# Solutions for CP Exercise Class 2

**1.** The number of critical references in the statements are seen to be:

$$a : 2, b : 1, c : 2, d : 1, e : 0, f : 2$$

Thus, only statements $b$, $d$, and $e$ can be considered atomic.

[Note that each occurrence of $x$ in $f$ should be counted as critical reference. An optimizing compiler may read it into a register only once, but we cannot assmume that in general.]

**2.** First question: NO. $x := x + 2$ and $x := x + 1$ can be executed in any sequential order, but are not atomic.

Second question: NO. $x := 1$ and $x := 2$ are each atomic, but the order is important.

**3.** The final value of $x$ may range from 2 (!) to 10.

Assuming the two processes to be called $P_1$ and $P_2$, this is how it can get as low as 2:

| | $x$ |
|---|---|
| Initially: | 0 |
| $P_1$ reads 0 from $x$. | 0 |
| $P_2$ increments $x$ four times. | 4 |
| $P_1$ writes 1. | 1 |
| $P_2$ reads 1 from $x$. | 1 |
| $P_1$ increments $x$ four times. | 5 |
| $P_2$ writes 2. | 2 |

It can be shown (using invariants — not imagination!) that this is the smallest result.

**4.** If a variable spans more than one memory word, it has to be accessed using several bus cycles. If these words are accessed by other processors or devices, intermediate memory states may be seen. Even if used only by a single processor, the access to a larger memory area (e.g. a record/structure) is likely to be divided into interruptable steps.

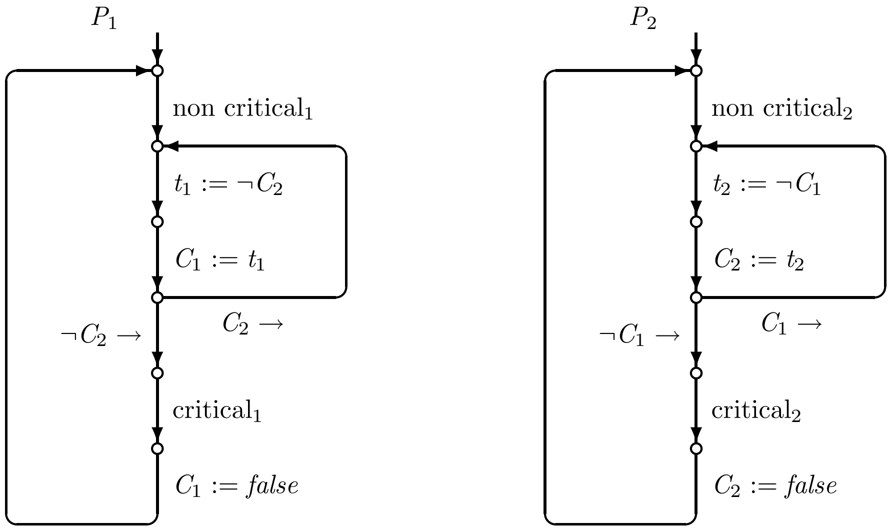
**5.** Usually the least addressable unit of memory is a byte. Thus to change a boolean variable represented as a bit of a byte, it is necessary to read the whole byte into a register, change the bit by masking and finally store the byte againg. This will not be atomic.

### 6. Solution for Share.2

(a) First we note that the statement $C_1 := \neg C_2$ is not atomic since $C_1$ is a shared variable and $C_2$ is a variable read by the other process. Rewriting to atomic actions we the following entry protocol for $P_1$:

**repeat**
  $< t_1 := \neg C_2 >$;
  $< C_1 := C_1 >$;
**until** $< \neg C_2 >$;

Correspondingly for process $P_2$. Transition diagrams:




(b) The algorithm *does not* ensure mutual exclusion. We now see that with the initializations given, an execution in which the atomic actions of the two processes alternate will first set both $C$-s to true and in the next repetition, both variables false after which both processes will enter their critical section!

(c) Since the idea of the algorithm is to set ones flag to the opposite of the flag of the other process, it is tempting to believe that the algoritm will work, if the statements $C_1 := \neg C_2$ and $C_2 := \neg C_1$ are executed atomically. But even assuming these to be atomic, the following execution is possible:

| | $C_1$ | $C_2$ |
|---|---|---|
| Initially: | *false* | *false* |
| $P_2$ executes $nc_2$, its entry-protocol and enters $cs_2$. | *false* | *true* |
| $P_1$ executes $nc_1$ and (atomically) sets $C_1 := \neg C_2$. | *false* | *true* |
| $P_2$ leaves $cs_2$ and executes $C_2 := \mathit{false}$. | *false* | *false* |
| $P_1$ tests $C_2$ and enters $cs_1$. | *false* | *false* |
| $P_2$ executes $nc_2$, enters its entry-protocol, sets $C_2$ to true, finds $C_1$ to be false and enters $cs_2$. | *false* | *true* |

Both processes are now in their critical sections!

The trouble is that the value of $C_2$ that is tested is not the same as the one that $C_1$ is set relative to (and vice versa).

If the **until**-test in $P_1$ is changed to $C_1$ and correspondingly in $P_2$ to $C_2$, the algoritm ensures mutual exclusion given atomic assignments.

To actually prove this we need the following auxiliary invariants:

$$
\begin{array}{lcll}
G_i & \triangleq & \mathit{in}\ cs_i \Rightarrow C_i & \qquad i = 1, 2 \\
I & \triangleq & \neg(C_1 \wedge C_2) &
\end{array}
$$

Now assume that both processes are in their critical sections

$$in\ cs_1 \wedge in\ cs_2$$

According to $G_i$ this would mean that both $C$ variables were true. This, however, would be in contradiction with $I$. Thus, if $G_i$ and $I$ are invariants, mutual exclusion is ensured.

We are now going to show the auxliary invariants. $G_1$ and $G_2$ are seen to be local invariants.

$I$ is shown by an inductive argument:

- Initially $I$ holds since both $C_1$ and $C_2$ are false.
- Since $e_1$ obviously preserves $I$, the only potentially dangerous action in $P_2$ is $a_1$:

  **$a_1$:** This actions will preserve $I$, as $C_1$ is set to the negation of $C_2$ and hence one of them will be false after the action.
- By symmetry, all actions in $P_2$ will also preserve $I$.

Thus $I$ is an invariant of the program.

**7.** In this course, we define a *critical region* to comprise a set of *critical sections* which are pieces of code among which there must be mutual exclusion. In the literature, this distinction is not always made.

**8.** Yes. The only constraint is that there cannot be two processes active within the same region at the same time.

**9.** Yes. For instance there may be a region protecting the use of a printer and a region protecting some shared variables. Critical regions may even overlap.

## 10. Solution for Andrews Ex. 3.3

(a)

```
var l : integer := 1;

process P[i : 1..n] =
  var r : integer := 0;
  repeat
    nc1:  non-critical section_i;
          repeat
              Swap(r, l);
          until r = 1;
    cs1:  critical section_i;
          Swap(r, l);
  forever;
```

We are now going to prove that the above solution does ensure mutual exclusion.

First, we assume that the local variables $r$ are renamed to global variables $r_i$ ($i = 1..n$) that are all initialized to 0;

Next, we prove some auxiliary invariants:

$$\begin{array}{lcll} F_i & \triangleq & \mathit{in}\ \mathit{cs}_i \Rightarrow r_i = 1 & \qquad i = 1..n \\ G & \triangleq & l \in 0,1 & \\ H_i & \triangleq & r_i \in 0,1 & \qquad i = 1..n \end{array}$$

Since $r_i$ is changed only in $P_i$, $F_i$ is a local invariant By induction, $H_i$ and $G$ are easily seen to be invariants since 0 and 1 are the only values being swapped around.

Now we define

$$I \triangleq r_1 + r_2 + \ldots + r_n + l = 1$$

This holds intially and since any of the variables are changed only by atomic swapping of two of them, their sum will remain constant. Therefore, $I$ is an invariant of the program.

Now, if two of the processes $P_i$ and $P_j$ $(j \neq i)$ should be in their critical sections at the same time, $F_i$ and together with $G$ and $H_i$ would give us

$$r_1 + r_2 + \ldots + r_n + l \geq 2$$

contradicting the invariant $I$. Thus, we conclude that this cannot be the case, ie. the algorithm ensures mutual exclusion.

If two or more processes execute $\mathit{Swap}(r_i, l)$ at the same time, one of the will get the "token" first and thereby obtain access to the region. Which of them it is not determined. Thus, the algorithm cannot deadlock nor livlock, but it is *not fair*. Starvation can occur if other process manages to enter the region inbetween a given process attempts to execute $\mathit{Swap}(r_i, l)$.

(b) To avoid memory contention by writing to $l$, its value may be checked before an attempt is made to change it with *Swap*:

**repeat**
    **while** $l = 0$ **do skip**;
    $\mathit{Swap}(r, l)$;
**until** $r = 1$;

This will not effect the proof in (a).

(c) *Not included*

**11.** Most likely. If your computer has an Intel 286 or above compatible processor, there is an atomic *XCHG* instructions that may excange a register (local variable) with a memory location (shared variable). Other (selected) instructions may be performed atomically by preceding them with the *LOCK* instruction.

**12.** See solution to point 10. above.

**13.** See solution to point 10. above.