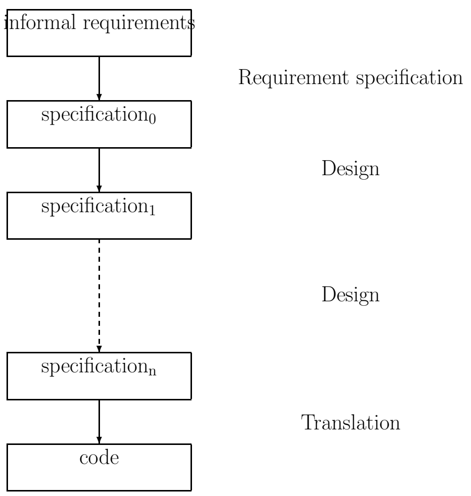
The RAISE Method
Contents

A Survey of the RAISE Method

RAISE Method Characteristics

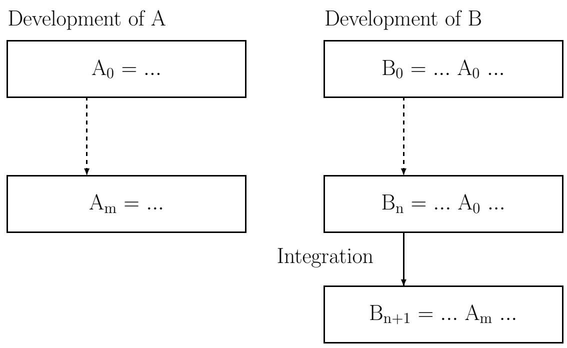
- stepwise development
- separate development
- invent and verify
- rigorous verification

Stepwise Development



specification$_0$ might build on a domain specification

Separate Development



Initial Specification

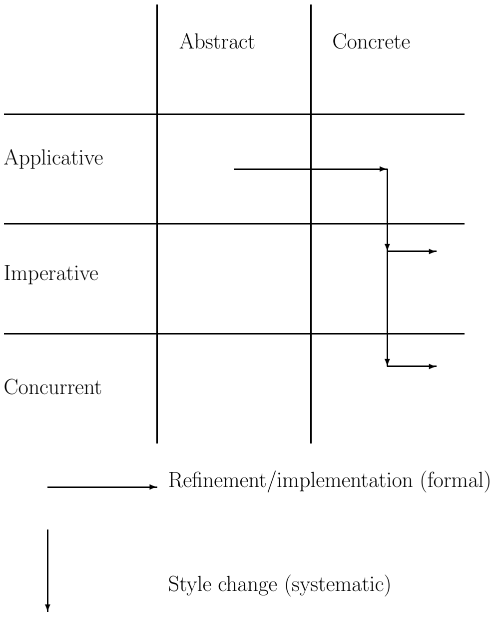
Should

- be modular (allows separate development)
- be as general as possible
- be abstract rather than detailed using sorts and important axioms and hiding what does not need to be exported

Design

- removing underspecification
  - replacing abstract types with concrete types
  - replacing signatures and axioms with explicit value definitions
- changing style
  - from applicative to imperative
  - from sequential to concurrent

Typical Development



Translation

- manual translation
- automatic translation (to ML, Ada and C++)

of low-level RSL (e.g. concrete types and explicit value definitions).

Invent and Verify

$$\begin{tikzcd} \boxed{\text{specification}_1} \arrow[d, "\text{development relation}"] \\ \boxed{\text{specification}_2} \end{tikzcd}$$

Possible development relations:

- implementation relation
- user-defined relation

Implementation Relation

$$\begin{tikzcd} \boxed{\text{class\_expr}_1} \arrow[d, "\text{implemented by}"] \\ \boxed{\text{class\_expr}_2} \end{tikzcd}$$

class_expr_2 implements class_expr_1

$\text{class\_expr}_2 \preceq \text{class\_expr}_1$

iff

1. class_expr_2 *statically implements* class_expr_1, i.e.: class_expr_2 defines at least all the names defined in class_expr_1 with same kinds and maximal types (statically decidable)
2. theory(class_expr_1) holds in class_expr_2 ($\Rightarrow$ implementation conditions)

theory(class_expr_1) is the collection of (explicit and implicit) axioms in class_expr_1.

Example: abstract specification

```
scheme REGISTRATION0 =
  class
    type
      Register,
      Name
    value
      enrol : Name × Register → Register,
      leave : Name × Register → Register,
      registered : Name × Register → Bool
    axiom
      [reg_enrol]
        ∀ n, n1 : Name, r : Register •
          registered(n, enrol(n1, r)) ≡
            n = n1 ∨ registered(n, r),
      [reg_leave]
        ∀ n, n1 : Name, r : Register •
          registered(n, leave(n1, r)) ≡
            n ≠ n1 ∧ registered(n, r)
  end
```


Example: Concrete Specification

```
scheme REGISTRATION1 =
  class
    type
      Register = Name-set,
      Name = Text

    value
      enrol : Name × Register → Register
      enrol(n,r) ≡ {n} ∪ r,

      leave : Name × Register → Register
      leave(n,r) ≡ r \ {n},

      registered : Name × Register → Bool
      registered(n,r) ≡ n ∈ r
  end
```

Example of a Development Relation

development_relation [REGISTRATION0_1]
REGISTRATION1 $\preceq$ REGISTRATION0

Static implementation can be checked by a tool.

Implementation conditions generated by a tool:

$\bullet \llcorner \forall$ n, n1 : Name, r : register $\bullet$
registered(n, enrol(n1, r)) $\equiv$
n = n1 $\vee$ registered(n, r)$\lrcorner$

$\bullet \llcorner \forall$ n, n1 : Name, r : register $\bullet$
registered(n, leave(n1, r)) $\equiv$
n $\neq$ n1 $\wedge$ registered(n, r)$\lrcorner$

These should be verified in the context of REGISTRATION1.

Rigorous Verification

justification = justification obligation + argument

Justification obligations:

- development relations between modules
- theorems about modules
- confidence conditions

Arguments are mixture of:

- formal arguments (proof rule applications)
- informal arguments

Examples of Proof Rules

Always available:

[isin_union]
e $\in$ (es1 $\cup$ es2) $\simeq$ e $\in$ es1 $\vee$ e $\in$ es2 **when** ...
[isin_singleton]
e $\in$ {e′} $\simeq$ e = e′
[is_annihilation]
e $\equiv$ e $\simeq$ **true**

Available in the context of REGISTRATION1:

[enrol_def] enrol(n,r) $\simeq$ {n} $\cup$ r **when** ...
[leave_def] leave(n,r) $\simeq$ r \ {n} **when** ...
[registered_def] registered(n,r) $\simeq$ n $\in$ r **when** ...

Example of a Justification

$\llcorner \forall$ n, n1 : Name, r : Register $\bullet$
registered(n, enrol(n1,r)) $\equiv$ n = n1 $\vee$ registered(n, r)$\lrcorner$
all_assumption_inf:
$\llcorner$registered(n, enrol(n1,r)) $\equiv$ n = n1 $\vee$ registered(n, r)$\lrcorner$
enrol_def:
$\llcorner$registered(n,{n1} $\cup$ r) $\equiv$ n = n1 $\vee$ registered(n, r)$\lrcorner$
registered_def:
$\llcorner$n $\in$ ({n1} $\cup$ r) $\equiv$ n = n1 $\vee$ registered(n, r)$\lrcorner$
registered_def:
$\llcorner$n $\in$ ({n1} $\cup$ r) $\equiv$ n = n1 $\vee$ n $\in$ r$\lrcorner$
isin_union:
$\llcorner$n $\in$ {n1} $\vee$ n $\in$ r $\equiv$ n = n1 $\vee$ n $\in$ r$\lrcorner$
isin_singleton:
$\llcorner$n = n1 $\vee$ n $\in$ r $\equiv$ n = n1 $\vee$ n $\in$ r$\lrcorner$
is_annihilation:
$\llcorner$**true**$\lrcorner$
qed

Implementation Relation

1. definition
2. use
3. proving static implementation using maximal signatures
4. proving (dynamic) implementation
5. compositionality properties

Implementation Relation

$$\begin{array}{c}\boxed{\text{class\_expr}_1}\\ \Big\downarrow \text{ implemented by}\\ \boxed{\text{class\_expr}_2}\end{array}$$

class_expr_2 implements class_expr_1

$\text{class\_expr}_2 \preceq \text{class\_expr}_1$

iff

1. class_expr_2 *statically implements* class_expr_1, i.e.: class_expr_2 defines at least all the names defined in class_expr_1 with same kinds and maximal types (statically decidable)
2. theory(class_expr_1) holds in class_expr_2 ($\Rightarrow$ implementation conditions)

theory(class_expr_1) is the collection of (explicit and implicit) axioms in class_expr_1.

Use of Implementation Relation

1. In development relations
   (a) between unparameterized schemes, S1 and S2

   S2 $\preceq$ S1

   (b) between parameterized schemes, S1 and S2

   in class object A : PAR **end** $\vdash$ S2(A) $\preceq$ S1(A)

   where PAR is the parameter requirement
2. Between formal and actual scheme parameters

   scheme ELEM = ...
   scheme S(X : ELEM) = ...
   scheme ACTUAL
   object A : ACTUAL

   Confidence condition for S(A): ACTUAL $\preceq$ ELEM

Showing implementation

1. max-sig(class_expr_1) $\subseteq$ max-sig(class_expr_2)
2. $\text{class_expr}_2 \vdash$theory($\text{class\_expr}_1$)

assuming no hiding (and local scheme defs) in class_expr1.

Maximal Signature

max−sig(**class** d1 ... dn **end**) ≃
  class max−sig(d1) ... max−sig(dn) **end**

max−sig(**type** id) ≃ **type** id

max−sig(**type** id = te) ≃ **type** id = max(te)

max−sig(**value** v : te) ≃ **value** v : max(te)

max−sig(**variable** v : te := ve) ≃ **variable** v : max(te)

max−sig(**channel** c : te) ≃ **channel** c : max(te)

max−sig(**object** O : ce) ≃ **object** O : max−sig(ce)

max−sig(**axiom** ax) ≃

Example

max−sig(
  class type T = **Nat value** v : T **axiom** v > 3 **end**
)
≃
  class type T = **Int value** v : **Int end**

Theory

theory(**class** d1 ... dn **end**) ≃
  theory(d1) ∧ ... ∧ theory(dn)

theory(**type** id) ≃ **true**

theory(**type** id = te) ≃ { x | x : id } = { x | x : te }

theory(**value** id : tmax) ≃ **true**

theory(**value** id : {| x : tmax • p(x) |}) ≃ p(id) ≡ **true**

...

theory(**axiom** ax) ≃ □ ax ≡ **true**

Example

theory(
  class type T = **Nat value** v : T **axiom** v > 3 **end**
)
≃
  { x | x : T } = { x | x : **Nat** } ∧ v ≥ 0 ∧ v > 3

Generation of Implementation Conditions

$\llcorner$ce2 ⪯ ce1$\lrcorner$
implementation_relation_expansion_inf :
- $\llcorner$p1$\lrcorner$
...
- $\llcorner$pn$\lrcorner$

where

theory(ce1) = p1 ∧ ... ∧ pn

If ce2 ⊢ pi is statically decidable, it is not generated.

Implementation Examples

S1 = **class value** x, y : **Int end**

S2 = **class value** x, y : **Int axiom** x > y **end**

S3 = **class value** x : **Int** = 1, y : **Int** = 0 **end**

S4 = **class value** x, y, z : **Int end**

S5 =
  class
    value x, y, z : **Int axiom** x > z ∧ z > y
  end

S3 ⪯ S2 ⪯ S1

S5 ⪯ S4 ⪯ S1

S5 ⪯ S2

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Implementation of type definitions

type id $\preceq$ **type** id

type id = te $\preceq$ **type** id

$$\frac{\mathrm{te2} \equiv \mathrm{te1}}{\textbf{type}\ \mathrm{id} = \mathrm{te2} \preceq \textbf{type}\ \mathrm{id} = \mathrm{te1}}$$

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Implementation of value definitions

There are a number of rules, e.g.

$$\frac{\mathrm{te2} \preceq \mathrm{te1}}{\textbf{value}\ \mathrm{id} : \mathrm{te2}\ \preceq\ \textbf{value}\ \mathrm{id} : \mathrm{te1}}$$

This is not the only possibility.

Example:

S1 = **class value** x : **Nat end**
S2 = **class value** x : **Int axiom** x = 2 **end**

S2 $\preceq$ S1

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Implementation of variable definitions

$$\frac{\mathrm{te2} \equiv \mathrm{te1}}{\textbf{variable}\ \mathrm{id} : \mathrm{te2}\ \preceq\ \textbf{variable}\ \mathrm{id} : \mathrm{te1}}$$

$$\frac{\mathrm{te2} \equiv \mathrm{te1}}{\textbf{variable}\ \mathrm{id} : \mathrm{te2} := \mathrm{e2}\ \preceq\ \textbf{variable}\ \mathrm{id} : \mathrm{te1}}$$

$$\frac{\mathrm{te2} \equiv \mathrm{te1},\ \mathrm{e2} \equiv \mathrm{e1}}{\textbf{variable}\ \mathrm{id} : \mathrm{te2} := \mathrm{e2}\ \preceq\ \textbf{variable}\ \mathrm{id} : \mathrm{te1} := \mathrm{e1}}$$

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Compositionality of Implementation Relation

- transitivity
  (allows for stepwise development)
- substitutivity
  (allows for separate development)

Requirements to Substitutions I

The result of a substitution must be well-formed.

Example

S1 = **class value** x : **Int end**
S2 = **class value** x, y : **Int end**

Substituting S1 with S2 in

extend S1 **with class value** y : **Int end**

gives an ill-formed class expression:

extend S2 **with class value** y : **Int end**

Requirements to Substitutions II

ce2 $\preceq$ ce1 $\Rightarrow$ F(ce2) $\preceq$ F(ce1) ?

Class expressions of local modules must only contain free names of parameter modules and global modules.

Example

Substitution of class(O) with

class value x : **Int** = y, y : **Int end**

in

class
  object O : **class value** x : **Int** = y **end**
  value y : **Int**
end

gives

class
  object O : **class value** x : **Int** = y, y : **Int end**
  value y : **Int**
end

which is not an implementation.

Development in the Large

Substitution with Implementations

Assume ce2 $\preceq$ ce1

Then

hide names **in** ce2 $\preceq$ **hide** names **in** ce1

use renamings **in** ce2 $\preceq$ **use** renamings **in** ce1

extend ce2 **with** ce $\preceq$ **extend** ce1 **with** ce

extend ce **with** ce2 $\preceq$ **extend** ce **with** ce1

class object O : ce2 **end** $\preceq$ **class object** O : ce1 **end**

Development in the Large

Substitution with Implementations

Development of unparameterized schemes:

Given

scheme S1 = ce1, S2 = ce2

If ce2 $\preceq$ ce1 then

S2 $\preceq$ S1

Development of parameterized schemes:

Given

scheme S1(X : ceX) = ce1, S2(X : ceX) = ce2
object A : ceA

If

ceA $\preceq$ ceX   and   **object** X : ceX $\vdash$ ce2 $\preceq$ ce1

then

S2(A) $\preceq$ S1(A)

Verification

- What to verify (justification obligations)
- How to verify (arguments)
- Proof rules and their application
- Available proof rules
- Informal arguments

Verification and Justifications

Justification obligations

A *justification obligation* is a logical expression that should be verified to be true. Kinds of justification conditions:

- development relations between modules, e.g.
  REGISTRATION1 $\preceq$ REGISTRATION0
- theorems about modules, e.g.
  REGISTRATION1 $\vdash$
  $\forall$ n : Name • registered(n, {}) = **false**

Justifications

A *justification* is a piece of syntax, consisting of
a justification obligation + an argument for its truth.

The RAISE tools comprise a justification editor.

Arguments

are mixtures of:

- formal arguments (proof rule applications)
- informal arguments

Verification

verification is the act of making a justification.

Example of a Justification

(Foil 16 once again.)

$\llcorner\forall$ n, n1 : Name, r : Register •
registered(n, enrol(n1,r)) $\equiv$ n = n1 $\vee$ registered(n, r)$\lrcorner$
all_assumption_inf:
$\llcorner$registered(n, enrol(n1,r)) $\equiv$ n = n1 $\vee$ registered(n, r)$\lrcorner$
enrol_def:
$\llcorner$registered(n,{n1} $\cup$ r) $\equiv$ n = n1 $\vee$ registered(n, r)$\lrcorner$
registered_def:
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registered_def:
$\llcorner$n $\in$ ({n1} $\cup$ r) $\equiv$ n = n1 $\vee$ n $\in$ r$\lrcorner$
isin_union:
$\llcorner$n $\in$ {n1} $\vee$ n $\in$ r $\equiv$ n = n1 $\vee$ n $\in$ r$\lrcorner$
isin_singleton:
$\llcorner$n = n1 $\vee$ n $\in$ r $\equiv$ n = n1 $\vee$ n $\in$ r$\lrcorner$
is_annihilation:
$\llcorner$**true**$\lrcorner$
qed

The context of this justification is REGISTRATION1.

Equivalence Rules

Simple form:

[name_of_rule]
term1 $\simeq$ term2

Examples:

[isin_singleton]
e $\in$ {e′} $\simeq$ e = e′
[is_annihilation]
e $\equiv$ e $\simeq$ **true**

state that two terms are equivalent

$\simeq$ is a generalisation of $\equiv$

Application of Equivalence Rules

can be applied in either direction
can be applied to subterms

⌞condition⌟
name_of_rule:
⌞resulting_condition⌟

Examples: see page 35

Equivalence Rules with Applicability Conditions

Form:

[name_of_rule]
term1 $\simeq$ term2 **when** condition

Application:

⌞condition⌟
name_of_rule:
⌞resulting_condition⌟
since
⌞side_condition⌟
argument_for_side_condition
end
argument_for_resulting_condition

Equivalence Rules with Applicability Conditions

Example

[equality_annihilation]
e = e $\simeq$ **true**
when convergent(e) $\wedge$ **readonly**(e)

Application:

⌞(5 = 5) $\Rightarrow$ ... ⌟
equality_annihilation:
⌞(**true**) $\Rightarrow$ ... ⌟
since
⌞ **convergent**(5) $\wedge$ **readonly**(5)⌟
<:*argument*:>
end
<:*argument*:>

Inference Rules

example

[and_split_inf]
$\dfrac{\text{ro\_eb, ro\_eb}'}{\text{ro\_eb} \wedge \text{ro\_eb}'}$

Application:

⌞x $\neq$ y $\wedge$ x = z⌟
and_split_inf:
- ⌞x $\neq$ y⌟
  <:*argument*:>
- ⌞x = z⌟
  <:*argument*:>

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Inference Rules

example

$$\frac{[\,\text{imply\_deduction\_inf}\,] \quad [\,\text{id}\,]\ \text{ro\_eb} \vdash \text{ro\_eb}'}{\text{ro\_eb} \Rightarrow \text{ro\_eb}'}$$

when convergent(ro_eb) $\wedge$ **pure**(ro_eb)

Application:

⌞x = 0 ⇒ x ≥ 0⌟
imply_deduction_inf :
[x_zero] x = 0 ⊢ ⌞x ≥ 0⌟
x_zero :
⌞0 ≥ 0⌟
<:*argument*:>

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Available Proof Rules

1. rule base (always available)
2. context rules (depend on context)

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Context Rules

Value definitions

Context:

value max : Int = 3

gives rise to the context rule

[max_def] max ≃ 3

Example of application:

⌞ ... max ... ⌟
max_def :
⌞ ... 3 ... ⌟

In the tools:

⌞ ... max ... ⌟
value_name_unfold :
⌞ ... 3 ... ⌟

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Context Rules

Function definitions without pre conditions

Context:

value
f : **Nat** → **Nat**
f(x) ≡ x + 1

gives rise to the context rule

[f_def] f(e) ≃ e + 1
when convergent(e) ∧ **pure**(e) ∧ **isin_subtype**(e, **Nat**)

Example of application:

⌞ ... f(7) ... ⌟
f_def :
⌞ ... 7 + 1 ... ⌟

In the tools:

⌞ ... f(7) ... ⌟
application_expr_unfold1 :
⌞ ... 7 + 1 ... ⌟

Context Rules
Function definitions with pre conditions

Context:

value
  f : **Int** $\xrightarrow{\sim}$ **Int**
  f(x) ≡ x + 1
  pre x > 0

gives rise to the context rule

[f_def] f(e) ≃ e + 1
when convergent(e) ∧ **pure**(e) ∧ e > 0

Example of application:

⌞ ... f(7) ... ⌟
f_def :
⌞ ... 7 + 1 ... ⌟
  since
    ⌞7 > 0⌟

  end

In the tools the rule application_expr_unfold2 is used instead of f_def.

Context Rules

Available in the context of REGISTRATION1:

[enrol_def] enrol(n,r) ≃ {n} ∪ r
when convergent(n, r) ∧ **pure**(n) ∧ **pure**(r) ∧
  isin_subtype(n, Name) ∧ **isin_subtype**(r, Register)

[leave_def] leave(n,r) ≃ r \ {n}
when ...

[registered_def] registered(n,r) ≃ n ∈ r
when ...

Informal Arguments

Explanation argument:

⌞⟨5⟩ ⌢ ⟨7⟩ ≡ ⟨5, 7⟩⌟
/∗ which is obviously true ∗/
qed

Replacement argument:

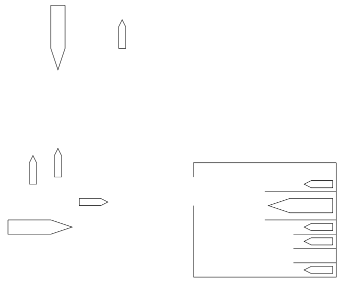
⌞7 ∈ {(⟨5,7,3⟩ ⌢ ⟨⟩)(2)}⌟
/∗ as ..., this is equivalent to ∗/
⌞7 ∈ {7}⌟

Commented Arguments

⌞x ≠ y ∧ x = z⌟
/∗ next we want to bring the left conjunct
  into an appropriate form ∗/
inequality_expansion:
⌞∼(x = y) ∧ x = z⌟

A case study: harbour

Example: a Harbour Master



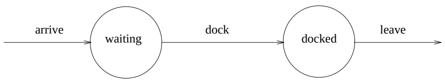
to control movement ships:

arrive
dock
leave

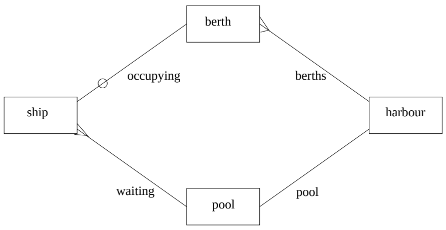
Some requirements:

1. Ships can arrive and will be registered.
2. Ships can be docked when a suitable berth is free.
3. Docked ships can leave.
4. Ships can only be docked in berths they fit.

State Transition Diagram



Entity Relationships



Initial Specification

```
scheme TYPES =
  class
    type
      Ship, Berth,
      Occupancy == vacant | occupied_by(occupant : Ship)
    value
      fits : Ship × Berth → Bool
  end

object T : TYPES

scheme A_HARBOUR0 = ...
```

Abstract Applicative Modules

1. Abstract type of interest T (**type** T)
2. Observers with range types not dependent on T (**value** obs : ... T → T2)
3. Constant(s) of types dependent on T (**value** c : T)
4. Generators with range types dependent on T (**value** gen : ... → T)
5. Guards for partial generators (**value** can_gen : ... → Bool)
6. Derived functions: defined from others
7. Axioms:
   (a) Observer-constant axioms
   (b) Observer-generator axioms
   (∀ ...• obs(..., gen(...)) ≡ ...)
   (c) Generator-result axioms
   (d) System axioms: preservation of invariants etc.

A_HARBOUR0

```
type
  /∗ type of interest ∗/
  Harbour

value
  /∗ observers ∗/
  waiting : T.Ship × Harbour → Bool,
  occupancy : T.Berth × Harbour → T.Occupancy,

  /∗ derived observer ∗/
  is_docked : T.Ship × Harbour → Bool
  is_docked(s, h) ≡
    (∃ b : T.Berth • occupancy(b, h) = T.occupied_by(s))
```


Invariants

1. at most one ship can be in any one berth
2. a ship can't be in two places at once
3. a ship can only be in a berth it fits

invariant no. 1 is ensured by:

value occupancy : T.Berth × Harbour → T.Occupancy

A_HARBOUR0

```
value
  /∗ invariant ∗/
  consistent : Harbour → Bool
  consistent(h) ≡
    (∀ s : T.Ship •
      ∼ (waiting(s, h) ∧ is_docked(s, h)) ∧
      (∀ b1, b2 : T.Berth •
        occupancy(b1, h) = T.occupied_by(s) ∧
        occupancy(b2, h) = T.occupied_by(s) ⇒
        b1 = b2
      ) ∧
    (∀ b : T.Berth •
      occupancy(b, h) = T.occupied_by(s) ⇒
        T.fits(s, b))
    )
```

A_HARBOUR0

```
value
  /∗ generators ∗/
  arrives : T.Ship × Harbour ~→ Harbour,
  docks : T.Ship × T.Berth × Harbour ~→ Harbour,
  leaves : T.Ship × T.Berth × Harbour ~→ Harbour,

  /∗ guards to express preconditions ∗/
  can_arrive : T.Ship × Harbour → Bool
  can_arrive(s, h) ≡ ∼ waiting(s, h) ∧ ∼ is_docked(s, h),

  can_dock : T.Ship × T.Berth × Harbour → Bool
  can_dock(s, b, h) ≡
    waiting(s, h) ∧ ∼ is_docked(s, h) ∧
    occupancy(b, h) = T.vacant ∧ T.fits(s, b),

  can_leave : T.Ship × T.Berth × Harbour → Bool
  can_leave(s, b, h) ≡ occupancy(b, h) = T.occupied_by(s)
```

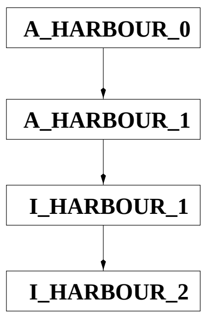

A_HARBOUR0

```
axiom
/∗ observer−generator axioms ∗/
  [waiting_arrives]
    ∀ h : Harbour, s1, s2 : T.Ship •
      waiting(s2, arrives(s1, h)) ≡
      s1 = s2 ∨ waiting(s2, h)
        pre can_arrive(s1, h),

  [occupancy_arrives]
    ∀ h : Harbour, s : T.Ship, b : T.Berth •
      occupancy(b, arrives(s, h)) ≡ occupancy(b, h)
        pre can_arrive(s, h),

  [waiting_docks]
    ∀ h : Harbour, s1, s2 : T.Ship, b : T.Berth •
      waiting(s2, docks(s1, b, h)) ≡
      s1 ≠ s2 ∧ waiting(s2, h)
        pre can_dock(s1, b, h),

  [occupancy_docks] ...,

  [waiting_leaves] ...,

  [occupancy_leaves] ...
```


A_HARBOUR0

```
axiom
  /∗ invariant preservation by generators ∗/
  [consistent_arrives]
    ∀ h : Harbour, s : T.Ship •
      arrives(s, h) as h′ post consistent(h′)
        pre consistent(h) ∧ can_arrive(s, h),
  [consistent_docks]
    ∀ h : Harbour, s : T.Ship, b : T.Berth •
      docks(s, b, h) as h′ post consistent(h′)
        pre consistent(h) ∧ can_dock(s, b, h),
  [consistent_leaves]
    ∀ h : Harbour, s : T.Ship, b : T.Berth •
      leaves(s, b, h) as h′ post consistent(h′)
        pre consistent(h) ∧ can_leave(s, b, h)
```


Development Plan



Development to Concrete Applicative

1. Replace sort definition with concrete type definition.
2. Give explicit definitions of constants and functions.
3. Remove axioms.

Extension of TYPES

type
  Index = {| i : **Int** • i ≥ min ∧ max ≥ i |}
value
  min, max : **Int**,
  indx : Berth → Index
axiom
  [index_not_empty] max ≥ min,
  [berths_indexable]
    ∀ b1, b2 : Berth • indx(b1) = indx(b2) ⇒ b1 = b2

A_HARBOUR1

scheme A_HARBOUR1 =
  class
    type Harbour =
        T.Ship-**set** ×
        {| bs : T.Index $\overrightarrow{m}$ T.Occupancy •
            (∀ idx : T.Index • idx ∈ **dom** bs) |}

    ... (see the following foils)
  end

A_HARBOUR1

value
  /* observers */
  waiting : T.Ship × Harbour → **Bool**
  waiting(s, (ws, bs)) ≡ s ∈ ws,

  occupancy : T.Berth × Harbour → T.Occupancy
  occupancy(b, (ws, bs)) ≡ bs(T.indx(b)),

A_HARBOUR1

```
/∗ generators ∗/
arrives : T.Ship × Harbour ∼→ Harbour
arrives(s, (ws, bs)) ≡
  (ws ∪ {s}, bs)
  pre can_arrive(s, (ws, bs)),

docks : T.Ship × T.Berth × Harbour ∼→ Harbour
docks(s, b, (ws, bs)) ≡
  (ws \ {s}, bs † [ T.indx(b) ↦ T.occupied_by(s) ])
  pre can_dock(s, b, (ws, bs)),

leaves : T.Ship × T.Berth × Harbour ∼→ Harbour
leaves(s, b, (ws, bs)) ≡
  (ws, bs † [ T.indx(b) ↦ T.vacant ])
   pre can_leave(s, b, (ws, bs)),
```


A_HARBOUR1

```
/∗ derived observer ∗/
is_docked : T.Ship × Harbour → Bool
is_docked(s, (ws, bs)) ≡ ...

/∗ invariant ∗/
consistent : Harbour → Bool
consistent((ws, bs)) ≡ ...

/∗ guards to express preconditions ∗/
...
```


A_HARBOUR0_1

```
development_relation [ A_HARBOUR0_1 ]
  A_HARBOUR1 ⪯ A_HARBOUR0
```


Part of proof

```
  ⌞∀ h : Harbour, s1, s2 : T.Ship •
    waiting(s2, arrives(s1, h)) ≡ s1 = s2 ∨ waiting(s2, h)
      pre can_arrive(s1, h)⌟
all_name_change :
  ⌞∀ (ws, bs) : Harbour, s1, s2 : T.Ship •
    waiting(s2, arrives(s1, (ws, bs))) ≡ s1 = s2 ∨ waiting(s2, (ws, bs))
      pre can_arrive(s1, (ws, bs))⌟
all_assumption_inf :
  ⌞waiting(s2, arrives(s1, (ws, bs))) ≡ s1 = s2 ∨ waiting(s2, (ws, bs))
    pre can_arrive(s1, (ws, bs))⌟
pre_deduction_inf :
[ assump ] can_arrive(s1, (ws, bs)) ⊢
  ⌞waiting(s2, arrives(s1, (ws, bs))) ≡ s1 = s2 ∨ waiting(s2, (ws, bs))⌟
arrives_def :
  ⌞waiting(s2, (ws ∪ {s1}, bs)) ≡ s1 = s2 ∨ waiting(s2, (ws, bs))⌟
  since
      ⌞can_arrive(s1, (ws, bs))⌟
    assump :
      ⌞true⌟
    qed
  end
```

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waiting_def :
  ⌞s2 ∈ ws ∪ {s1} ≡ s1 = s2 ∨ waiting(s2, (ws, bs))⌟
waiting_def :
  ⌞s2 ∈ ws ∪ {s1} ≡ s1 = s2 ∨ s2 ∈ ws⌟
isin_union :
  ⌞s2 ∈ ws ∨ s2 ∈ {s1} ≡ s1 = s2 ∨ s2 ∈ ws⌟
isin_singleton :
  ⌞s2 ∈ ws ∨ s2 = s1 ≡ s1 = s2 ∨ s2 ∈ ws⌟
or_commutativity :
  ⌞s2 = s1 ∨ s2 ∈ ws ≡ s1 = s2 ∨ s2 ∈ ws⌟
equality_commutativity :
  ⌞s1 = s2 ∨ s2 ∈ ws ≡ s1 = s2 ∨ s2 ∈ ws⌟
is_annihilation :
  ⌞**true**⌟
qed

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I_HARBOUR1

I_HARBOUR1 is as A_HARBOUR1 except:

- variables are introduced

  variable
    ws : T.Ship-**set**,
    bs : {| bs : T.Index $\overrightarrow{m}$ T.Occupancy •
        (∀ idx : T.Index • idx ∈ **dom** bs) |}

- type Harbour is removed
- Function signatures:
  Harbour removed and **read/write any** added
- Function bodies changed accordingly

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Examples:

  waiting : T.Ship × Harbour → **Bool**
  waiting(s, (ws, bs)) ≡ s ∈ ws,

  arrives : T.Ship × Harbour $\xrightarrow{\sim}$ Harbour
  arrives(s, (ws, bs)) ≡ (ws ∪ {s}, bs)
    pre can_arrive(s, (ws, bs))

changed to:

  waiting : T.Ship → **read any Bool**
  waiting(s) ≡ s ∈ ws,

  arrives : T.Ship $\xrightarrow{\sim}$ **write any Unit**
  arrives(s) ≡ ws := ws ∪ {s} **pre** can_arrive(s)

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I_HARBOUR2

I_HARBOUR2 is as I_HARBOUR1 except:

  is_docked : T.Ship → **read any Bool**
  is_docked(s) ≡
    (∃ b : T.Berth • bs(T.indx(b)) = T.occupied_by(s))

is changed to

value
  is_docked : T.Ship → **read any Bool**
  is_docked(s) ≡
    local
      variable
        found : **Bool** := **false**,
        indx : T.Index := T.min
    in
      while ∼ found ∧ indx ≤ T.max **do**
        found := bs(indx) = T.occupied_by(s) ;
        indx := indx + 1
      end ;
      found
    end

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I_HARBOUR1_2

development_relation [I_HARBOUR1_2]
I_HARBOUR2 $\preceq$ I_HARBOUR1

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Further development of TYPES

- concrete types for Ship, Berth
- explicit definition of fits, min, max and indx

left as an exercise for you!