02153 Declarative Programming
Programming Exercise 1

1. Declare a function `g: int -> int`, where $\texttt{g}(n) = n + 4$.

2. Declare a recursive function `f: int -> int`, where

$$\texttt{f}(n) = 1 + 2 + \cdots + (n-1) + n$$

for $n \geq 0$. (Hint: use two clauses with `0` and `n` as argument patterns.)

State the recursion formula corresponding to the declaration.

3. Declare a recursive function `sum: int*int -> int`, where

$$\texttt{sum}(m, n) = m + (m+1) + (m+2) + \cdots + (m + (n-1)) + (m+n)$$

for $m \geq 0$ and $n \geq 0$. (Hint: use two clauses with `(m,0)` and `(m,n)` as argument patterns.)

Give the recursion formula corresponding to the declaration.

4. The sequence $\mathrm{F}_0, \mathrm{F}_1, \mathrm{F}_2, \ldots$ of Fibonacci numbers is defined by:

$$\begin{array}{lcl} \mathrm{F}_0 & = & 0 \\ \mathrm{F}_1 & = & 1 \\ \mathrm{F}_n & = & \mathrm{F}_{n-1} + \mathrm{F}_{n-2} \end{array}$$

Thus, the first members of the sequence are $0, 1, 1, 2, 3, 5, 8, 13, \ldots$.

Declare an SML function to compute F_n. Use a declaration with three clauses, where the argument patterns correspond to the three cases of the above definition.

5. The following figure gives the first part of Pascal's triangle:

$$
\begin{array}{c}
1 \\
1\ \ 1 \\
1\ \ 2\ \ 1 \\
1\ \ 3\ \ 3\ \ 1 \\
1\ \ 4\ \ 6\ \ 4\ \ 1 \\
\ldots\ldots\ldots\ldots\ldots
\end{array}
$$

The entries of the triangle are called *binomial* coefficients. The k'th binomial coefficient of the n'th row is denoted $\binom{n}{k}$, for $n \geq 0$ and $0 \leq k \leq n$, e.g. $\binom{2}{1} = 2$ and $\binom{4}{2} = 6$. The first and last binomial coefficients, i.e. $\binom{n}{0}$ and $\binom{n}{n}$, of row n are both 1. A binomial coefficient inside a row is the sum of the two binomial coefficients immediately above it. These properties can be expressed as follows:

$$\left(\begin{array}{c} n \\ 0 \end{array} \right) = \left(\begin{array}{c} n \\ n \end{array} \right) = 1$$

and

$$\left(\begin{array}{c} n \\ k \end{array} \right) = \left(\begin{array}{c} n-1 \\ k-1 \end{array} \right) + \left(\begin{array}{c} n-1 \\ k \end{array} \right) \text{ if } n \neq 0,\ k \neq 0, \text{ and } n > k.$$

Declare an SML function `bin: int*int -> int` to compute binomial coefficients.

Hint: You cannot use an argument pattern `(n,n)` for the case $\left(\begin{array}{c} n \\ n \end{array} \right) = 1$, as SML does not allow multiple occurrences of an identifier in a pattern. Use an `if - then - else -` expression instead.