The following problems are selected examination questions from 1999 and 2000 in the course 49409 Automata, Languages and Computability.

Problem 1

Let

$$L_2 = \Big\{ w \in \{0,1\}^* \ : \ \text{there exists } x \in \{0,1\}^*, k \geq 0 : \ w = x1^k \text{ and the length of } x \text{ is } k \Big\}$$

For example, $0011, 101111$ and 1111 are all elements in L_2, while 000011 and 1110111 are not elements in L_2.

1. Prove that L_2 is not regular.
2. Is L_2 context-free? Justify your answer.

Problem 2

In this problem, we will consider languages over the alphabet $\Sigma = \{a, b\}$.

1. Let $L_1 = \{w : n_a(w) < n_b(w)\}$, where $n_a(w)$ and $n_b(w)$ denote the number of a's and b's in w, respectively. Is L_1 regular? Justify your answer.
2. Let L be a regular language over Σ and let $start(L)$ be the language defined as

   $$start(L) = \{x : x = yz \text{ for some } y \in L \text{ and } z \in \Sigma^* \}.$$

   Is $start(L)$ regular? Justify your answer.

Problem 3

In mini-LISP, a well-formed list is either the single "atom" a, or a sequence of one or more well-formed lists separated by commas and enclosed in parentheses. For example, "a" is a list, as are "(a)" and "$(a, a, ((a)))$", but "$((), aa)$" is not (for two reasons).
Give a context-free grammar defining mini-LISP lists.

Problem 4

Let $G = (V, \Sigma, R, S)$ be a context-free grammar, where $\Sigma = \{w, c, \{, \}, ;, a\}$, $V = \{S\}$, and R contains the rules:

$$\begin{aligned} S &\rightarrow w\,c\,S \\ S &\rightarrow \{\,S\,\} \\ S &\rightarrow S\,;\,S \\ S &\rightarrow a \end{aligned}$$

(This grammar models some statements in C-like languages, where w represents **while**, c represents a Boolean condition, $\{\ldots\}$ represents a compound statement, and a represents a non-**while** statement.)

Is G ambiguous?