MRH 10.12.05

1 Problem 1

1.1) We prove that L_2 is not regular by using the pumping lemma (Theorem 4.1):

- Assume L_2 is regular.
- Let n be given by the pumping lemma.
- Let $w = 0^n1^n$. Observe that $w \in L_2$ and $|w| \geq n$.
- According to the pumping lemma, w can be written as $w = xyz$ where $|xy| \leq n$ and $|y| \geq 1$. It follows that $xy = 0^p$ for some $p \leq n$ and $y = 0^j$ for some j, $1 \leq j \leq p$.
- According to the pumping lemma, $xy^kz \in L_2$ for all $k \geq 0$. Consider the case $k = 2$:

  $$xy^2z = 0^{n+j}1^n$$

  Since $j > 0$, $xy^2z \notin L_2$. The only assumption made is that L_2 is regular, thus we conclude that L_2 is non-regular.

1.2) Yes, L_2 is defined by a CFG with the following three production rules:

$$S \rightarrow \epsilon \qquad S \rightarrow 0S1 \qquad S \rightarrow 1S1$$

2 Problem 2

2.1) We prove that L_1 is not regular by using the pumping lemma (Theorem 4.1):

- Assume L_1 is regular.
- Let n be given by the pumping lemma.
- Let $w = a^nb^{n+1}$. Observe that $w \in L_1$ and $|w| \geq n$.
- According to the pumping lemma, w can be written as $w = xyz$ where $|xy| \leq n$ and $|y| \geq 1$. It follows that $xy = a^p$ for some $p \leq n$ and $y = a^j$ for some j, $1 \leq j \leq p$.
- According to the pumping lemma, $xy^kz \in L_1$ for all $k \geq 0$. Consider the case $k = 2$:

  $$xy^2z = a^{p-j}a^{2j}a^{n-p}b^{n+1} = a^{p-j+2j+n-p}b^{n+1} = a^{j+n}b^{n+1}$$

  Since $j > 0$, $xy^2z \notin L_1$. The only assumption made is that L_1 is regular, thus we conclude that L_1 is non-regular.

2.2) We can write *start*(L) as $L\Sigma^*$. Since Σ^* is a regular language and regular languages are closed under concatenation, *start*(L) is regular.

3 Problem 3

$G = (V, \Sigma, R, S)$, where $V = \{S, L\}$, $\Sigma = \{(,), a, \text{“,”}\}$, and R contains the rules:

$$\begin{array}{lcl@{\qquad\qquad}lcl} S & \rightarrow & a & L & \rightarrow & S \\ S & \rightarrow & (L) & L & \rightarrow & L, S \end{array}$$

4 Problem 4

Two leftmost derivations of the string $w\,c\,a\,;\,a$ are:

$$\begin{array}{c} S \Rightarrow S\,;\,S \Rightarrow w\,c\,S\,;\,S \Rightarrow w\,c\,a\,;\,S \Rightarrow w\,c\,a\,;\,a \\ S \Rightarrow w\,c\,S \Rightarrow w\,c\,S\,;\,S \Rightarrow w\,c\,a\,;\,S \Rightarrow w\,c\,a\,;\,a \end{array}$$

Since there are two different leftmost derivations, the grammar is ambiguous.