# Distributed Termination

Robin Sharp

robin@imm.dtu.dk

*Systems Security Group*

Informatics and Mathematical Modelling

Technical University of Denmark

DK-2800 Kgs. Lyngby, Denmark.

# DISTRIBUTED TERMINATION

- GIVEN:
  - A set of disjoint processes (no shared variables!).
  - Possibility of communication between processes.
- REQUIRED:
  - A technique to decide when a task, which the processes cooperate to solve, is finished.
- NOTE: This is not trivial in a distributed system!

# ASSUMPTIONS

- Global termination when a post-condition $B(\bar{y})$ holds, where $\bar{y}$ is *GLOBAL STATE*.
- $\bar{y}$ can be divided into $n$ $(\geq 2)$ disjoint states $\bar{y}_1,\ \bar{y}_2, \ldots, \bar{y}_n$
- There are $n$ predicates $B_i(\bar{y}_i),\ i = 1, 2, \ldots, n$ such that:

$$\left(\bigwedge_{i=1}^{n} B_i(\bar{y}_i)\right) \Rightarrow B(\bar{y})$$

- There are $n$ processes $P_1, P_2, \ldots, P_n$ with state vectors $\bar{y}_1,\ \bar{y}_2, \ldots, \bar{y}_n$ which can reach states where $B_i(\bar{y}_i),\ i = 1, 2, \ldots, n$ hold after finite time.
The necessary communication to achieve this is called *BASIC COMMUNICATION*.

# FRANCEZ' CLAIM

NOTE: Uses old-fashioned CSP (1978) notation!!

- Program $P :: [P_1 \parallel P_2 \parallel \ldots \parallel P_n]$ will be a solution if termination can be forced as soon as each $P_i$ reaches a state where $B_i$ holds. This state is called a *FINAL STATE*.
- Each $P_i$ is repetitive, of form:

$$
\begin{aligned}
P_i \quad = \quad & *[g_{i1} \rightarrow S_{i1} \\
& \;\; \| g_{i2} \rightarrow S_{i2} \\
& \;\; \| \ldots \\
& \;\; \| g_{ik_i} \rightarrow S_{ik_i}
\end{aligned}
$$

  where each guard $g_{ij}$ may involve Boolean conditions and *BASIC COMMUNICATION*

- In stable state $\forall_i \cdot B_i(\bar{y}_i)$, all $P_i$ are in their outer level without any ready guard. So we assume:

  | No processes in FINAL STATES perform *BASIC COMM.* |
  |---|

# TERMINATION CONDITIONS

- Processes $P_i$ establish termination by using *CONTROL COMMUNICATION* in addition to their *BASIC COMMUNICATION*.
- Assume that *CONTROL COMMUNICATION* does not require extra *COMM. CHANNELS*!
- For given process $P_i$ there are two termination possibilities:

**ENDOTERMINATION:** Reachability of a final state is determined by a predicate over $P_i$'s initial state $B_i(\bar{y_{0i}})$.

**EXOTERMINATION:** Termination depends on each member of a *TERMINATION DEPENDENCY SET*:

$$T = \{P_{i_1}, P_{i_2}, \ldots, P_{i_k}\}, \quad k > 0$$

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  having terminated.
- We can similarly define:

  **ENDONONTERMINATION:** No final state can be reached.

  **EXONONTERMINATION:** At least one process in the *TDS* does not terminate.

# GRAPHS

- COMMUNICATION GRAPH, $G_P$ defined for system $P \; :: \; [P_1 \parallel P_2 \parallel \ldots \parallel P_n]$ by:
  - Each process $P_i$ corresponds to a node in $G_P$.
  - Each process pair $< P_i, P_j >$ for which communication can take place from $P_i$ to $P_j$, corresponds to an edge in $G_P$.

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  NOTE: $G_P$ can be determined syntactically from $P$.
- TERMINATION GRAPH, $T_P$ defined for system $P$ by:
  - Each process $P_i$ corresponds to a node in $G_P$.
  - For each edge $(P_i, P_j)$ in $G_P$:

$$(P_i, P_j) \in T_P \quad \Leftrightarrow \quad P_i \in TDS_j$$
$$(P_j, P_i) \in T_P \quad \Leftrightarrow \quad P_j \in TDS_i$$

    NOTE: $T_P$ reflects all termination dependencies in $P$, and may depend on initial state.
  - All nodes corresponding to ENDOTERMINATING processes are sources in $T_P$ (i.e. with in-degree=0).

# GRAPHS (2)

THEOREM: $P$ TERMINATES FOR $(\bar{y}_1, \ldots, \bar{y}_n)$ ONLY IF $T_P$ IS ACYCLIC.

**PROOF:** If $T_P$ contains a loop, deadlock can take place, as all nodes on loop correspond to processes which are EXONONTERMINATING for $\bar{y}$.

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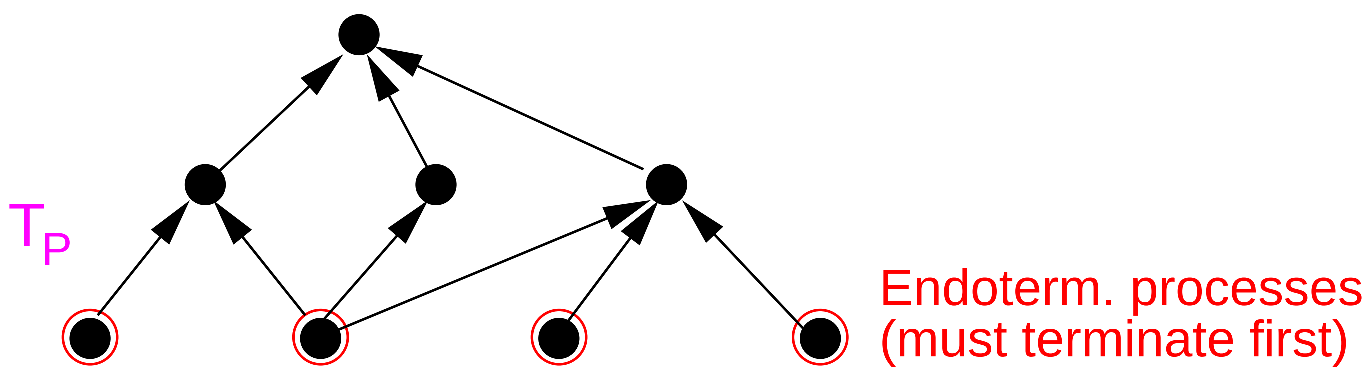
- What should we conclude from all this?
- If $P$ terminates for $\bar{y}$, a partial ordering is defined such that when all ENDOTERMINATING processes terminate, then all processes in their $TDS$s terminate, etc.




This partial ordering defines a *TERMINATION WAVE*...

# STRATEGY

Basic idea for ensuring orderly termination:

1. Choose termination dependencies among $P_1, P_2, \ldots, P_n$ such that $T_P$ is acyclic.
2. Designate arbitrary $P_{i0}$ which collects information about whether all others are in a state where $B_i(\bar{y_i})$ holds.
3. When this happens, $P_{i0}$ must terminate, and this initiates *TERMINATION WAVE*:

   All processes in $P_{i0}$'s $TDS$ terminate,
   then all processes in *their* $TDS$s
   and so on.

# STRATEGY (2)

- Need to find a *SPANNING TREE* in undirected graph corresponding to $G_P$. Then:
  1. Root process in tree initiates *CONTROL WAVE* to all its successors in the tree.
  2. Wave propagates past $P_j$ if $B_j(\bar{y_j})$ holds.
     Passage of wave *FREEZES* all *BASIC COMMUNICATION*.
  3. Each node in tree reports back to its parent whether its successors have terminated.
  4. When root process receives *POSITIVE ANSWER*, it terminates and this initiates *TERMINATION WAVE*.
  5. If root process receives *NEGATIVE ANSWER*, an *UNFREEZING WAVE* is propagated to allow *BASIC COMMUNICATION* again.

  At least one more *BASIC COMMUNICATION* must then take place before a new *CONTROL WAVE* can be initiated.

# WHAT MUST $P$ LOOK LIKE?

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- In general, this will not be true of the system $P$ from the original algorithm whose termination is desired.
- To construct the final system, say $\bar{P}$:
  1. Choose $T^*_P$= arbitrary spanning tree in undirected $G'_P$.
  2. Modify $P_1, P_2, \ldots, P_n$ to $\bar{P}_1, \bar{P}_2, \ldots, \bar{P}_n$, where:
     - $TDS_i = \{$parent of $\bar{P}_i$ in $T^*_P\}$.
     - Root of $T^*_P$ is ENDOTERMINATING.
     - Each $\bar{P}_i$ is derived from $P_i$ by adding a *CONTROL SECTION*, $C_i$, consisting of further alternatives in $P_i$.

     *CONTROL SECTION* deals with *CONTROL WAVE*, *TERMINATION WAVE*, *UNFREEZING WAVE* etc. For details, see Francez' paper.

# DISTRIBUTED PARTITION SORT

- Disjoint partitioning of $S = S_1 + S_2 + \ldots + S_n$, where cardinality of $S_i$, $|S_i| = m_i$.
- After sorting:

$$\begin{array}{lll} \forall i,j\ (1 \leq i < j \leq n) & \cdot & (\forall p,q \cdot (p \in S_i \wedge q \in S_j \Rightarrow p < q)) \\ \wedge \quad \forall i\ (1 \leq i \leq n) & \cdot & (|S_i| = m_i) \end{array}$$

Or, equivalently:

$$\begin{array}{lll} \forall i,j\ (1 \leq i < n) & \cdot & (\max(S_i) < \min(S_{i+1})) \\ \wedge \quad \forall i\ (1 \leq i \leq n) & \cdot & (|S_i| = m_i) \end{array}$$

Or, alternatively $\bigwedge_{i=1}^{n} B_i(S_i, lin_i)$

$$\text{where} \begin{cases} B_i(S_i, lin_i) \stackrel{\text{def}}{=} \max(S_i) \leq lin_i \wedge |S_i| = m_i \quad (1 \leq i < n) \\ B_n \stackrel{\text{def}}{=} \text{true} \end{cases}$$

where $lin_i$ is the latest value received from $P_{i+1}$

# DPS ALGORITHM

$$
\begin{array}{lll}
P_i & :: & update; lin := -\infty; \\
 & & *[mx > lin; P_{i+1}!mx \rightarrow S_i := S_i - \{mx\}; P_{i+1}?lin; \\
 & & \qquad S_i := S_i + \{lin\}; update \\
 & & \;[\!] P_{i-1}?l \rightarrow S_i := S_i + \{l\}; update; P_{i-1}!mn; \\
 & & \qquad S_i := S_i - \{mn\}; update; P_{i-1}!mn; \\
 & & \;[\!] P_{i+1}?lin \rightarrow \mathsf{skip}] \\
P_1 & :: & update; lin := -\infty; \\
 & & *[mx > lin; P_2!mx \rightarrow S_1 := S_1 - \{mx\}; P_2?lin; \\
 & & \qquad S_1 := S_1 + \{lin\}; update \\
 & & \;[\!] P_2?lin \rightarrow \mathsf{skip}] \\
P_n & :: & update; \\
 & & *[P_{n-1}?l \rightarrow S_n := S_n + \{l\}; update; P_{n-1}!mn; \\
 & & \qquad S_n := S_n - \{mn\}; update; P_{n-1}!mn]
\end{array}
$$

where $update \stackrel{\text{def}}{=} mx := \max(S_i); mn := \min(S_i);$