

5 ☐ $\frac{1}{72}$

6 ☐ Don't know

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Exercise 20

Two cinemas of equal size compete for 1000 customers. Assume that each customer chooses independently and with equal probability between the two cinemas. Let N denote the number of seats in each cinema. We now consider one of the two cinemas.

Question 20

Find an expression, possibly approximate, for N that will guarantee that the probability of rejecting a customer (due to a full house) is less than 1%.

1 ☐ $\sum_{x=N+1}^{500} \binom{500}{x} \left(\frac{1}{2}\right)^x < 0.01$

2 ☐ $1 - \Phi\left(\frac{N-499.5}{\sqrt{250}}\right) < 0.01$

3 ☐ $\sum_{x=N+1}^{1000} \binom{1000}{x} 2^x \left(1-\frac{1}{2}\right)^{1000-x} < 0.01$

4 ☐ $\Phi\left(\frac{N-499.5}{\sqrt{500}}\right) > 0.01$

5 ☐ $\left(\frac{1}{2}\right)^{500} \sum_{x=N+1}^{500} \binom{500}{x} < 0.01$

6 ☐ Don't know

Exercise 21

From the random variable $X \sim \textit{binomial}\left(6,\frac{1}{2}\right)$ one gets $Y = |X-3|$.

Question 21

In the range of Y one finds

1 ☐ $\mathsf{P}(Y=y) = 2\mathsf{P}(X=3-y)$

2 ☐ $\mathsf{P}(Y=y) = \mathsf{P}(X=3-y) + \mathsf{P}(X=y-3)$

3 ☐ $\mathsf{P}(Y=0) = \frac{5}{16}, \quad \mathsf{P}(Y=1) = \frac{15}{32}, \quad \mathsf{P}(Y=2) = \frac{3}{16}, \quad \mathsf{P}(Y=3) = \frac{1}{32}$

4 ☐ $\mathsf{P}(Y=0) = \frac{1}{32}, \quad \mathsf{P}(Y=1) = \frac{3}{16}, \quad \mathsf{P}(Y=2) = \frac{15}{32}, \quad \mathsf{P}(Y=3) = \frac{5}{16}$

5 ☐ $\mathsf{P}(Y=y) = \binom{3}{y}\frac{1}{8}$

6 ☐ Don't know

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Exercise 22

A three dimensional random variable (X_1,X_2,X_3), with $X_1+X_2+X_3=20$, has the joint distribution $P(X_1=x_1,X_2=x_2,X_3=x_3)=\frac{20!}{x_1!x_2!x_3!}\left(\frac{1}{2}\right)^{x_1}\left(\frac{1}{5}\right)^{x_2}\left(\frac{3}{10}\right)^{x_3}$. One observes $X_1=10$.

Question 22

The conditional distribution of (X_2,X_3) for $X_1=10$ is

1 ☐ $P(X_2=x_2,X_3=x_3|X_1=10)=\frac{\frac{20!}{10!x_2!x_3!}\left(\frac{1}{2}\right)^{10}\left(\frac{1}{5}\right)^{x_2}\left(\frac{3}{10}\right)^{x_3}}{\sum_{x_1=0}^{20}\frac{20!}{x_1!x_2!x_3!}\left(\frac{1}{2}\right)^{x_1}\left(\frac{1}{5}\right)^{x_2}\left(\frac{3}{10}\right)^{x_3}}$ with $x_2+x_3=10$

2 ☐ $P(X_2=x_2,X_3=x_3|X_1=10)=\frac{10!}{x_1!x_2!}\left(\frac{1}{5}\right)^{x_1}\left(\frac{3}{10}\right)^{x_2}e^{-\left(\frac{1}{5}+\frac{3}{10}\right)}$ with $x_2+x_3=10$

3 ☐ $P(X_2=x_2,X_3=x_3|X_1=10)=\frac{\frac{10!}{x_2!x_3!}\left(\frac{2}{5}\right)^{x_2}\left(\frac{3}{5}\right)^{x_3}}{\sum_{x_1=0}^{20}\frac{20!}{x_1!x_2!x_3!}\left(\frac{1}{2}\right)^{x_1}\left(\frac{1}{5}\right)^{x_2}\left(\frac{3}{10}\right)^{x_3}}$ with $x_2+x_3=10$

4 ☐ $P(X_2=x_2,X_3=x_3|X_1=10)=\binom{10}{x_2}\left(\frac{2}{5}\right)^{x_2}\left(\frac{3}{5}\right)^{x_3}$ with $x_2+x_3=10$

5 ☐ $P(X_2=x_2,X_3=x_3|X_1=10)=\frac{20!}{10!x_2!x_3!}\left(\frac{1}{2}\right)^{10}\left(\frac{1}{5}\right)^{x_2}\left(\frac{3}{10}\right)^{x_3}$, with $x_2+x_3=10$

6 ☐ Don't know

Exercise 23

The continuous random variable X is uniformly distributed on the interval]1,3[while the continuous random variable Y is uniformly distributed on the interval [2,4[. The two random variables X and Y are independent.

Question 23

In the range for (X,Y) the joint density $f(x,y)$ is

1 ☐ $f(x,y)=\frac{1}{4}$

2 ☐ $f(x,y)=\frac{xy}{4}$

3 ☐ $f(x,y)=1$

4 ☐ $f(x,y)=\frac{xy}{4}-\frac{x}{2}-\frac{y}{4}+\frac{1}{2}$

5 ☐ Not enough information has been provided for the task to be solved.

6 ☐ Don't know

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Exercise 24

Let R be a Rayleigh distributed random variable.

Question 24

The median in the distribution for R is

1 ☐ $\sqrt{2\log(2)}$

2 ☐ $\frac{1}{2}$

3 ☐ 1

4 ☐ $\sqrt{\frac{\log(2)}{2}}$

5 ☐ The solution to the equation $re^{-\frac{r^2}{2}} = \frac{1}{2}$

6 ☐ Don't know

Exercise 25

Consider the pair (X,Y) of uniformly distributed random variables in the area bounded by the lines $x = 0, y = 0, 2x + y = 3$ and $x + 2y = 3$. Then define $Z = Y/X$.

Question 25

The density $f_Z(z)$ for Z is

1 ☐ $f_Z(z) = \int_0^{\frac{3}{1+z}} \frac{2}{9} x \mathrm{d}x$

2 ☐ $f_Z(z) = \begin{cases} \frac{15}{8(\frac{3}{2}+z)^2} & \text{for } 0 \leq z < 1 \\ \frac{15}{8(1+\frac{3}{2}z)^2} & \text{for } 1 \leq z \end{cases}$

3 ☐ $f_Z(z) = \begin{cases} \frac{5}{9(\frac{2}{3}+z)^2} & \text{for } 0 \leq z < 1 \\ \frac{5}{9(1+\frac{2}{3}z)^2} & \text{for } 1 \leq z \end{cases}$

4 ☐ $f_Z(z) = \begin{cases} \frac{3}{(2+z)^2} & \text{for } 0 \leq z < 1 \\ \frac{3}{(1+2z)^2} & \text{for } 1 \leq z \end{cases}$

5 ☐ $f_Z(z) = \int_0^{\frac{3}{1+2z}} \frac{4}{9} x \mathrm{d}x$

6 ☐ Don't know

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Exercise 26

Measurements of wind speeds are satisfactorily described by a Rayleigh distribution. Three independent wind measurements are made at a measuring station.

Question 26

The density $f(x)$ for the second largest of the measurements is

1 ☐ $f(x) = xe^{-\frac{x^2}{2}}$

2 ☐ $f(x) = 2xe^{-x^2}$

3 ☐ $f(x) = 2xe^{-\frac{x^2}{2}}\left(1 + e^{-\frac{x^2}{2}}\right)$

4 ☐ $f(x) = 6xe^{-x^2} - 6xe^{-\frac{3}{2}x^2}$

5 ☐ $f(x) = xe^{-\frac{x^2}{2}} - 2xe^{-x^2} + \frac{3}{2}xe^{-\frac{3}{2}x^2}$

6 ☐ Don't know

Exercise 27

The pair (X,Y) has a bivariate normal distribution with $\mathsf{E}(X) = 0, \mathsf{E}(Y) = \mu_Y, \mathsf{Var}(X) = \sigma_X^2, \mathsf{Var}(Y) = \sigma_Y^2$ and $\mathsf{Cov}(X,Y) = \rho\sigma_X\sigma_Y$.

Question 27

The probability that Y is between the values y_1 and y_2, when X is known to be x is

1 ☐ $\Phi\left(\frac{y_2-\mu_y-\rho x}{\sigma_Y\sqrt{1-\rho^2}}\right) - \Phi\left(\frac{y_1-\mu_y-\rho x}{\sigma_Y\sqrt{1-\rho^2}}\right)$

2 ☐ $\Phi\left(\frac{y_2-\mu_Y}{\sigma_Y}\right) - \Phi\left(\frac{y_1-\mu_Y}{\sigma_Y}\right)$

3 ☐ $\Phi\left(\frac{y_2-\mu_y-\rho x}{\sqrt{\sigma_Y^2+\sigma_X^2}\sqrt{1-\rho^2}}\right) - \Phi\left(\frac{y_1-\mu_y-\rho x}{\sqrt{\sigma_Y^2+\sigma_X^2}\sqrt{1-\rho^2}}\right)$

4 ☐ $\Phi\left(\frac{y_2-\mu_y-\rho\frac{\sigma_Y}{\sigma_X}x}{\sigma_Y\sqrt{1-\rho^2}}\right) - \Phi\left(\frac{y_1-\mu_y-\rho\frac{\sigma_Y}{\sigma_X}x}{\sigma_Y\sqrt{1-\rho^2}}\right)$

5 ☐ $\Phi\left(\frac{y_2-\mu_y-\rho\frac{\sigma_X}{\sigma_Y}x}{\sigma_Y\sqrt{1-\rho^2}}\right) - \Phi\left(\frac{y_1-\mu_y-\rho\frac{\sigma_X}{\sigma_Y}x}{\sigma_Y\sqrt{1-\rho^2}}\right)$

6 ☐ Don't know

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Exercise 28

The random variable X is $beta(2{,}1)$ distributed, while the random variable Y for given $X = x$ is $binomial(4{,}x)$ distributed.

Question 28

One finds

1 ☐ $\mathsf{P}(Y = 3) = \frac{4}{15}$

2 ☐ $\mathsf{P}(Y = 3) = \frac{32}{81}$

3 ☐ $\mathsf{P}(Y = 3) = \frac{1}{4}$

4 ☐ $\mathsf{P}(Y = 3) = \frac{1}{5}$

5 ☐ $\mathsf{P}(Y = 3) = \frac{2}{7}$

6 ☐ Don't know

Exercise 29

A public institution has sent a major IT task out to tender. It is expected that there will be a total of 4 offers, and that the individual offers can be described independently of each other using a distribution function $F(x) = 1 - \exp(-(x-a)b)$ for $x \geq a$, where a and b are appropriate constants. The institution wants and is obliged to choose the lowest offer.

Question 29

The probability that the public institution will have to pay at least $2a$ to have the IT task carried out is

1 ☐ $\exp(-ab)$

2 ☐ $\exp(-4ab)$

3 ☐ $\exp(-8ab)$

4 ☐ $(1 - \exp(-ab))^4$

5 ☐ $1 - (1 - \exp(-ab))^4$

6 ☐ Don't know

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Exercise 30

A container ship of the feeder type must be loaded with 20-foot and 40-foot containers respectively. The number of containers to be loaded can be reasonably described by a bivariate normal distribution with mean values 1000 and 500 for 20-foot and 40-foot containers, respectively. The associated standard deviations are 100 (20-foot containers) and 50 (40-foot containers). The correlation coefficient in the distribution is calculated to be $-\frac{4}{5}$. The volume of the two container types is 33 m^3 and 66 m^3 .

Question 30

The probability that the total volume of the container load exceeds 69 000 m^3 is

1 ☐ $\Phi\left(-\frac{5\sqrt{10}}{11}\right)$

2 ☐ $1-\Phi\left(\frac{5\sqrt{2}}{11}\right)$

3 ☐ $\Phi\left(-\frac{5\sqrt{10}}{33}\right)$

4 ☐ $1-\Phi\left(\frac{25\sqrt{2}}{11}\right)$

5 ☐ $\Phi\left(-\frac{\sqrt{2}}{11}\right)$

6 ☐ Don't know

End of exam.