Informatik and Mathematical Modelling 10/8 04
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02140 Languages and Parsing
Programming Exercise 1

1. Declare a function `g: int -> int`, where $\texttt{g}(n) = n + 4$.

2. Declare a recursive function `f: int -> int`, where

$$\texttt{f}(n) = 1 + 2 + \cdots + (n-1) + n$$

   for $n \geq 0$. (Hint: use two clauses with `0` and `n` as argument patterns.)

   State the recursion formula corresponding to the declaration.

3. Declare a recursive function `sum: int*int -> int`, where

$$\texttt{sum}(m, n) = m + (m+1) + (m+2) + \cdots + (m + (n-1)) + (m+n)$$

   for $m \geq 0$ and $n \geq 0$. (Hint: use two clauses with `(m,0)` and `(m,n)` as argument patterns.)

   Give the recursion formula corresponding to the declaration.

4. The sequence $\mathrm{F}_0, \mathrm{F}_1, \mathrm{F}_2, \ldots$ of Fibonacci numbers is defined by:

$$\begin{array}{rcl} \mathrm{F}_0 & = & 0 \\ \mathrm{F}_1 & = & 1 \\ \mathrm{F}_n & = & \mathrm{F}_{n-1} + \mathrm{F}_{n-2} \end{array}$$

   Thus, the first members of the sequence are $0, 1, 1, 2, 3, 5, 8, 13, \ldots$.

   Declare an SML function to compute F_n. Use a declaration with three clauses, where the argument patterns correspond to the three cases of the above definition.

5. The following figure gives the first part of Pascal's triangle:

$$
\begin{array}{c}
1 \\
1 \quad 1 \\
1 \quad 2 \quad 1 \\
1 \quad 3 \quad 3 \quad 1 \\
1 \quad 4 \quad 6 \quad 4 \quad 1 \\
\cdots\cdots\cdots\cdots
\end{array}
$$

The entries of the triangle are called *binomial* coefficients. The k'th binomial coefficient of the n'th row is denoted $\binom{n}{k}$, for $n \geq 0$ and $0 \leq k \leq n$, e.g. $\binom{2}{1} = 2$ and $\binom{4}{2} = 6$. The first and last binomial coefficients, i.e. $\binom{n}{0}$ and $\binom{n}{n}$, of row n are both 1. A binomial coefficient inside a row is the sum of the two binomial coefficients immediately above it. These properties can be expressed as follows:

$$\binom{n}{0} = \binom{n}{n} = 1$$

and

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k} \quad \text{if } n \neq 0,\ k \neq 0, \text{ and } n > k.$$

Declare an SML function `bin: int*int -> int` to compute binomial coefficients.

Hint: You cannot use an argument pattern `(n,n)` for the case $\binom{n}{n} = 1$, as SML does not allow multiple occurrences of an identifier in a pattern. Use an `if - then - else -` expression instead.

6. Determine a type for each of the expressions:

```
(Math.pi, fact ~1)
fact(fact 4)
power(Math.pi,fact 2)
(power, fact)
```

7. Consider the declarations:

```
val a = 5;

fun f a = a + 1;

fun g b = (f b) + a;
```

Find the environment obtained from these declarations and give evaluations for the expressions `f 3` and `g 3`.