

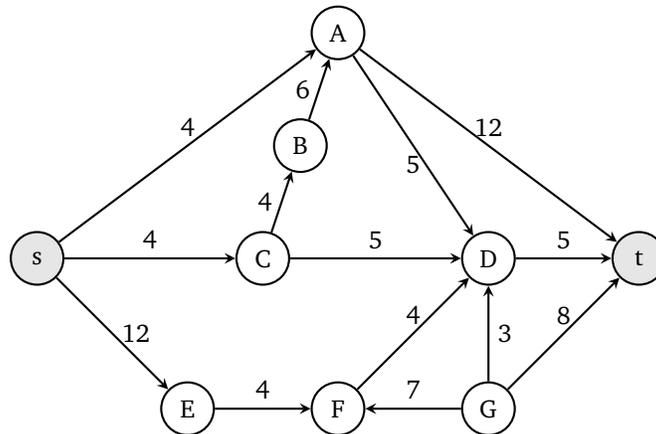
Reading material

No reading material for this week.

Exercises

1 [w] **Maximum flow and minimum cut** Consider the network N below with capacities on the edges.

Give a maximum flow from s to t in the network N (write the flow for each edge), give the value of the flow, and give a minimum $s - t$ cut (give the partition of the vertices).

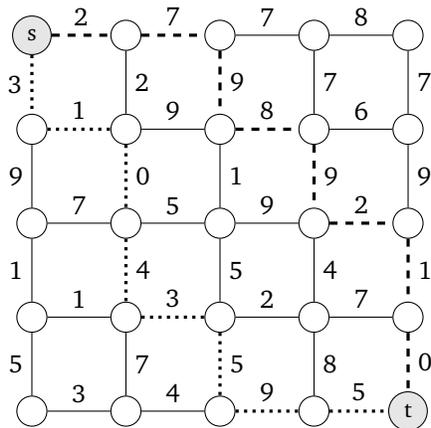


2 [w] **Min-cut max-flow** Solve KT 7.3.

3 **Manhattan Tourists 1** You are helping the tourist guide company "Manhattan Tourists", that are arranging guided tours of the city. They want to find a walk between two points on the map that is both interesting and short. The map is a square grid graph. The square grid graph has n rows with n nodes in each row. Let node $v_{i,j}$ denote the j th node on row i . For $1 \leq i < n$ and for $1 \leq j \leq n$ node $v_{i,j}$ is connected to $v_{i+1,j}$. And for $1 \leq i \leq n$ and for $1 \leq j < n$ node $v_{i,j}$ is connected to $v_{i,j+1}$. The edges have non-negative edge weights that indicate how interesting that street is. See the graph below for an example of a 5×5 grid graph.

They want to find a short interesting walk from the upper left corner ($s = v_{1,1}$) to the lower right corner ($t = v_{n,n}$). More precisely, they want to find a path with the possible smallest number of edges, and among all paths with this number of edges they want the path with the maximum weight (the weight of a path is the sum of weights on the path).

All shortest paths have $2n - 2$ edges and go from s to t by walking either down or right in each step. In the example below two possible shortest paths (of length 8) are indicated. The dashed path has weight 38 and the dotted path has weight 30.



3.1 Let $W[i, j]$ be the maximal weight you can get when walking from s to $v_{i,j}$ walking either down or right in each step. Let $D[i, j]$ be the weight of the edge going down from $v_{i,j}$ and let $R[i, j]$ be the weight of the edge going right from $v_{i,j}$.

Fill out the table for the example above:

$W_{i,j}$	1	2	3	4	5
1					
2					
3					
4					
5					

3.2 Give a recurrence for $W[i, j]$. Remember to argue that it is correct.

3.3 Write pseudocode for an algorithm based on dynamic programming and the recurrence from Question 4.2 that computes the maximum weight a shortest path can have in the grid graph.

Analyze the space usage and running time of your algorithm in terms of n .

4 **Manhattan Tourists 2** The tourist guide company "Manhattan Tourists" are arranging several tours of the city at the same time. As in Question 4 the map is a square grid graph, but now some of the streets are blocked, so they can't walk on these. They want to arrange several tours from s to t that do not overlap.

4.1 The company wants to arrange two different tours from s to t that are disjoint in the sense that they don't share any streets. In this question you don't care whether the tours are interesting (the weight on the path does not matter). The tours should still only walk down and right in each step and you cannot walk on the blocked streets. Give an efficient algorithm that determines whether it is possible to make two such tours. Analyze the asymptotic running time of your algorithm in terms of n . Remember to argue that your algorithm is correct.

4.2 It turned out that the tours were not different enough. The company now wants to arrange two different tours from s to t that are disjoint in the sense that the two tours only meet in s and t . In this question

you don't care whether the tours are interesting (the weight on the path does not matter). The tours should still only walk down and right in each step and you cannot walk on the blocked streets. Give an algorithm that determines whether it is possible to make two such tours. Analyze the asymptotic running time of your algorithm in terms of n . Remember to argue that your algorithm is correct.

- 5 **CSES: Police Chase** Solve <https://cses.fi/problemset/task/1695>