02240 Computability and Semantics Michael R. Hansen
Informatics and Mathematical Modelling 19/4 2005

Mandatory Assignment 4

This mandatory assignment must be solved in groups of 1, 2 or 3 persons. The solution must be handed in no later than Wednesday, May 4.

The assignment consists mainly of old examination questions in computability.

Question A — from summer 2003

Give a transition diagram for a Turing Machine accepting the language:

$$\{0^n 1^n 0^m \mid m \geq n\}$$

Give an informal explanation of the main ideas of your construction.

Question B — from summer 2003

Answer True or False to the following claims, and give a brief explanation to each answer.

a) There is an instance of Post's Correspondence Problem which has precisely two solutions.

b) The following language is recursively enumerable:

$$\{(M, w) \mid \text{Turing machine } M \text{ accepts } w \text{ in a even number of computation steps}\}$$

c) If $L_1 \cup L_2$ is recursive, then L_1 and L_2 are both recursively enumerable.

d) If L_1 and L_2 are recursively enumerable, then their concatenation $L_1\,L_2$ is recursively enumerable.

e) The following problem is undecidable:

Given Turing machine M: Does M accept an even number of strings?

Problem C — from summer 2003

Show that the following problem is undecidable:

Given two Turing machines M_1 and M_2: Is the language accepted by M_1 equal to the complement of the language accepted by M_2, i.e. $L(M_1) = \overline{L(M_2)}$?

Problem D — from summer 2002

Let $\Sigma = \{0,1\}$. Consider the following two instances of Post's Correspondence Problem:

Instance 1:

	List A	List B
i	w_i	x_i
1	100	10
2	101	01
3	110	1010

Instance 2:

	List A	List B
i	w_i	x_i
1	1	10
2	01	101
3	0	101
4	001	0

For each instance, either give a solution or prove that none exists.

Problem E — from summer 2002

Answer True or False to the following claims, and give a brief explanation to each answer.

a) If P_1 reduces to P_2 and P_2 reduces to P_3, then P_1 reduces to P_3.

b) Given Turing machine M, input string w, it is decidable whether there exist a $k \geq 0$ such that M uses at most k tape cells on input w.

c) Define $L_1 \uplus L_2 = \{w \in \Sigma^* \mid (w \in L_1 \wedge w \notin L_2) \vee (w \in L_2 \wedge w \notin L_1)\}$, where Σ is an alphabet containing at least to elements.

   (a) If L_1 and L_2 are recursive, then so is $L_1 \uplus L_2$.

   (b) If L_1 and L_2 are recursive enumerable, then so is $L_1 \uplus L_2$.

Problem F — from summer 2002

Show that the following problem is undecidable:

Given Turing machine M, does M accept the empty string ϵ?

Is the language consisting of all those encoded Turing machines accepting the empty string a recursively enumerable language? (Justify your answer.)