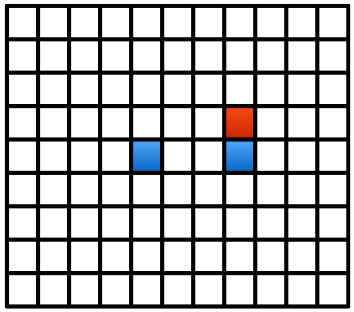
Subset Sum

- Recurrence:

$$\text{OPT}(i, w) = \begin{cases} \text{OPT}(i-1, w) & \text{if } w < w_i \\ \max(\text{OPT}(i-1, w), w_i + \text{OPT}(i-1, w-w_i)) & \text{otherwise} \end{cases}$$

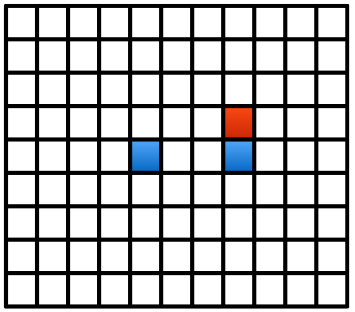


Subset Sum

- Recurrence:

$$\text{OPT}(i,w) = \begin{cases} \text{OPT}(i-1,w) & \text{if } w < w_i \\ \max(\text{OPT}(i-1,w), w_i + \text{OPT}(i-1,w-w_i)) & \text{otherwise} \end{cases}$$

```
Subset-Sum(n,W)
  Array M[0…n][0…W]
  Initialize M[0][w] = 0 for each w = 0,1,…,W
  for i = 1 to n
    for w = 0 to W
      if w < wi
        M[i][w] = M[i-1][w]
      else
        M[i][w] = max(M[i-1][w], wi + M[i-1][w-wi])
  return M[n,W]
```



Subset Sum

- Recurrence:

$$\text{OPT}(i, w) = \begin{cases} \text{OPT}(i-1, w) & \text{if } w < w_i \\ \max(\text{OPT}(i-1, w), w_i + \text{OPT}(i-1, w-w_i)) & \text{otherwise} \end{cases}$$

- Example
  - {1, 2, 5, 8, 9} and W = 12

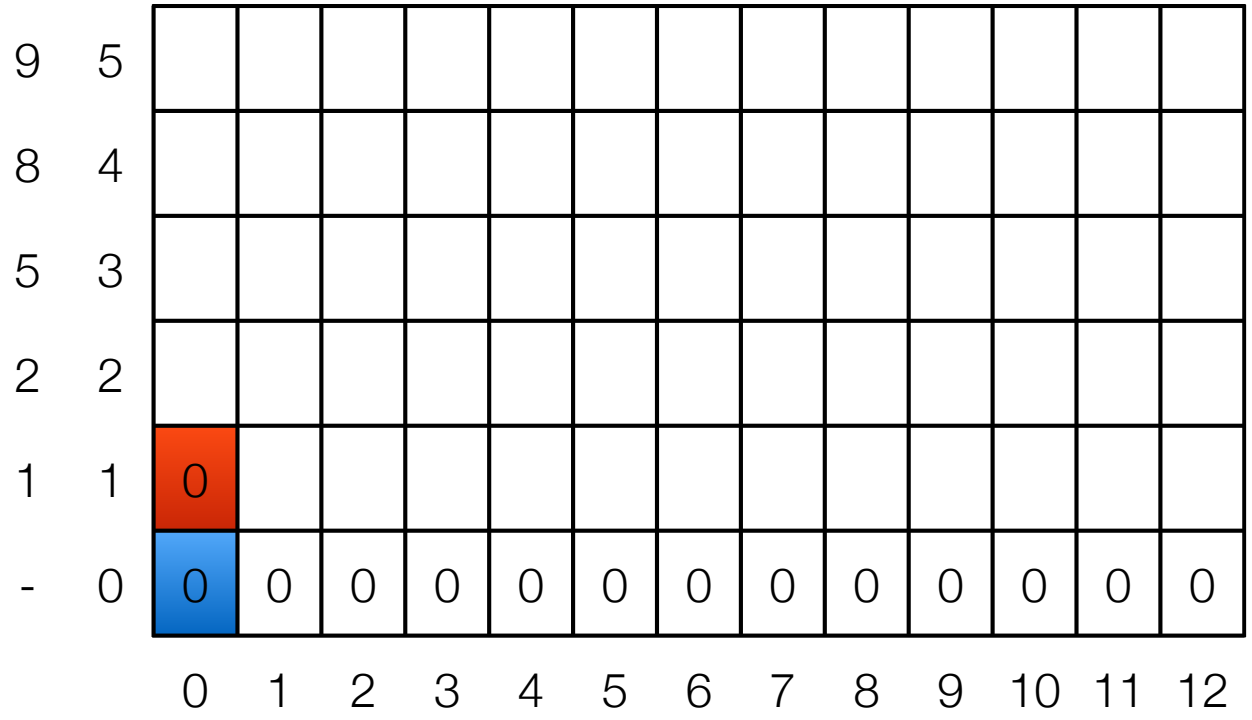
9	5												
8	4												
5	3												
2	2												
1	1												
-	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	1	2	3	4	5	6	7	8	9	10	11	12

Subset Sum

- Recurrence:

$$\text{OPT}(i, w) = \begin{cases} \text{OPT}(i-1, w) & \text{if } w < w_i \\ \max(\text{OPT}(i-1, w), w_i + \text{OPT}(i-1, w-w_i)) & \text{otherwise} \end{cases}$$

- Example
  - {1, 2, 5, 8, 9} and W = 12

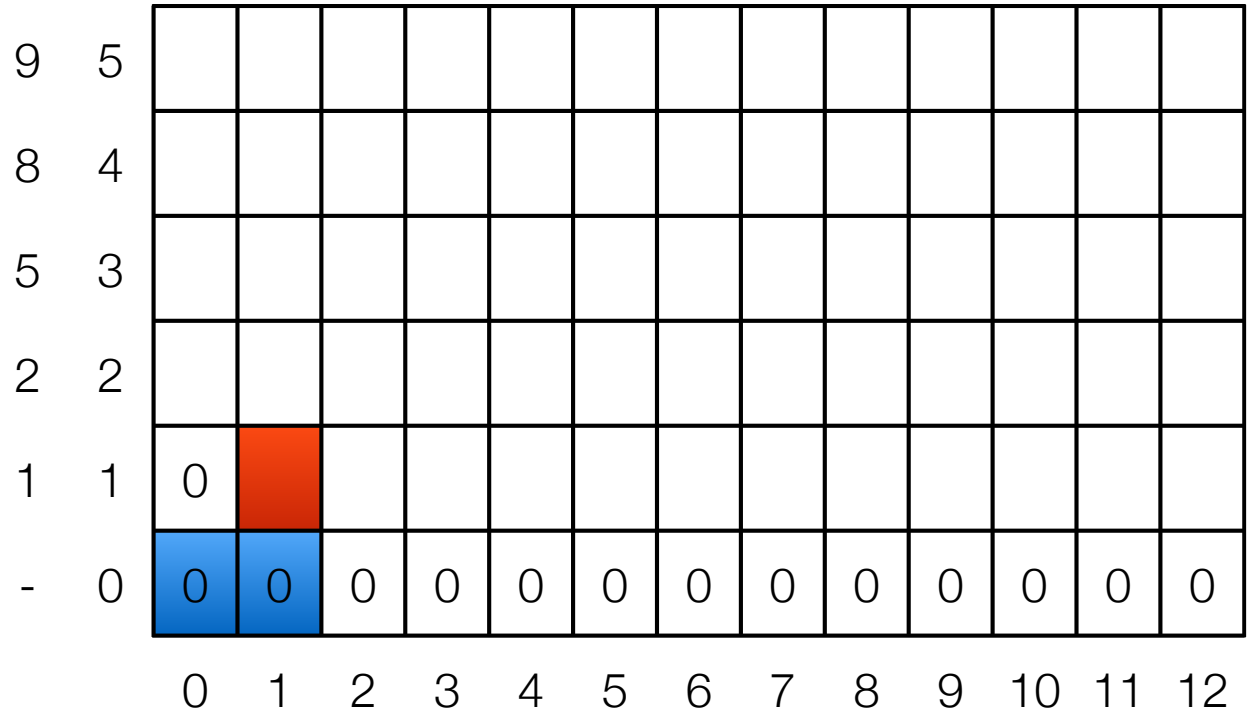


Subset Sum

- Recurrence:

$$\text{OPT}(i, w) = \begin{cases} \text{OPT}(i-1, w) & \text{if } w < w_i \\ \max(\text{OPT}(i-1, w), w_i + \text{OPT}(i-1, w-w_i)) & \text{otherwise} \end{cases}$$

- Example
  - {1, 2, 5, 8, 9} and W = 12



Subset Sum

- Recurrence:

$$\text{OPT}(i, w) = \begin{cases} \text{OPT}(i-1, w) & \text{if } w < w_i \\ \max(\text{OPT}(i-1, w), w_i + \text{OPT}(i-1, w-w_i)) & \text{otherwise} \end{cases}$$

- Example
  - {1, 2, 5, 8, 9} and W = 12

9	5												
8	4												
5	3												
2	2												
1	1	0	1										
-	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	1	2	3	4	5	6	7	8	9	10	11	12

Subset Sum

- Recurrence:

$$\text{OPT}(i, w) = \begin{cases} \text{OPT}(i-1, w) & \text{if } w < w_i \\ \max(\text{OPT}(i-1, w), w_i + \text{OPT}(i-1, w-w_i)) & \text{otherwise} \end{cases}$$

- Example
  - {1, 2, 5, 8, 9} and W = 12

9	5												
8	4												
5	3												
2	2												
1	1	0	1	1									
-	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	1	2	3	4	5	6	7	8	9	10	11	12

Subset Sum

- Recurrence:

$$\text{OPT}(i, w) = \begin{cases} \text{OPT}(i-1, w) & \text{if } w < w_i \\ \max(\text{OPT}(i-1, w), w_i + \text{OPT}(i-1, w-w_i)) & \text{otherwise} \end{cases}$$

- Example
  - {1, 2, 5, 8, 9} and W = 12

9	5												
8	4												
5	3												
2	2												
1	1	0	1	1	1	1	1	1	1	1	1	1	1
-	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	1	2	3	4	5	6	7	8	9	10	11	12

Subset Sum

- Recurrence:

$$\text{OPT}(i, w) = \begin{cases} \text{OPT}(i-1, w) & \text{if } w < w_i \\ \max(\text{OPT}(i-1, w), w_i + \text{OPT}(i-1, w-w_i)) & \text{otherwise} \end{cases}$$

- Example
  - {1, 2, 5, 8, 9} and W = 12

9	5												
8	4												
5	3												
2	2	0											
1	1	0	1	1	1	1	1	1	1	1	1	1	1
-	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	1	2	3	4	5	6	7	8	9	10	11	12

Subset Sum

- Recurrence:

$$\text{OPT}(i, w) = \begin{cases} \text{OPT}(i-1, w) & \text{if } w < w_i \\ \max(\text{OPT}(i-1, w), w_i + \text{OPT}(i-1, w-w_i)) & \text{otherwise} \end{cases}$$

- Example
  - {1, 2, 5, 8, 9} and W = 12

		0	1	2	3	4	5	6	7	8	9	10	11	12
9	5													
8	4													
5	3													
2	2	0	1											
1	1	0	1	1	1	1	1	1	1	1	1	1	1	1
-	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Subset Sum

- Recurrence:

$$\text{OPT}(i, w) = \begin{cases} \text{OPT}(i-1, w) & \text{if } w < w_i \\ \max(\text{OPT}(i-1, w), w_i + \text{OPT}(i-1, w-w_i)) & \text{otherwise} \end{cases}$$

- Example
  - {1, 2, 5, 8, 9} and W = 12

9	5												
8	4												
5	3												
2	2	0	1										
1	1	0	1	1	1	1	1	1	1	1	1	1	1
-	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	1	2	3	4	5	6	7	8	9	10	11	12

Subset Sum

- Recurrence:

$$\text{OPT}(i, w) = \begin{cases} \text{OPT}(i-1, w) & \text{if } w < w_i \\ \max(\text{OPT}(i-1, w), w_i + \text{OPT}(i-1, w-w_i)) & \text{otherwise} \end{cases}$$

- Example
  - {1, 2, 5, 8, 9} and W = 12

9	5												
8	4												
5	3												
2	2	0	1	2									
1	1	0	1	1	1	1	1	1	1	1	1	1	1
-	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	1	2	3	4	5	6	7	8	9	10	11	12

Subset Sum

- Recurrence:

$$\text{OPT}(i, w) = \begin{cases} \text{OPT}(i-1, w) & \text{if } w < w_i \\ \max(\text{OPT}(i-1, w), w_i + \text{OPT}(i-1, w-w_i)) & \text{otherwise} \end{cases}$$

- Example
  - {1, 2, 5, 8, 9} and W = 12

9	5												
8	4												
5	3												
2	2	0	1	2	3								
1	1	0	1	1	1	1	1	1	1	1	1	1	1
-	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	1	2	3	4	5	6	7	8	9	10	11	12

Subset Sum

- Recurrence:

$$\text{OPT}(i, w) = \begin{cases} \text{OPT}(i-1, w) & \text{if } w < w_i \\ \max(\text{OPT}(i-1, w), w_i + \text{OPT}(i-1, w-w_i)) & \text{otherwise} \end{cases}$$

- Example
  - {1, 2, 5, 8, 9} and W = 12

9	5												
8	4												
5	3												
2	2	0	1	2	3	3							
1	1	0	1	1	1	1	1	1	1	1	1	1	1
-	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	1	2	3	4	5	6	7	8	9	10	11	12

Subset Sum

- Recurrence:

$$\text{OPT}(i, w) = \begin{cases} \text{OPT}(i-1, w) & \text{if } w < w_i \\ \max(\text{OPT}(i-1, w), w_i + \text{OPT}(i-1, w-w_i)) & \text{otherwise} \end{cases}$$

- Example
  - {1, 2, 5, 8, 9} and W = 12

9	5												
8	4												
5	3												
2	2	0	1	2	3	3	3	3	3	3	3	3	3
1	1	0	1	1	1	1	1	1	1	1	1	1	1
-	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	1	2	3	4	5	6	7	8	9	10	11	12

Subset Sum

- Recurrence:

$$\text{OPT}(i, w) = \begin{cases} \text{OPT}(i-1, w) & \text{if } w < w_i \\ \max(\text{OPT}(i-1, w), w_i + \text{OPT}(i-1, w-w_i)) & \text{otherwise} \end{cases}$$

- Example
  - {1, 2, 5, 8, 9} and W = 12

9	5												
8	4												
5	3	0	1	2	3	3	5						
2	2	0	1	2	3	3	3	3	3	3	3	3	3
1	1	0	1	1	1	1	1	1	1	1	1	1	1
-	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	1	2	3	4	5	6	7	8	9	10	11	12

Subset Sum

- Recurrence:

$$\text{OPT}(i, w) = \begin{cases} \text{OPT}(i-1, w) & \text{if } w < w_i \\ \max(\text{OPT}(i-1, w), w_i + \text{OPT}(i-1, w-w_i)) & \text{otherwise} \end{cases}$$

- Example
  - {1, 2, 5, 8, 9} and W = 12

9	5												
8	4												
5	3	0	1	2	3	3	5	6					
2	2	0	1	2	3	3	3	3	3	3	3	3	3
1	1	0	1	1	1	1	1	1	1	1	1	1	1
-	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	1	2	3	4	5	6	7	8	9	10	11	12

Subset Sum

- Recurrence:

$$\text{OPT}(i, w) = \begin{cases} \text{OPT}(i-1, w) & \text{if } w < w_i \\ \max(\text{OPT}(i-1, w), w_i + \text{OPT}(i-1, w-w_i)) & \text{otherwise} \end{cases}$$

- Example
  - {1, 2, 5, 8, 9} and W = 12

9	5												
8	4												
5	3	0	1	2	3	3	5	6	7	8	8	8	8	8
2	2	0	1	2	3	3	3	3	3	3	3	3	3	3
1	1	0	1	1	1	1	1	1	1	1	1	1	1	1
-	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	1	2	3	4	5	6	7	8	9	10	11	12

Subset Sum

- Recurrence:

$$\text{OPT}(i, w) = \begin{cases} \text{OPT}(i-1, w) & \text{if } w < w_i \\ \max(\text{OPT}(i-1, w), w_i + \text{OPT}(i-1, w-w_i)) & \text{otherwise} \end{cases}$$

- Example
  - {1, 2, 5, 8, 9} and W = 12

9	5												
8	4	0	1	2	3	3	5	6	7	8	9		
5	3	0	1	2	3	3	5	6	7	8	8	8	8
2	2	0	1	2	3	3	3	3	3	3	3	3	3
1	1	0	1	1	1	1	1	1	1	1	1	1	1
-	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	1	2	3	4	5	6	7	8	9	10	11	12

Subset Sum

- Recurrence:

$$\text{OPT}(i, w) = \begin{cases} \text{OPT}(i-1, w) & \text{if } w < w_i \\ \max(\text{OPT}(i-1, w), w_i + \text{OPT}(i-1, w-w_i)) & \text{otherwise} \end{cases}$$

- Example
  - {1, 2, 5, 8, 9} and W = 12

9	5	0	1	2	3	3	5	6	7	8	9	10	11	12
8	4	0	1	2	3	3	5	6	7	8	9	10	11	11
5	3	0	1	2	3	3	5	6	7	8	8	8	8	8
2	2	0	1	2	3	3	3	3	3	3	3	3	3	3
1	1	0	1	1	1	1	1	1	1	1	1	1	1	1
-	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	1	2	3	4	5	6	7	8	9	10	11	12