













Parallel Merge Sort
<pre>e chan in1(int), in2(int), out(int); process Merge { int v1, v2; receive in1(v1); # get first two input values receive in2(v2); # send smaller value to output channel and repeat while (v1 != EOS and v2 != EOS) { if (v1 <= v2) { send out(v1); receive in1(v1); } else # (v2 < v1) { send out(v2); receive in2(v2); } } # consume the rest of the non-empty input channel if (v1 == EOS) while (v2 != EOS) { send out(v2); receive in2(v2); } else # (v2 == EOS) while (v1 != EOS) { send out(v1); receive in1(v1); } else # (v2 == EOS) while (v1 != EOS) { send out(v1); receive in1(v1); } # append a sentinel to the output channel send out(EOS); } }</pre>









Resource Allocator: Monitor

```
monitor Resource_Allocator {
  int avai1 = MAXUNITS;
  set units = initial values;
  cond free;
              # signaled when a process wants a unit
 procedure acquire(int &id) {
    if (avail == 0)
      wait(free);
    else
      avail = avail-1;
    remove(units, id);
  }
 procedure release(int id) {
    insert(units, id);
    if (empty(free))
      avail = avail+1;
    else
      signal(free);
 }
}
```



Resource Allocator: Server







```
Exchanging Values – symmetric
          chan values[n](int);
            process P[i = 0 \text{ to } n-1] {
                        # assume v has been initialized
               int v;
               int new, smallest = v, largest = v; # initial state
               # send my value to the other processes
               for [j = 0 \text{ to } n-1 \text{ st } j != i]
                 send values[j](v);
               # gather values and save the smallest and largest
               for [j = 1 \text{ to } n-1] {
                 receive values[i](new);
                 if (new < smallest)</pre>
                   smallest = new;
                 if (new > largest)
                   largest = new;
               }
             }
```

	g II
<pre> process P[i = 1 to n-1] { int v; # assume v has been initialized int smallest, largest; # receive smallest and largest so far, then update # them by comparing their values to v receive values[i](smallest, largest) if (v < smallest) smallest = v; if (v > largest) largest = v; # send the result to the next processes, then wait # to get the global result send values[(i+1) mod n](smallest, largest); receive values[i](smallest, largest); } </pre>	<pre>to n-1] { sume v has been initialized largest; lest and largest so far, then update omparing their values to v s[i](smallest, largest) st) = v; st) v; sult to the next processes, then wait global result i+1) mod n](smallest, largest); s[i](smallest, largest);</pre>

Communication Sequential Processes (CSP)

• Language proposal by C.A.R. Hoare 1978

Idea

- A simple imperative langauge based on *Dijkstra's Guarded Commands*
- Plus a few simple concepts for concurrent programming
 - No shared variables
 - Synchronous communication
 - ► Selection

Impact

- Further eveloped into *Theoretical CSP* notation
- Realized in the Occam language
- Seminal for many theories of concurrency, esp. process algebras
- Inspiration for concurrency in Go

CSP: Basic Co	onstructs			
Dijkstra's Guarded	Commands			
• if $b_1 \rightarrow S_1$ [] $b_2 \rightarrow S_2$ [] [] $b_n \rightarrow S_n$ fi				
Synchronous Communication				
•	process A	process B		
Output	: B ! e :	: A?x :	Input	
 A process has re Another process They name each The output exp No buffering ne 	eached an output (!) of s has reached an input h other in the operation ression <i>e</i> matches the t eded!	peration (?) operation ns cype of the input variabl	e <i>x</i>	























Theoretical CSP

- A formal notation for desribing processes (*process algebra*) based on CSP
 Syntax
- Process expressions $P ::= \text{stop} \mid a \rightarrow P \mid P \mid Q \mid P \mid Q \dots$
- Example: $p \stackrel{\Delta}{=} a \rightarrow (b \rightarrow p [] c \rightarrow \text{stop})$

Semantics

- Basic trasistion: $P \xrightarrow{a} Q$
- Axioms, e.g. $(a \rightarrow P) \xrightarrow{a} P$
- Inference rules, e.g.

$P \xrightarrow{a} P'$	$P \xrightarrow{a} P'$	$Q \stackrel{a}{\longrightarrow} Q'$
$P \llbracket Q \xrightarrow{a} P'$	$P \mid\mid Q \xrightarrow{a}$	• P' Q'

• Forms the basis for defining process equivalence, refinement etc.

Occam

- Programming language developed in the 80'ies by the Inmos chip company
- Based directly on the CSP notions (albeit using typed channels). e.g.

```
PROC Copy (CHAN OF BYTE West, East)
BYTE c:
WHILE TRUE
SEQ
West ? c
East ! c
```

• Named after Ockham's Razor

Entia non sunt mulitiplicanda praeter necessitatem

— William of Ockham (1287–1347)

Transputer

- Specialized processor supporting Occam with four communication links
- Transputers are readily connected in a *mesh* providing a parallel machine