

First Order Logic: Resolution-based Automated Theorem Proving

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Resolution for first-order logic with equality

Additional rules for equational reasoning are needed.

- Paramodulation:

$$\text{Par : } \frac{C \vee s_1 = s_2, \quad D \vee Q(t, t_1, \dots, t_n)}{(C \vee D \vee Q(s_2, t_1, \dots, t_n))\sigma} \quad (\sigma = \text{MGU}(s_1, t))$$

- Superposition.

First-order resolution: some optimizations

- Tautology deletion.
- Subsumption resolution.
- Clause ordering.
- Indexing techniques.

First-order resolution: some strategies

- Unit resolution.
- Set-of-support resolution.
- Hyper-resolution.
- Linear resolution.
- Selective Linear Definite clause (SLD)-resolution.

Resolution-based Logic Programming: PROLOG

- PROLOG: Language for Logic Programming based on SLD-resolution plus backtracking.
- Because of the depth-first search strategy, the pure PROLOG does not always terminate, even when a derivation exists, and hence is incomplete.
- To remedy that problem, PROLOG employs some non-logical features, such as `cut` and `fail`, implementing 'Negation as Failure'.

Resolution-based automated reasoning in FOL: some tool implementations

- OTTER, Prover9, Mace (Argonne National Laboratory)
- SPASS (Max Planck Institute for Informatics, Saarbrücken)
- Vampire (University of Manchester)
- E equational theorem prover (Technical University of Munich)
- SETHEO, based on model generation, and E-SETHEO (Technical University of Munich)
- CADE ATP System Competition (CASC).
- Thousand Problems for Theorem Provers (TPTP) library:
www.tptp.org
- See more references and links on the course website.

SPASS

- SPASS: automated theorem prover for first-order logic with equality.
- Website: <http://www.spass-prover.org/>
- Downloadable and online versions. WebSPASS
- Easy user interface. Tutorial.
- The last problem in Assignment 1 uses SPASS.

Automated reasoning in AI

- Automated reasoning: major area of AI
- Logic-based knowledge representation
- Automated deduction. Automated theorem provers: mostly for first-order logic.
- Interactive theorem proving. Automated proof assistants for first-order, non-classical, and higher-order logics: HOL, Isabelle, Coq, Mizar, etc.