Dynamic Programming

Algorithm Design 6.1, 6.2, 6.4

Thank you to Kevin Wayne for inspiration to slides

Applications

- In class (today and next time)

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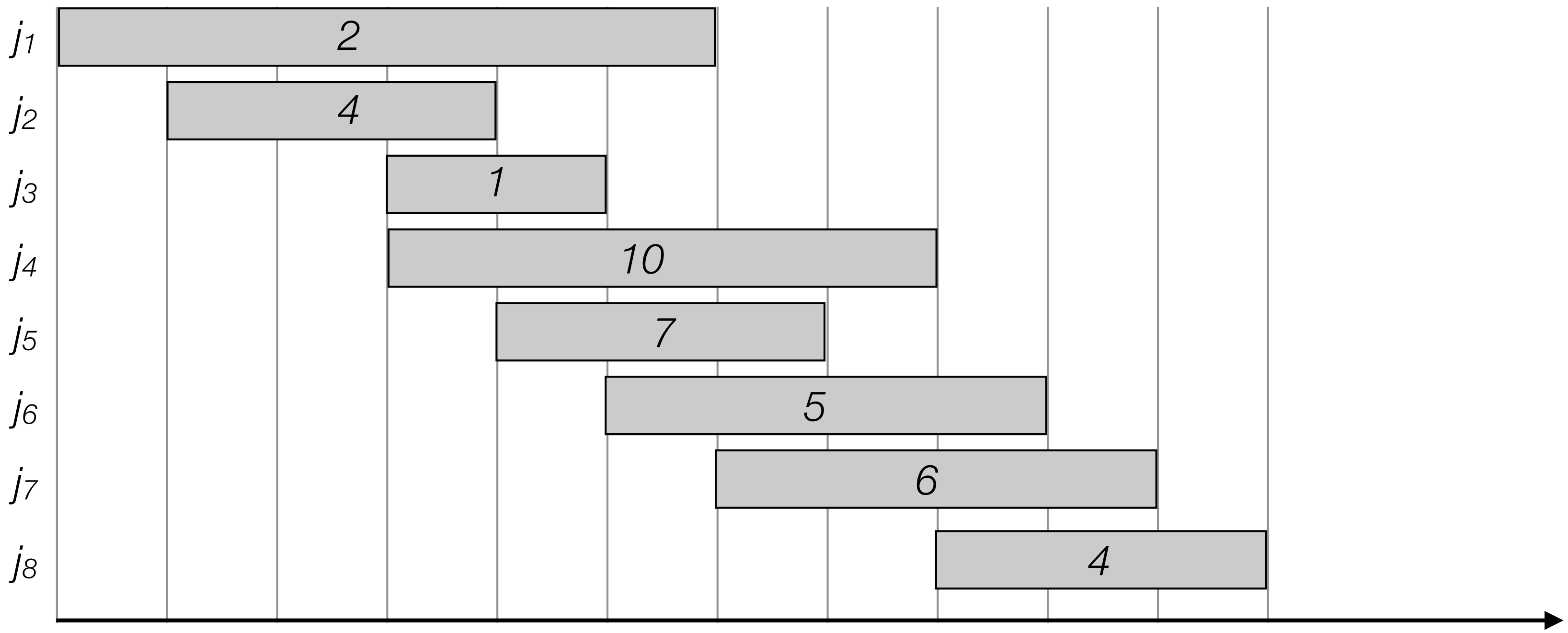
  - Weighted interval scheduling
    - Set of weighted intervals with start and finishing times
    - Goal: find maximum weight subset of non-overlapping intervals



Applications

- Today and next time
  - Weighted interval scheduling
  - Subset Sum and Knapsack
    - Set of items each having a weight and a value
    - Knapsack with a bounded capacity
    - Goal: fill knapsack so as to maximise the total value.

value	10	8	2	5	15	4
weight	2	3	1	2	5	4

Capacity 8

Applications

- Today and next time
  - Weighted interval scheduling
  - Subset Sum and Knapsack
  - Sequence alignment
    - Given two strings A and B how many edits (insertions, deletions, relabelings) is needed to turn A into B?

```
A C A A G T C            A C A A - G T C

- C A T G T -            - C A - T G T -

1 mismatch, 2 gaps        0 mismatches, 4 gaps
```

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  - Can be used when the problem have "optimal substructure":
    - ✦ *Solution can be constructed from optimal solutions to subproblems*
    - ✦ *Use dynamic programming when subproblems overlap.*

Computing Fibonacci numbers

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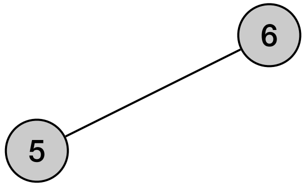

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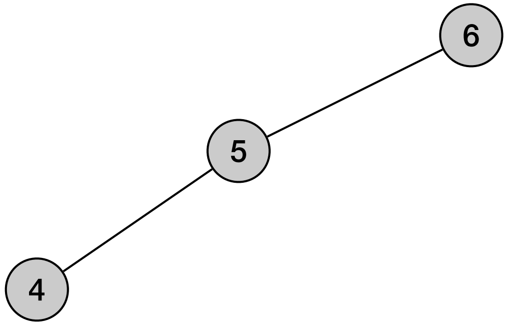

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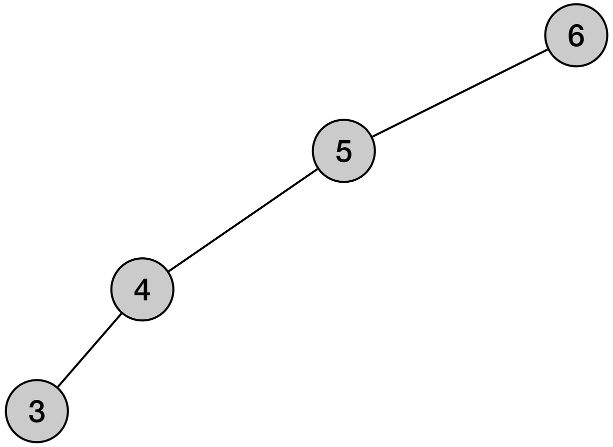

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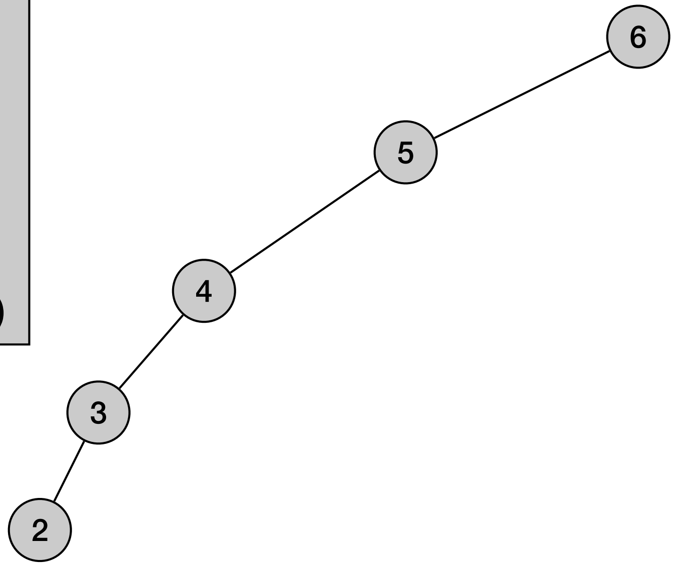

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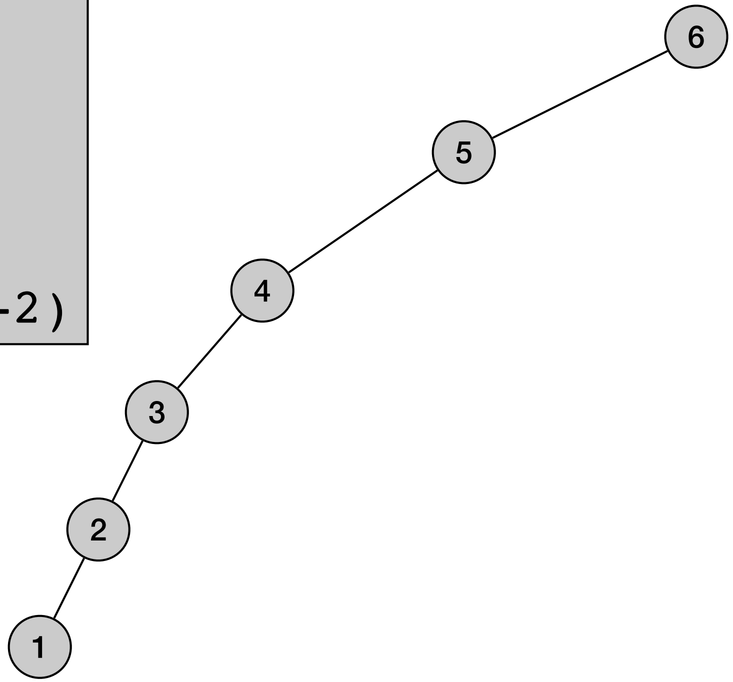

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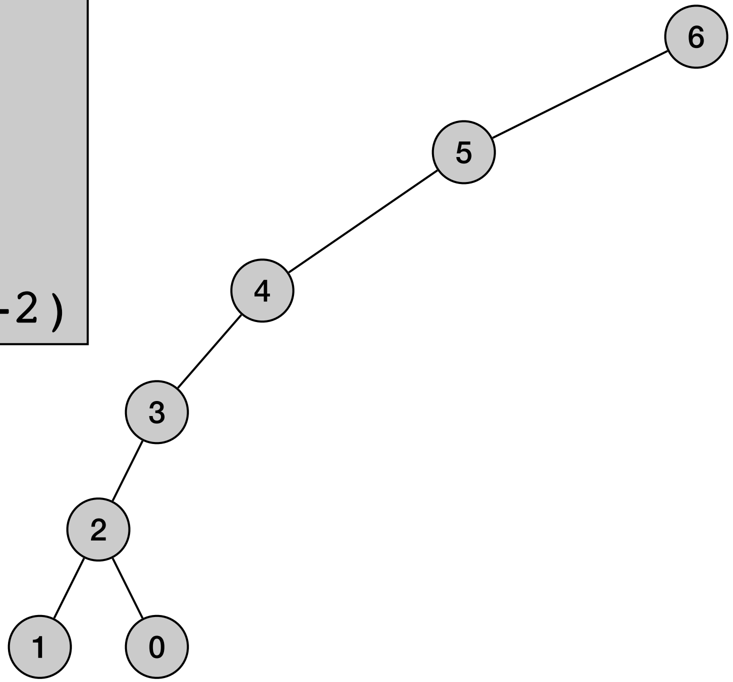

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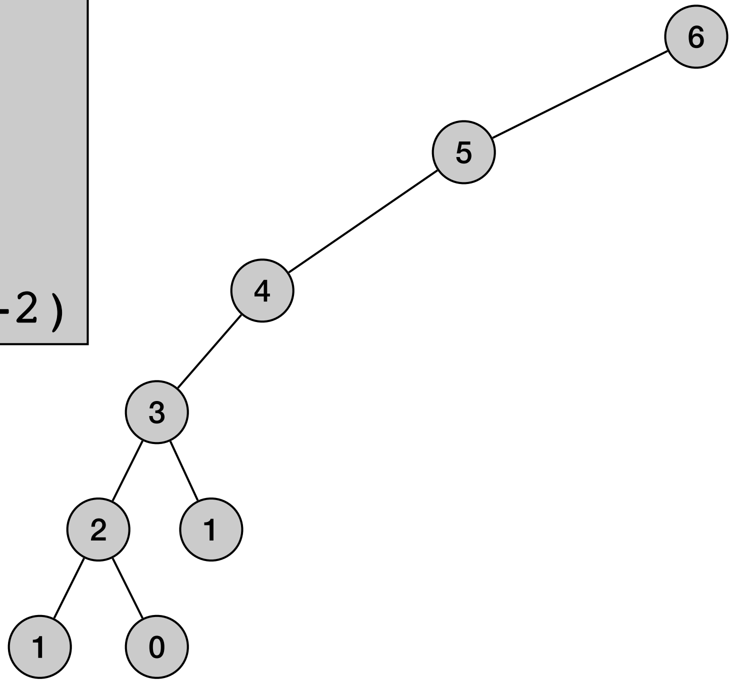

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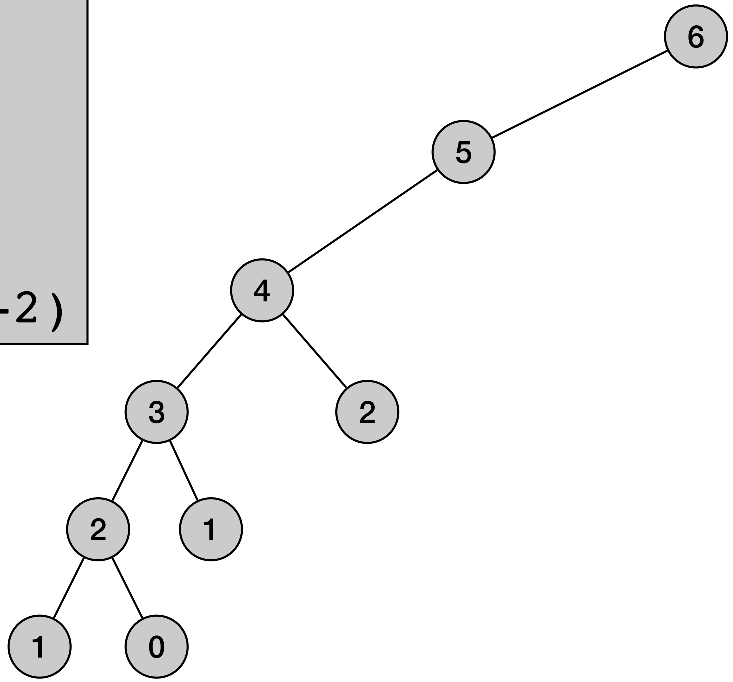

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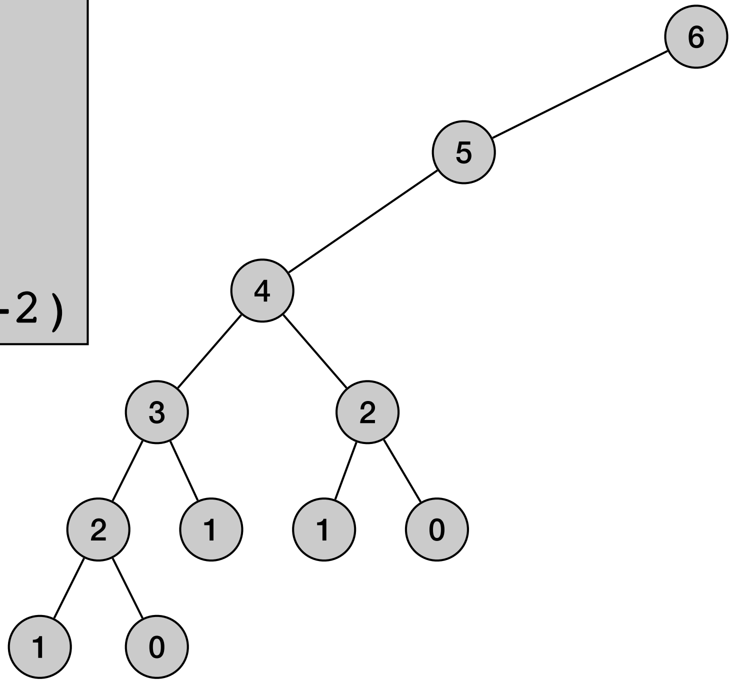

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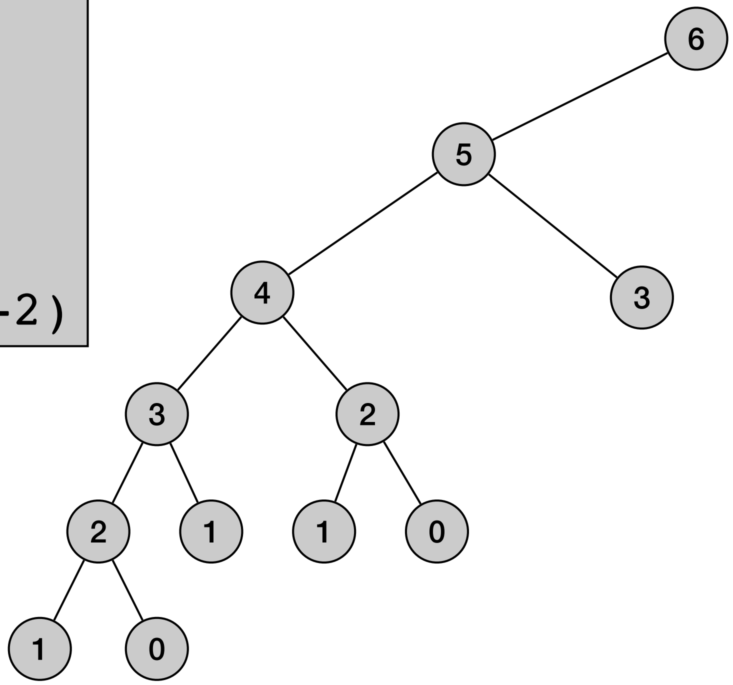

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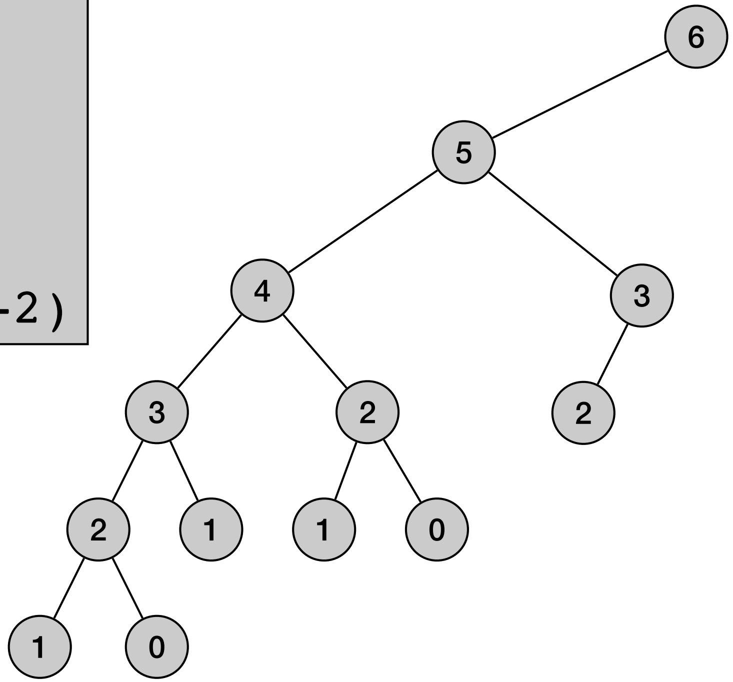

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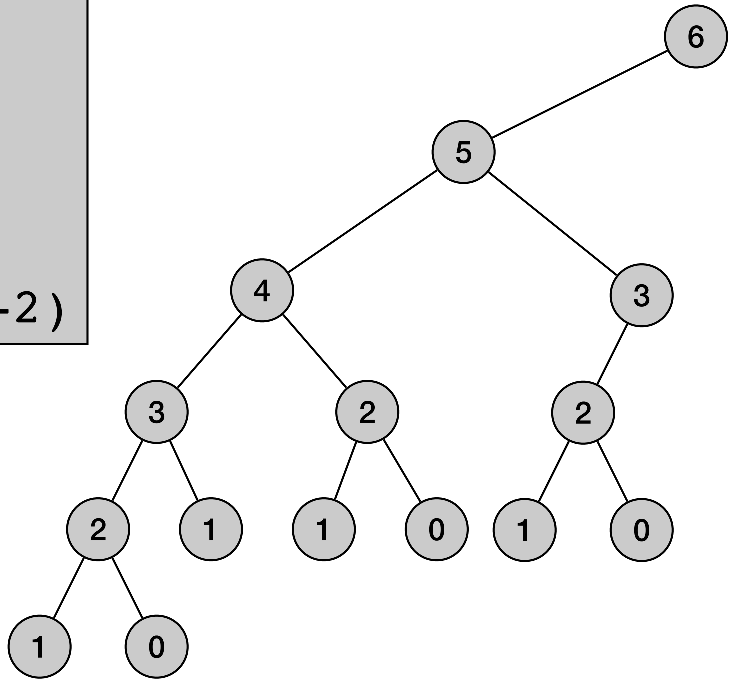

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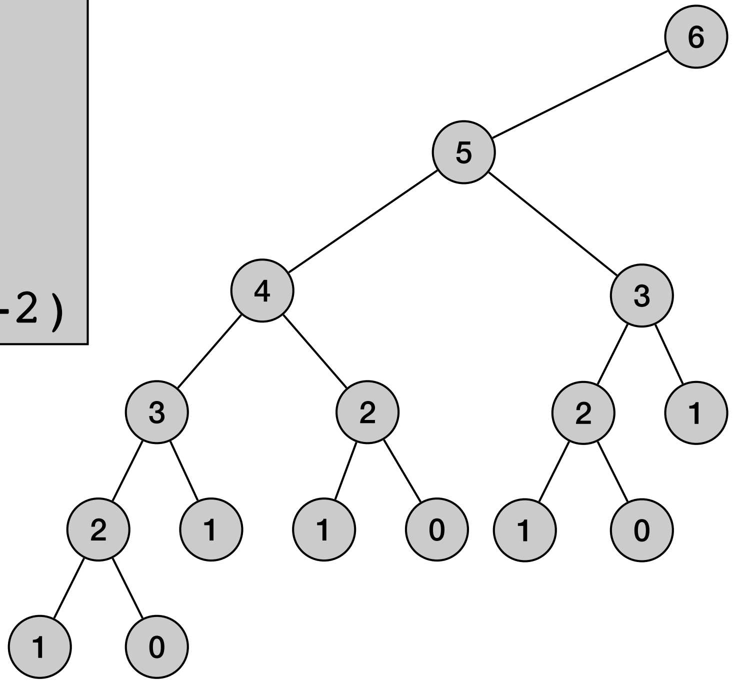

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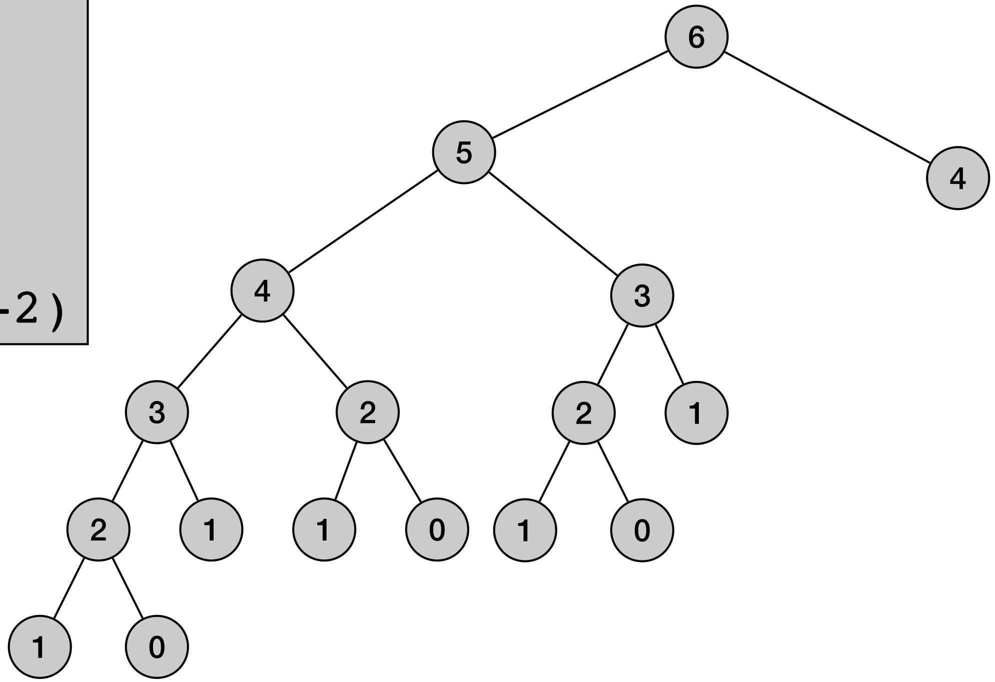

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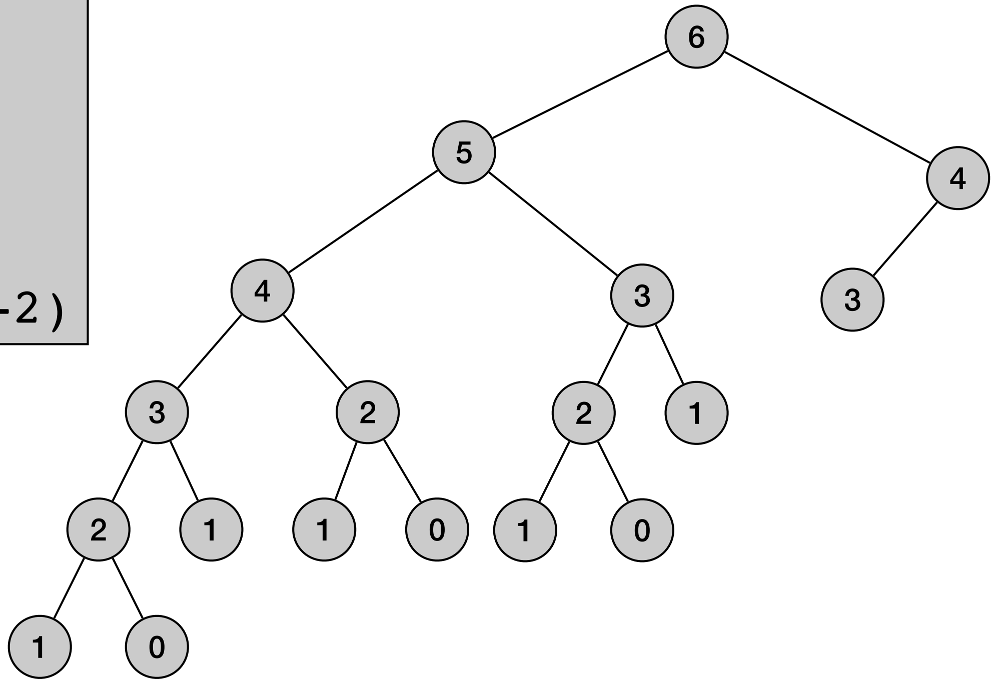

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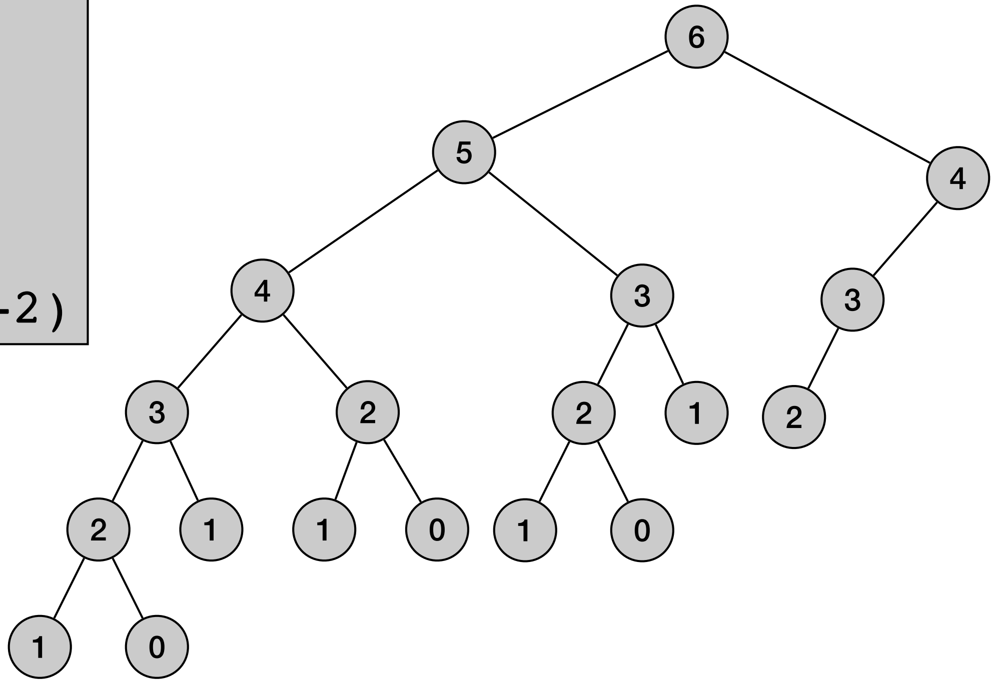

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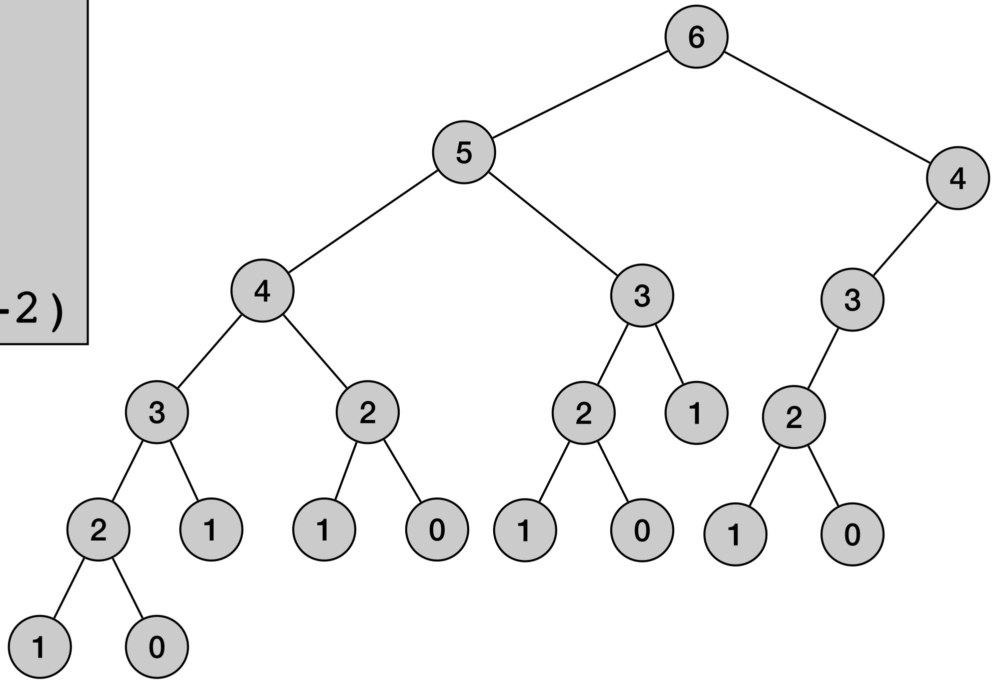

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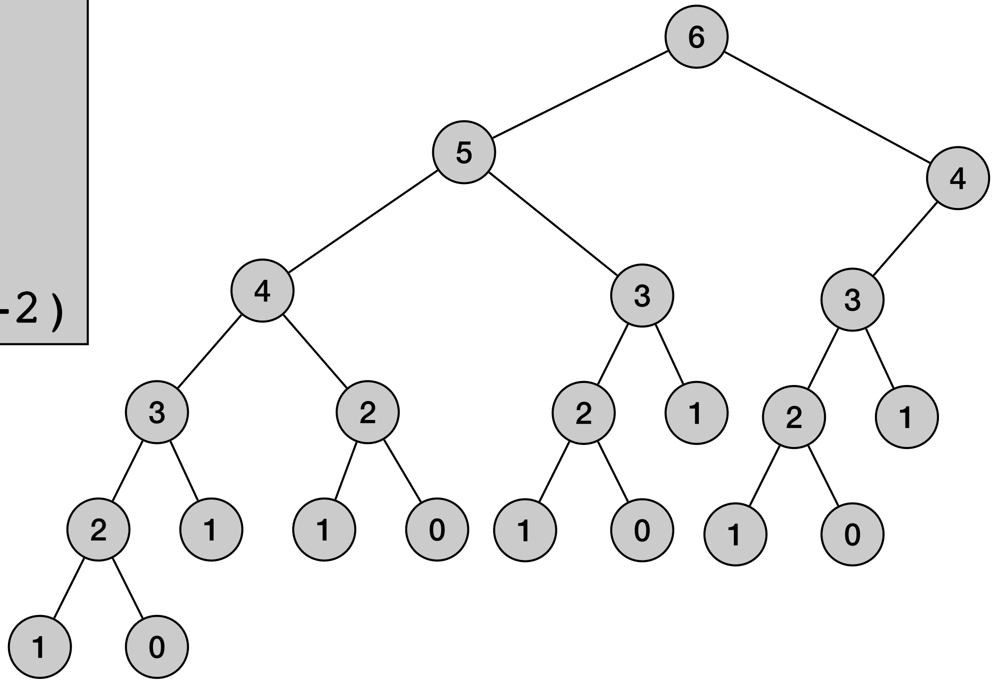

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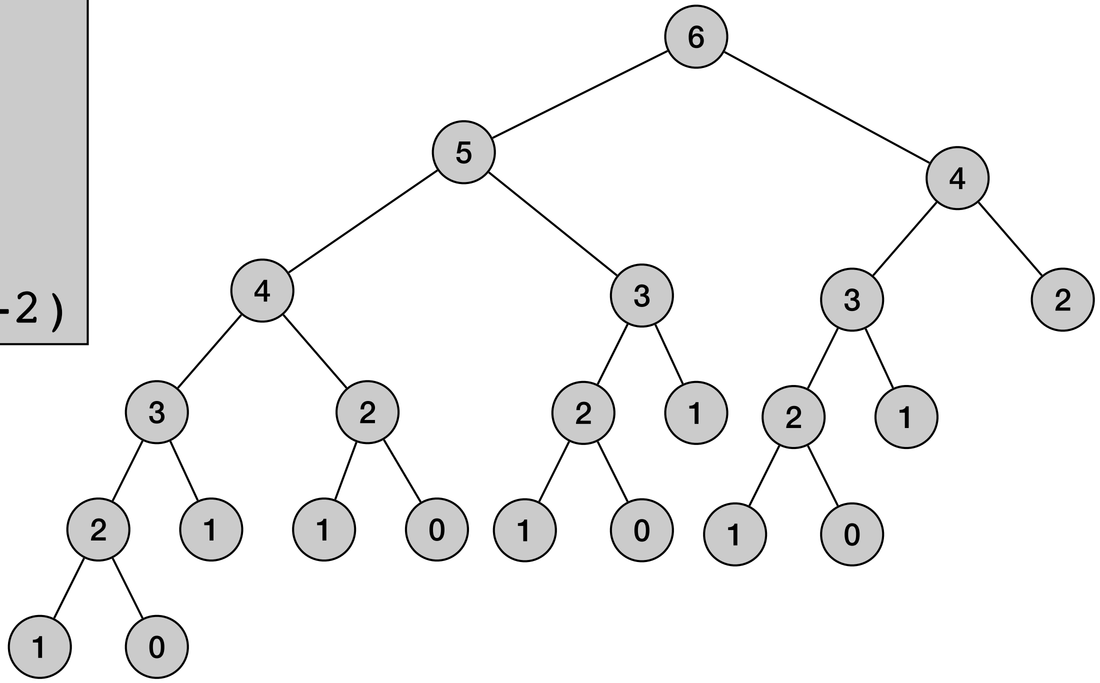

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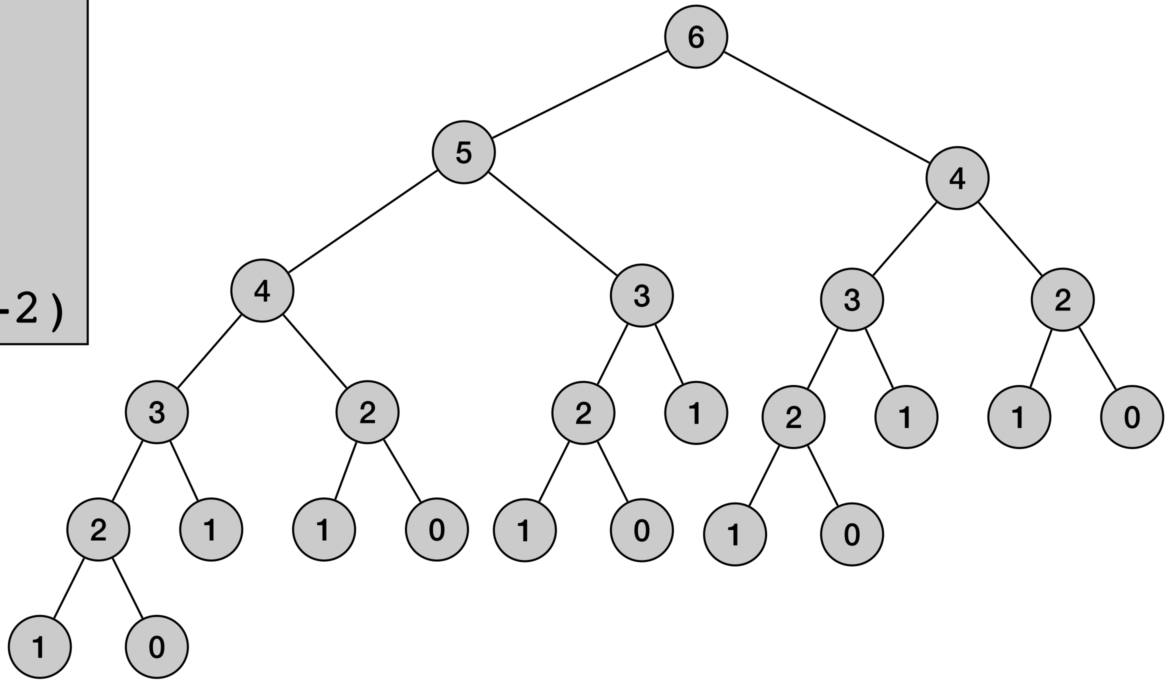

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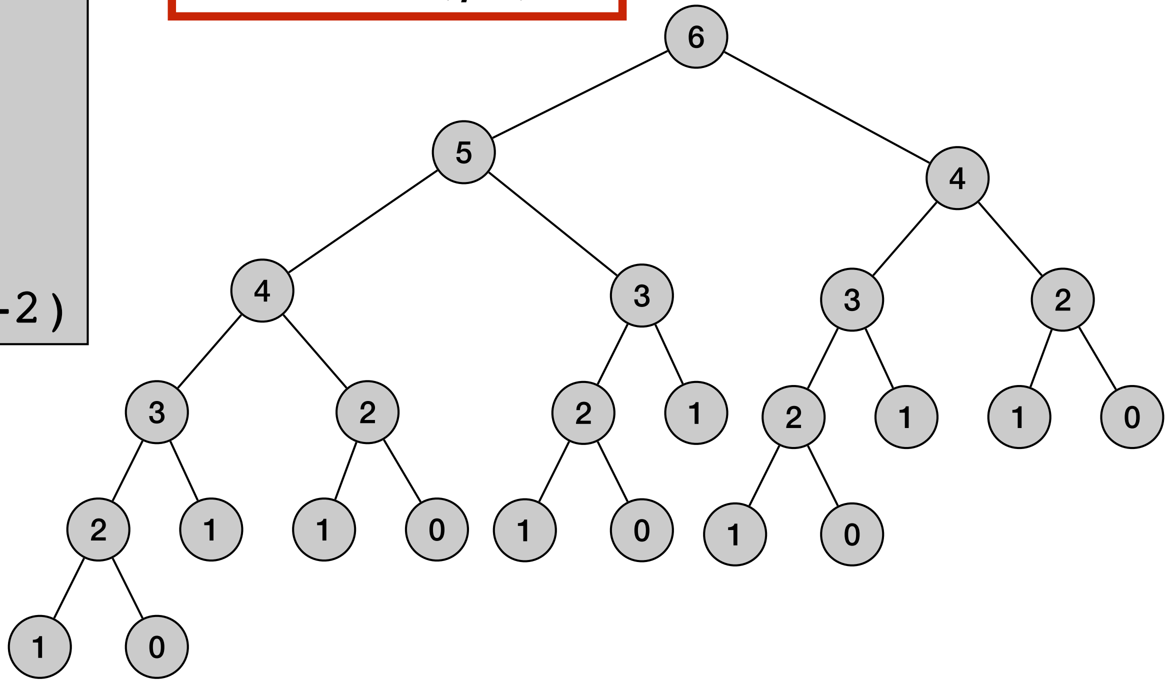
time $\Theta(\phi^n)$



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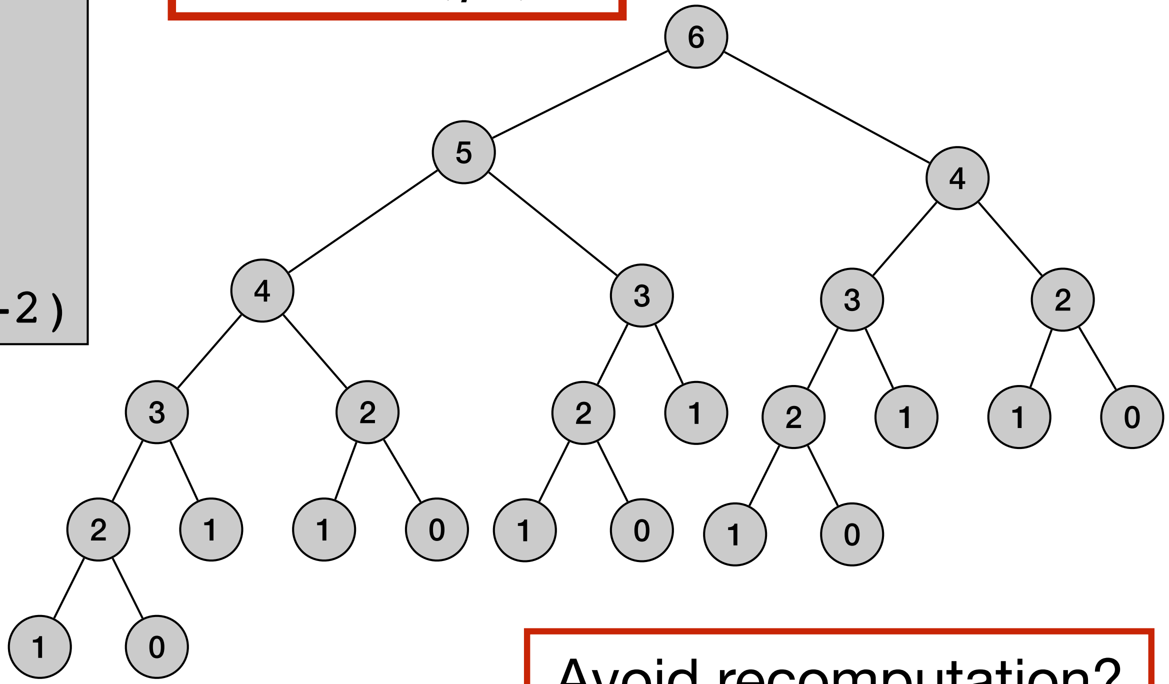

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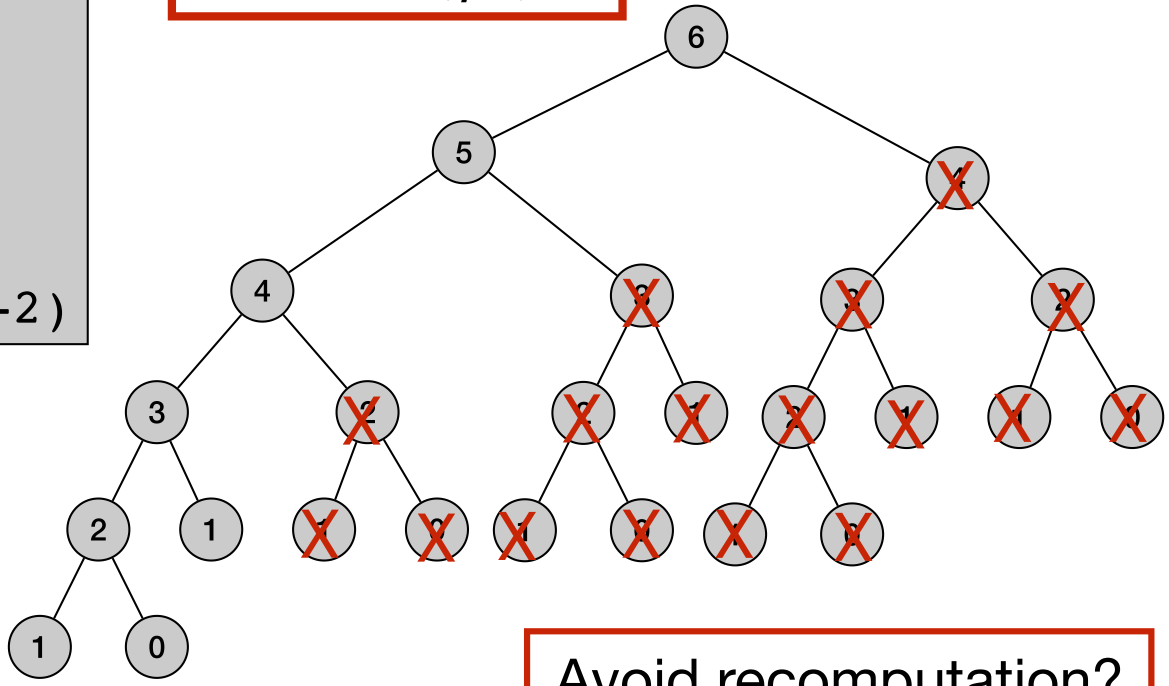

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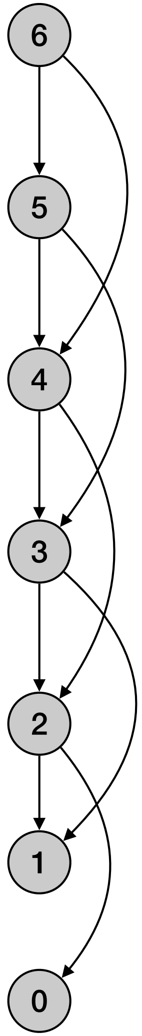

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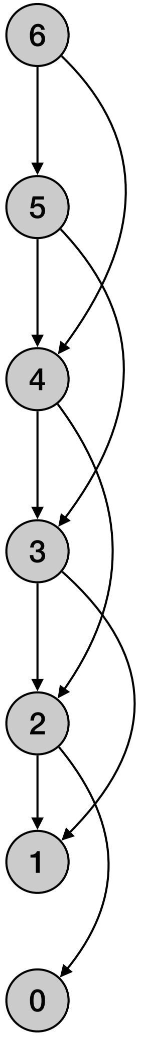

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time $\Theta(n)$

space $\Theta(n)$

Bottom-up Fibonacci numbers - save space

- Fibonacci numbers:

$$F_n = \begin{cases} 0 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ F_{n-1} + F_{n-2} & \text{otherwise} \end{cases}$$

- Remember last two computed values:

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- Remember last two computed values:

```
Iter-Fib(n)
previous = 0
current = 1
for i = 1 to n
  next = previous + current
  previous = current
  current = next
return current
```

Bottom-up Fibonacci numbers - save space

- Fibonacci numbers:

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```
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```

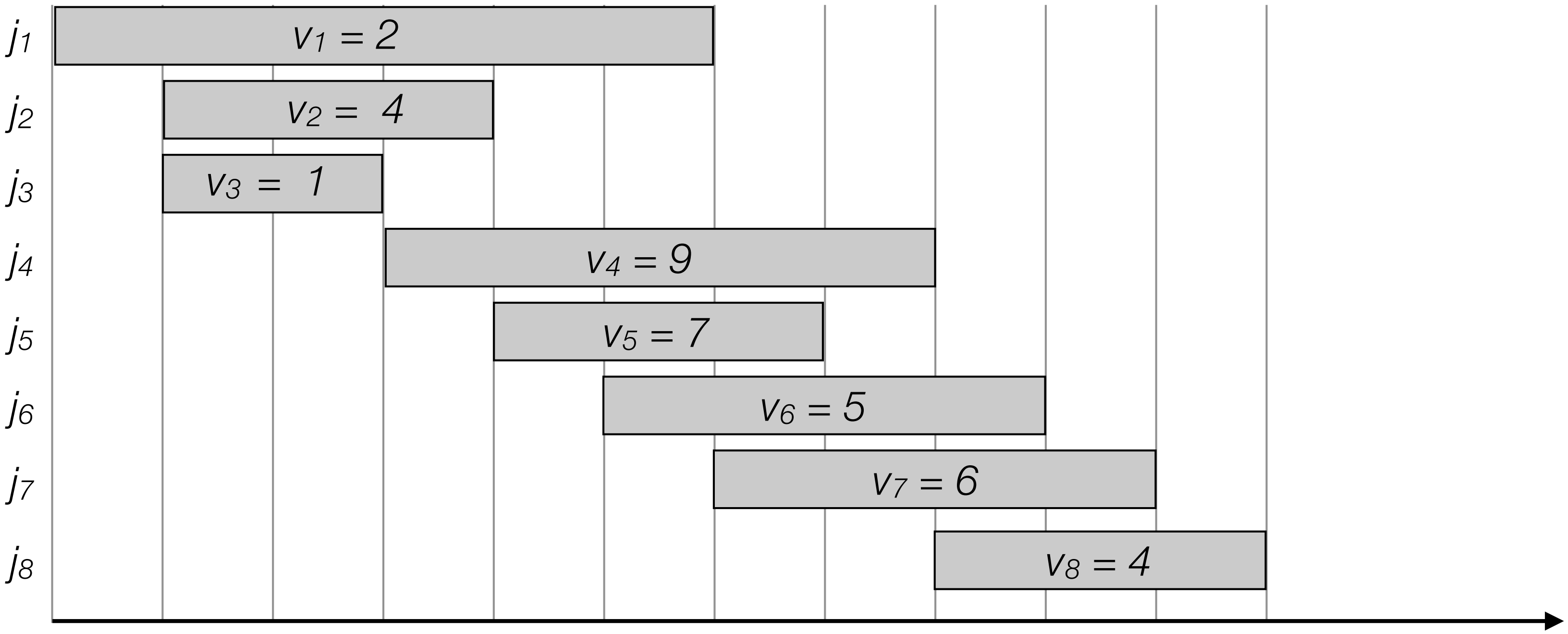
time $\Theta(n)$

space $\Theta(1)$

Weighted Interval Scheduling

Weighted interval scheduling

- Weighted interval scheduling problem
  - n jobs (intervals)
  - Job *i* starts at s_i, finishes at f_i and has weight/value v_i.
  - Goal: Find maximum weight subset of non-overlapping (compatible) jobs.



Weighted interval scheduling

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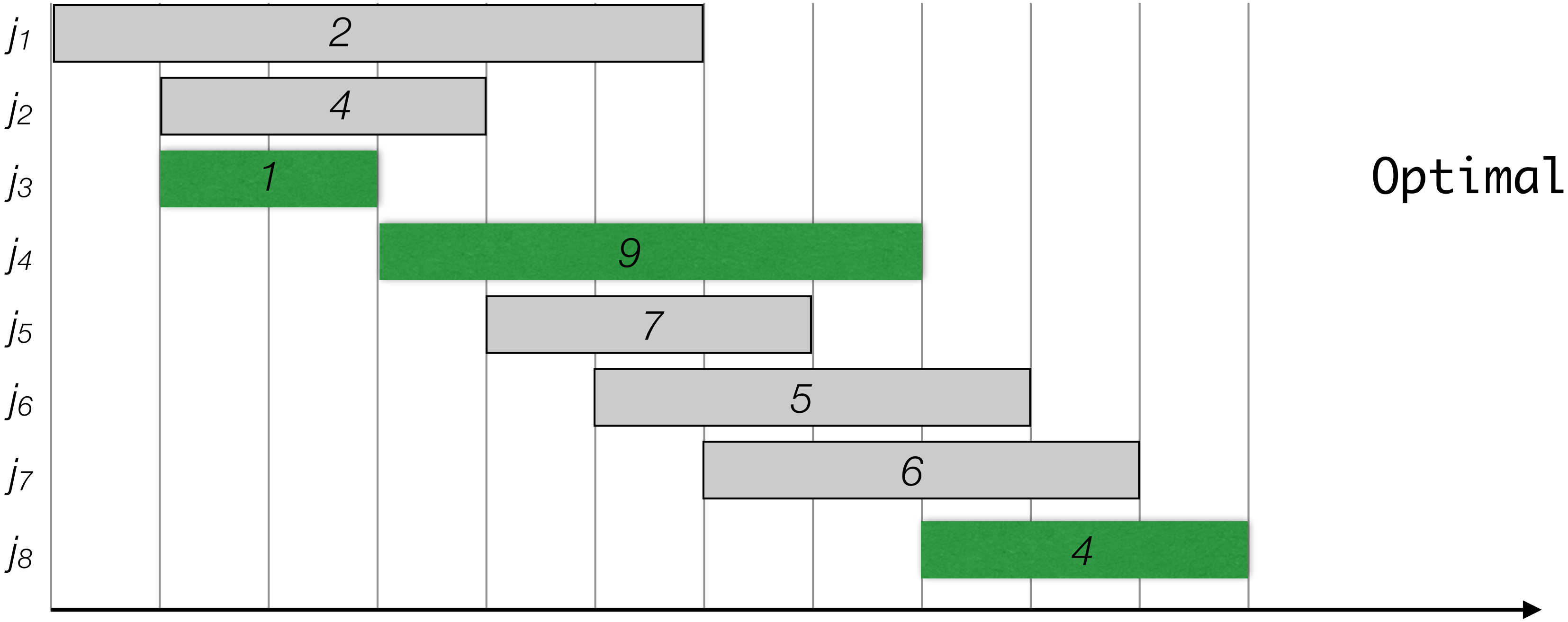

Optimal?

Weighted interval scheduling

- Weighted interval scheduling problem
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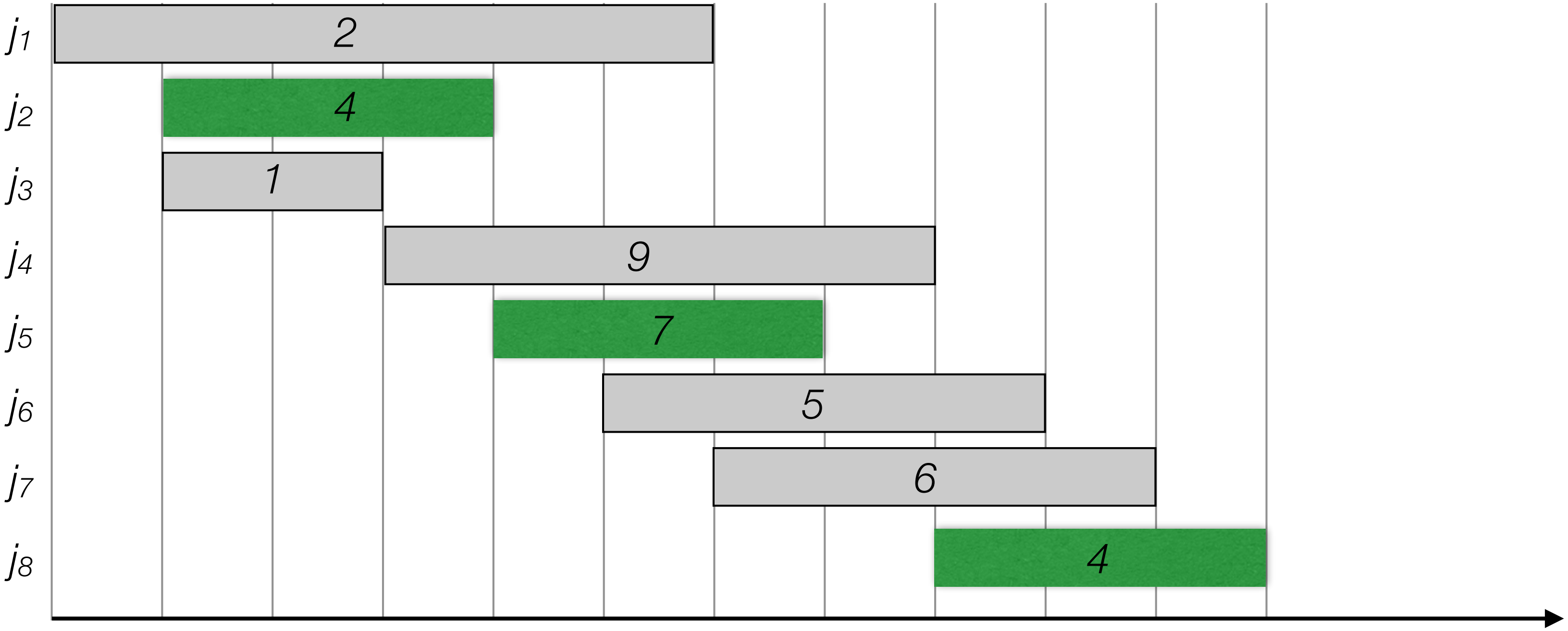
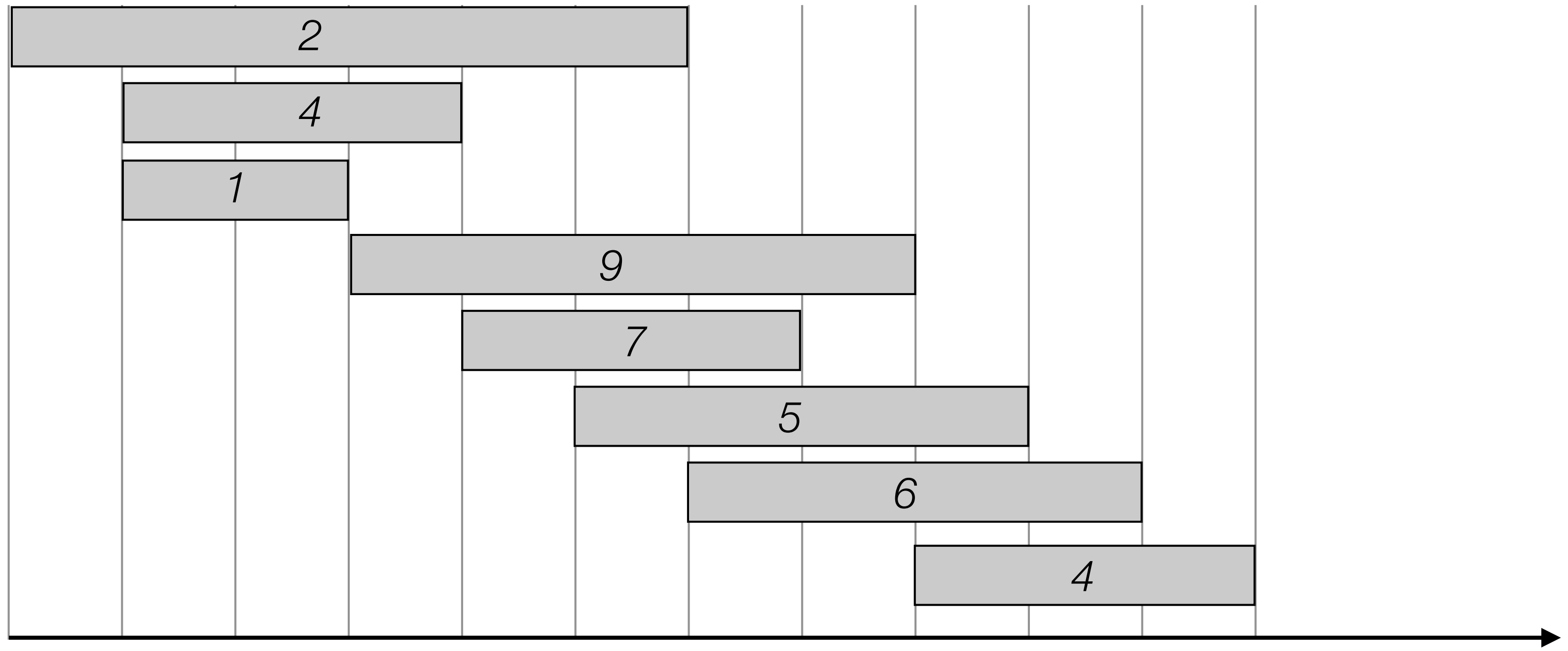
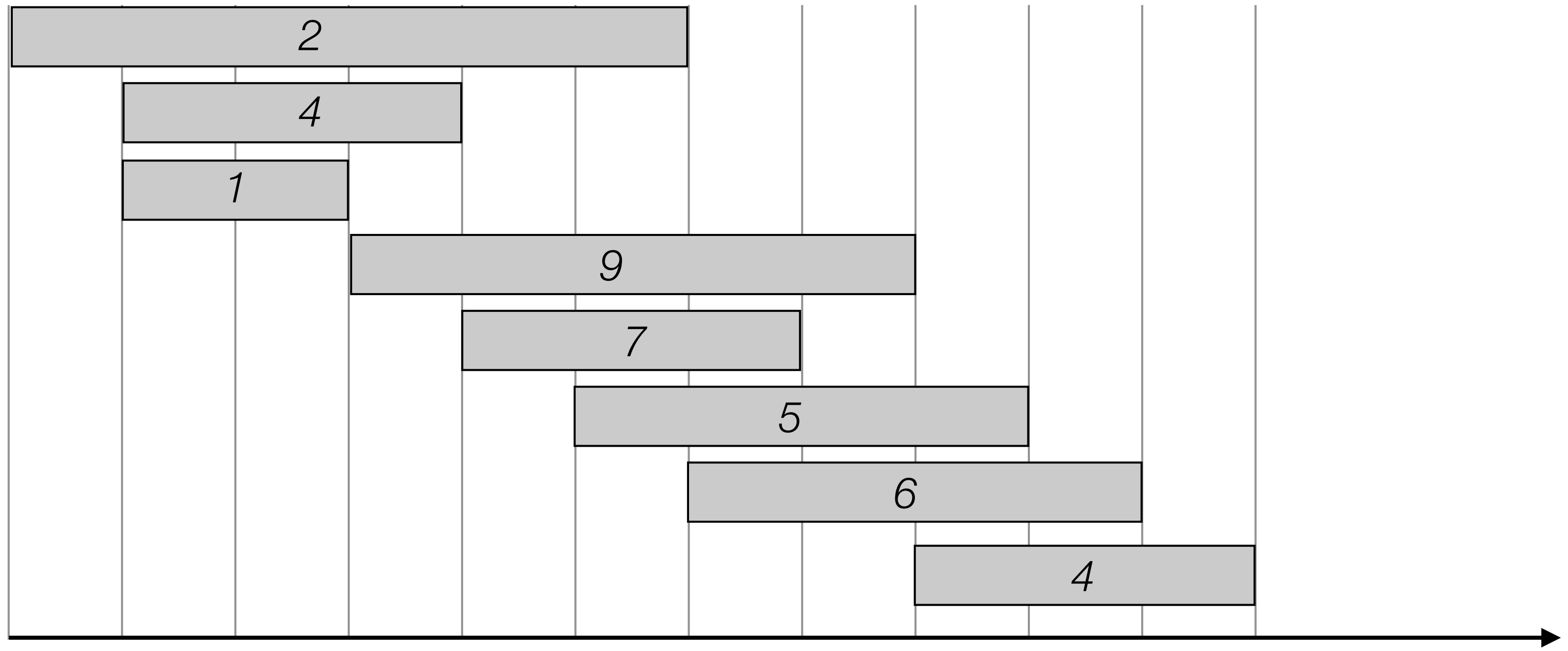


Weighted interval scheduling



Weighted interval scheduling

- Label/sort jobs by finishing time: $f_1 \leq f_2 \leq \ldots \leq f_n$



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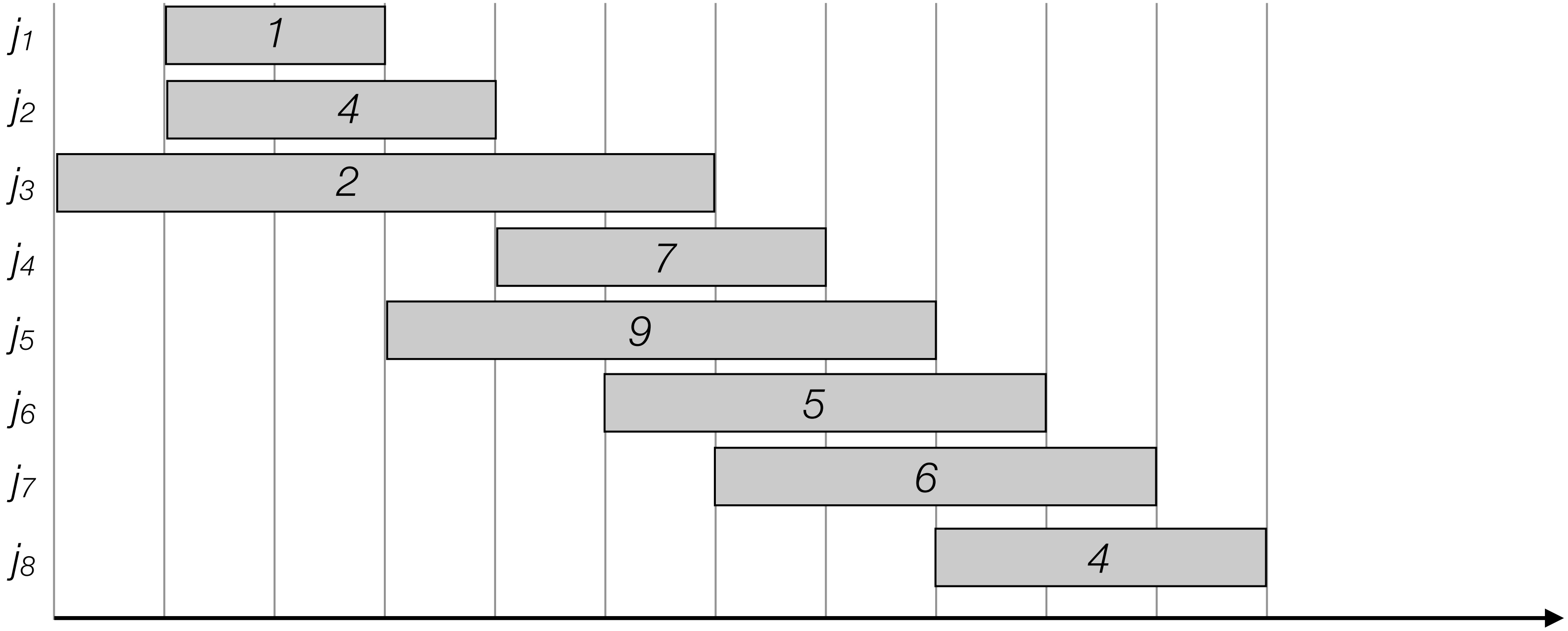
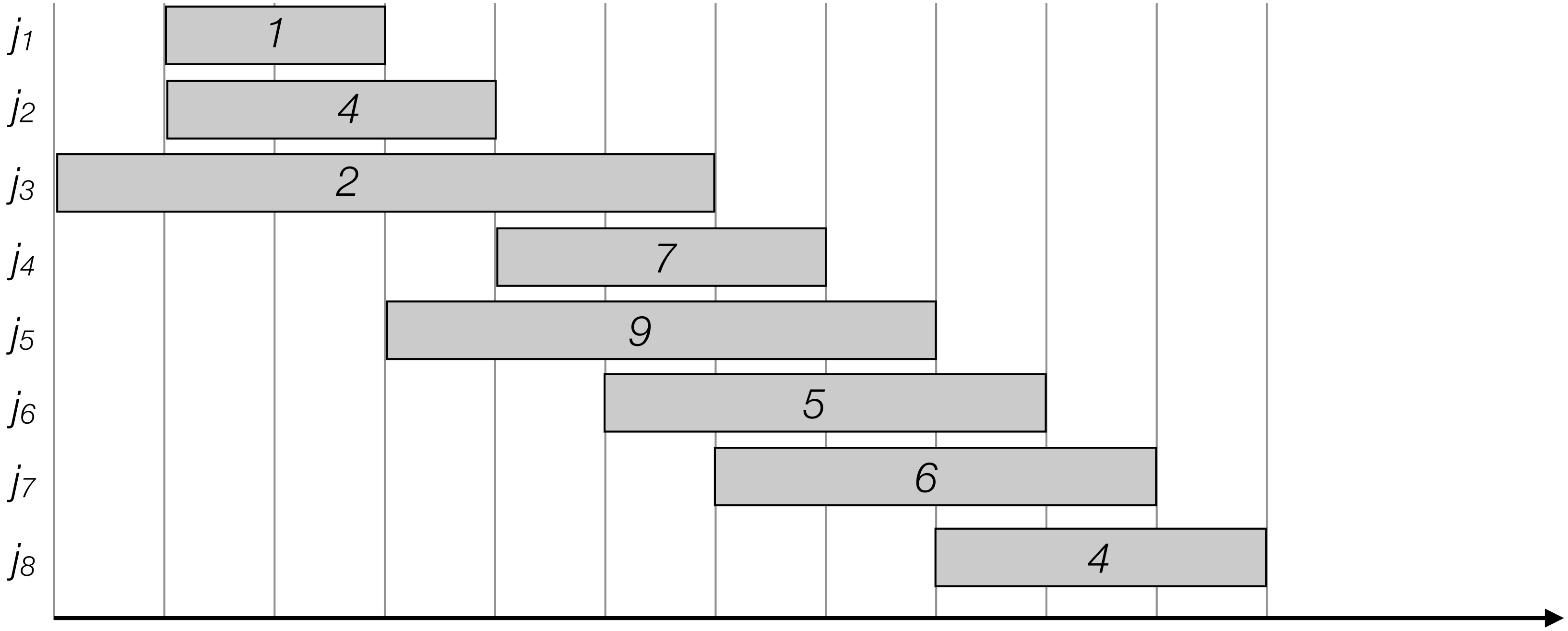


Weighted interval scheduling

- Label/sort jobs by finishing time: $f_1 \leq f_2 \leq \ldots \leq f_n$
- Greedy?



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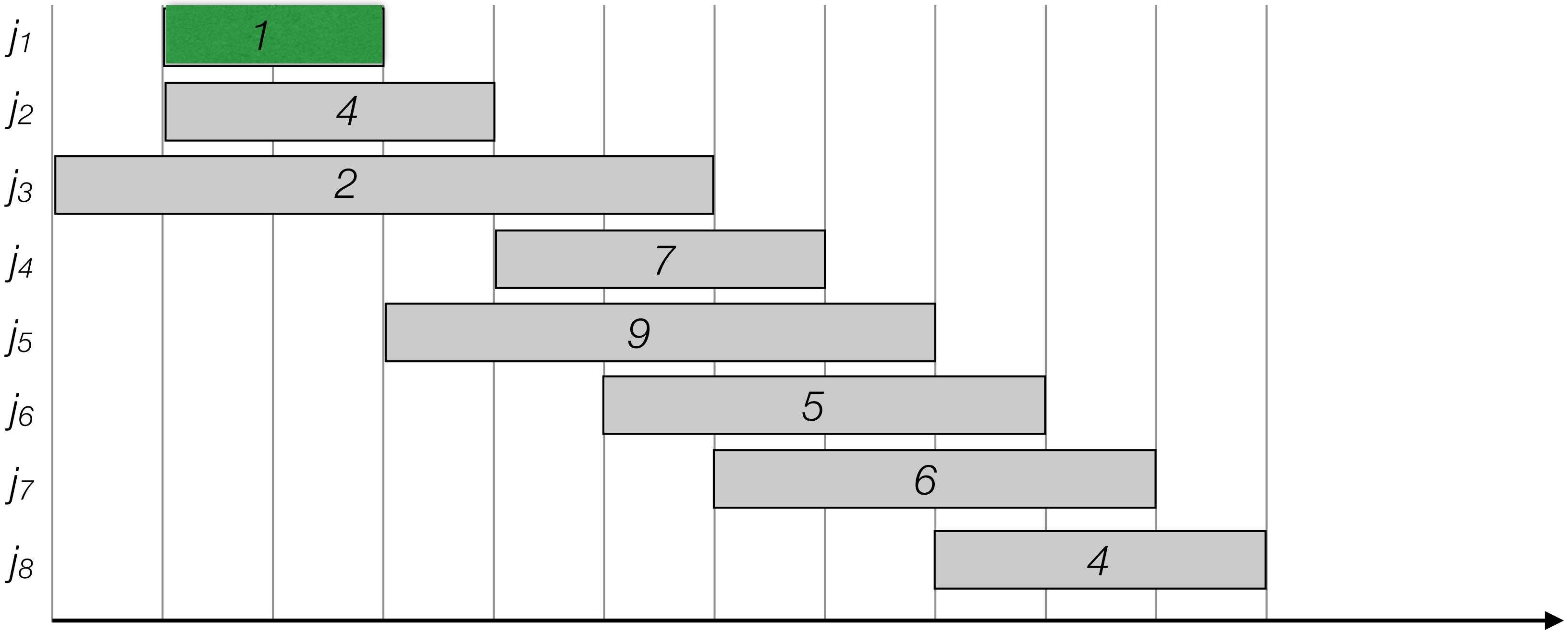

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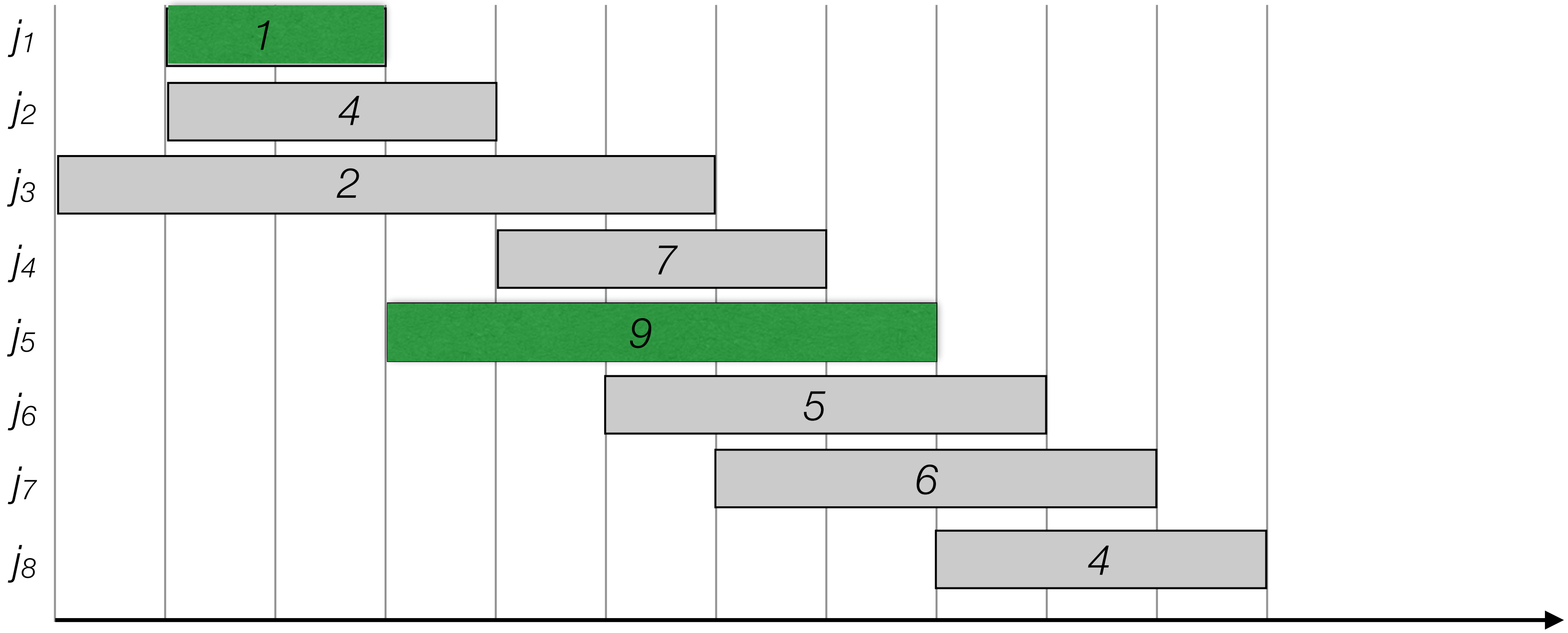

Weighted interval scheduling

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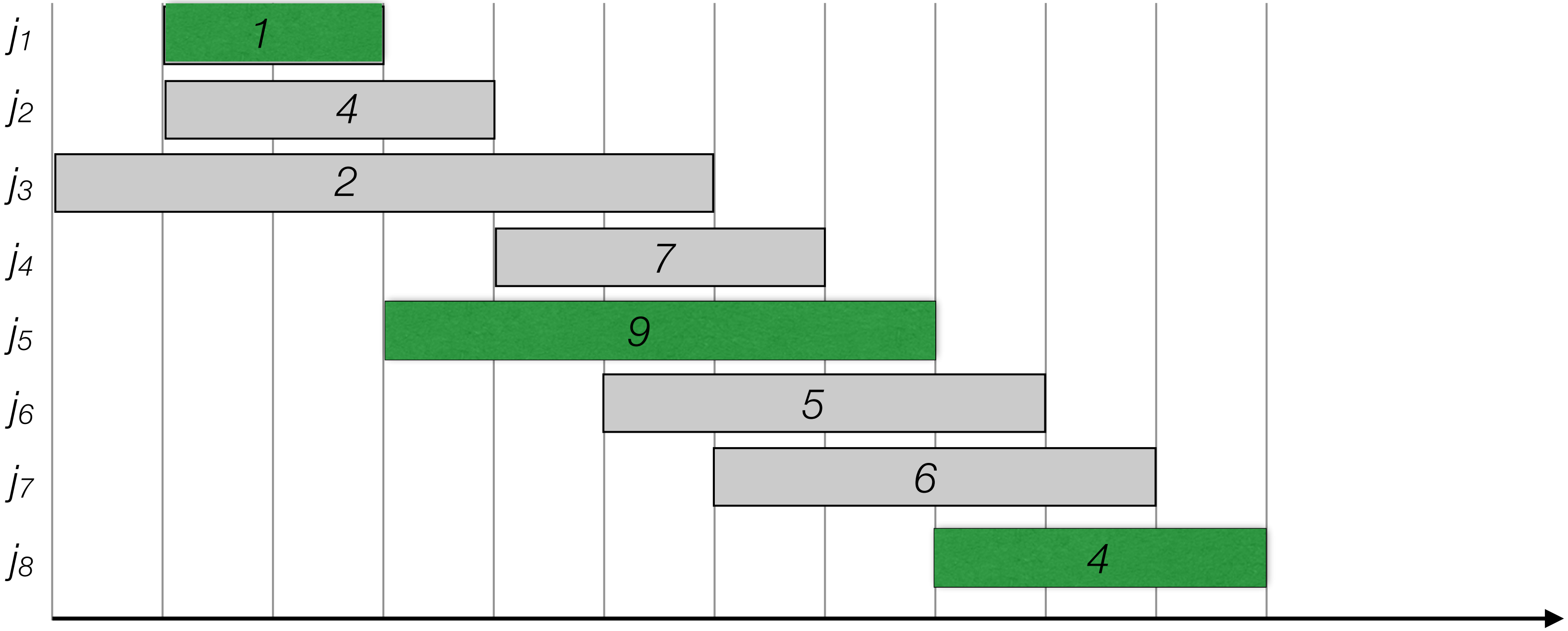

Weighted interval scheduling

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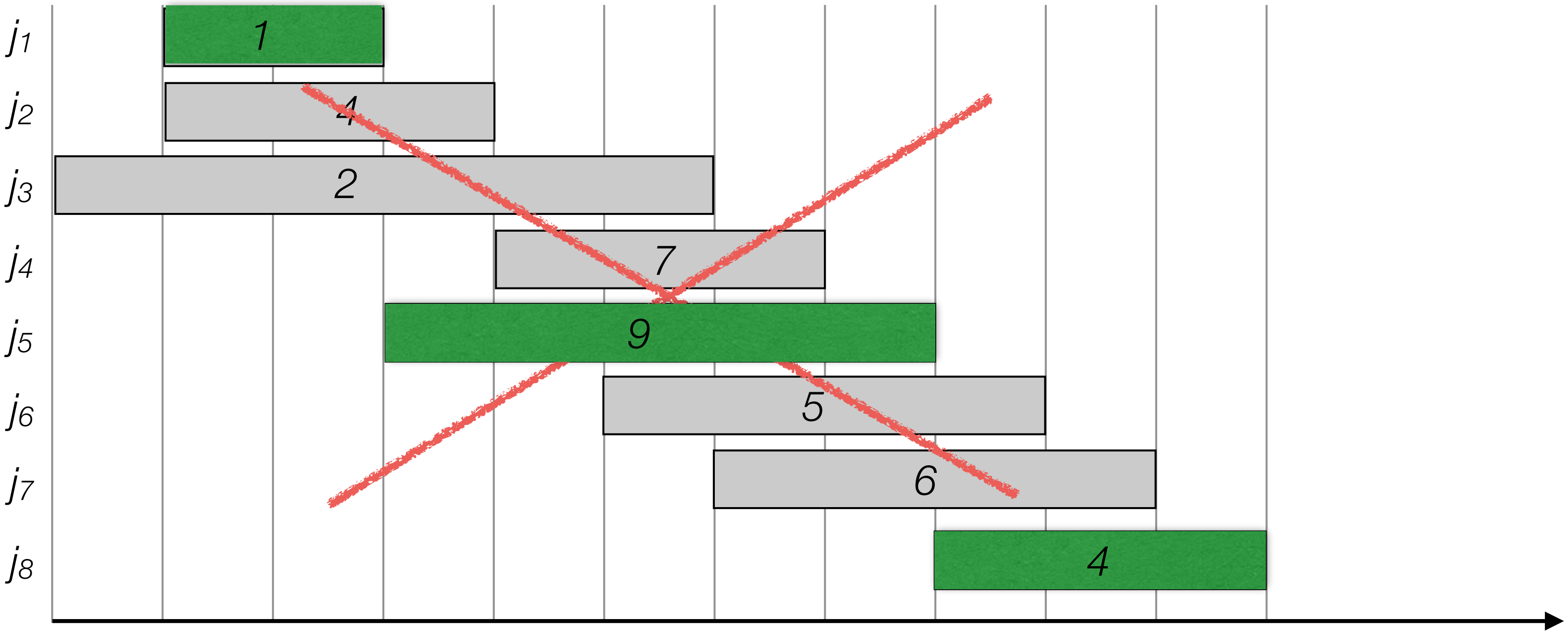

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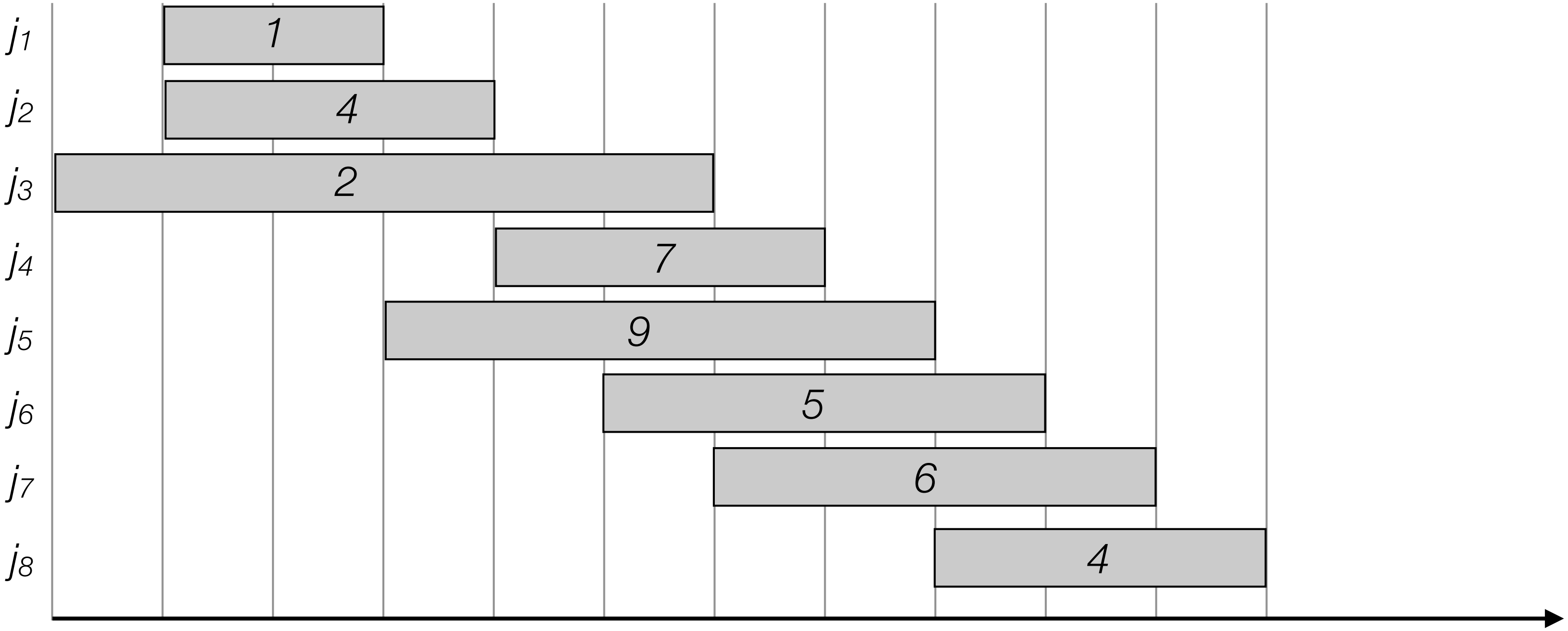

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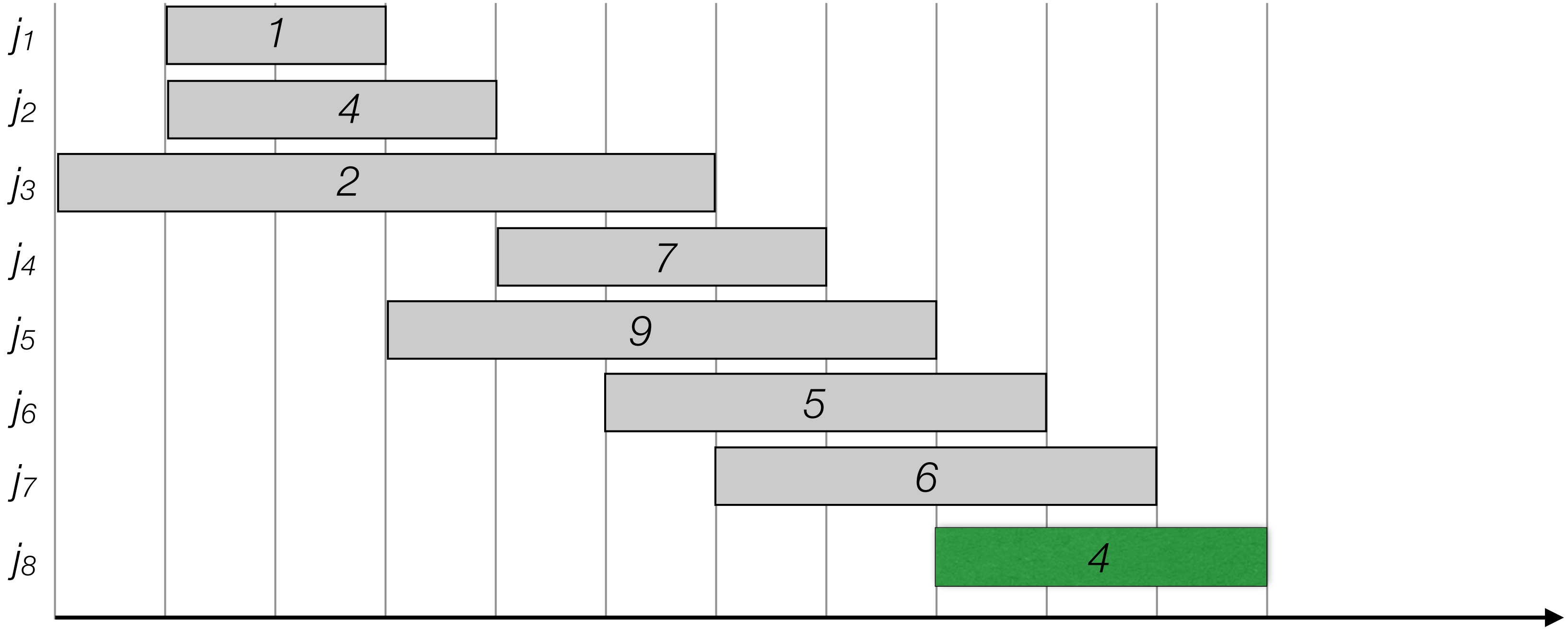

Weighted interval scheduling

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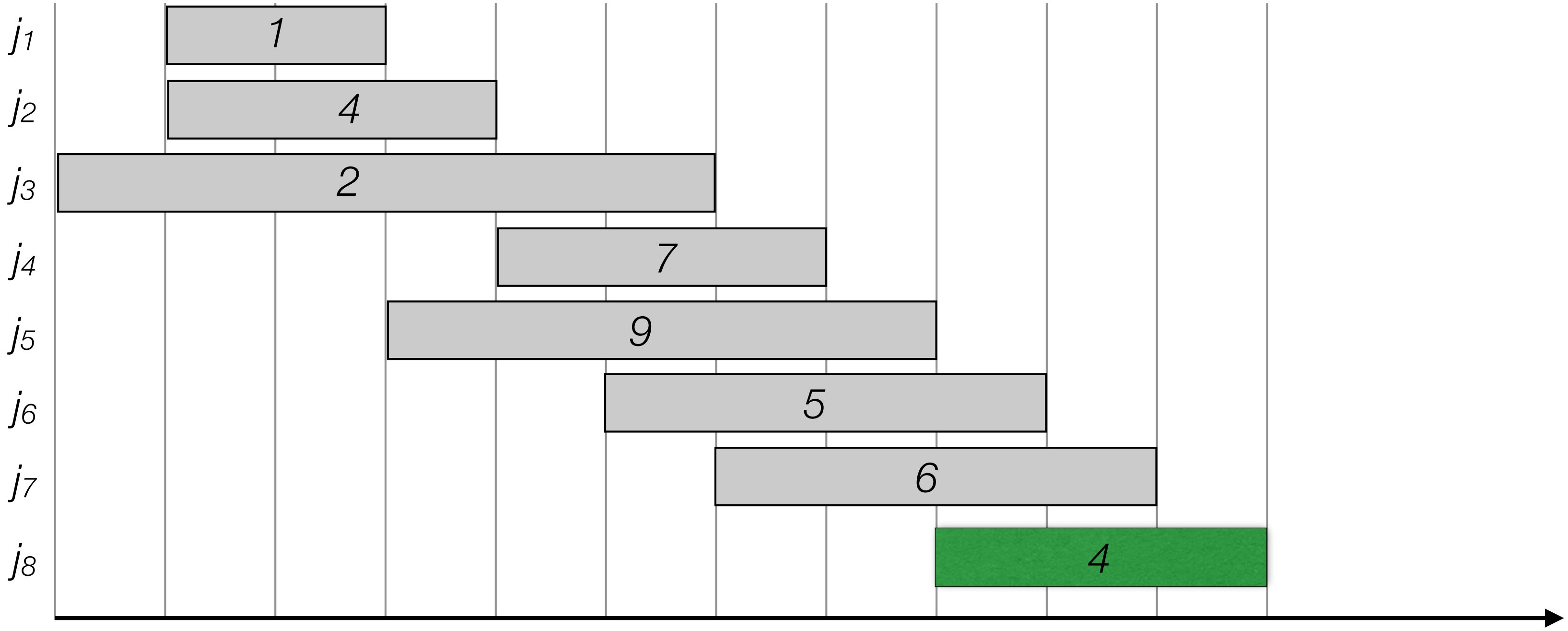
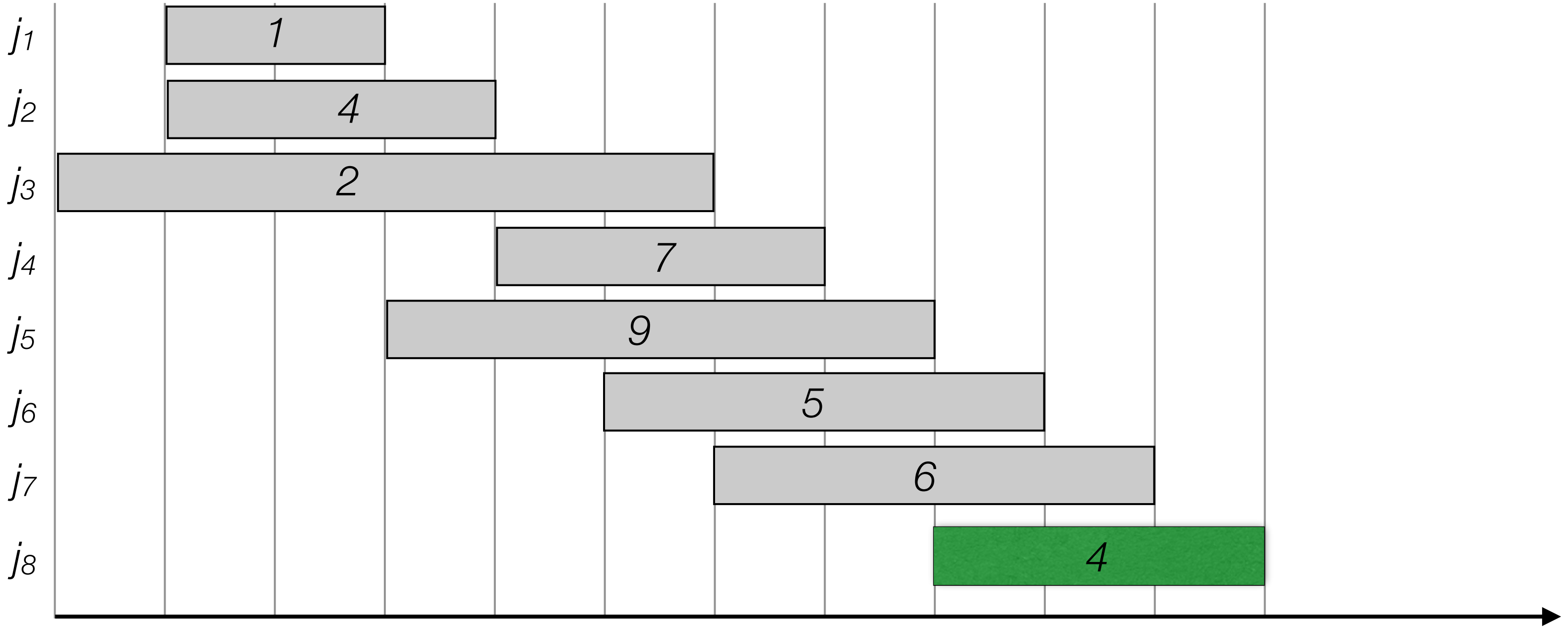
- Optimal solution OPT:



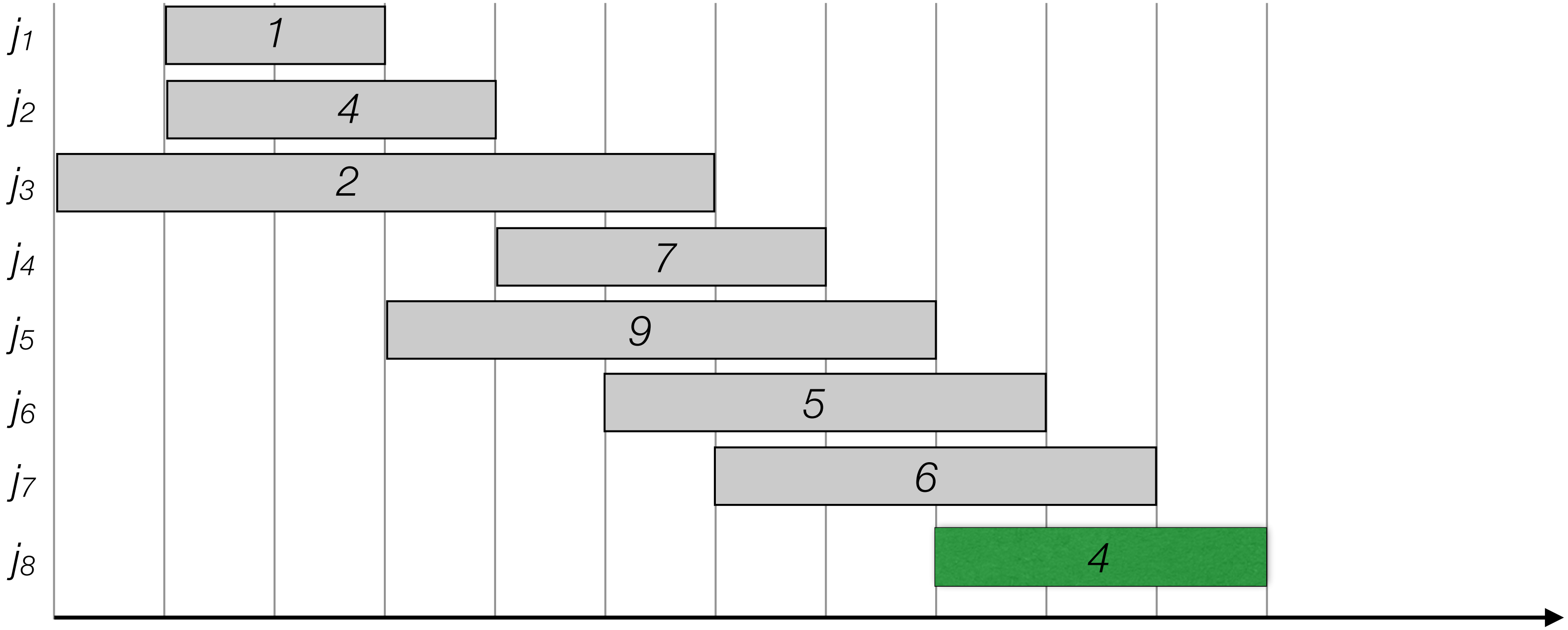
Weighted interval scheduling

- Label/sort jobs by finishing time: $f_1 \leq f_2 \leq \ldots \leq f_n$
- Optimal solution OPT:
  - Case 1. OPT selects last job



Weighted interval scheduling

- Label/sort jobs by finishing time: $f_1 \leq f_2 \leq \ldots \leq f_n$
- Optimal solution OPT:
  - Case 1. OPT selects last job
  - Case 2. OPT does not select last job



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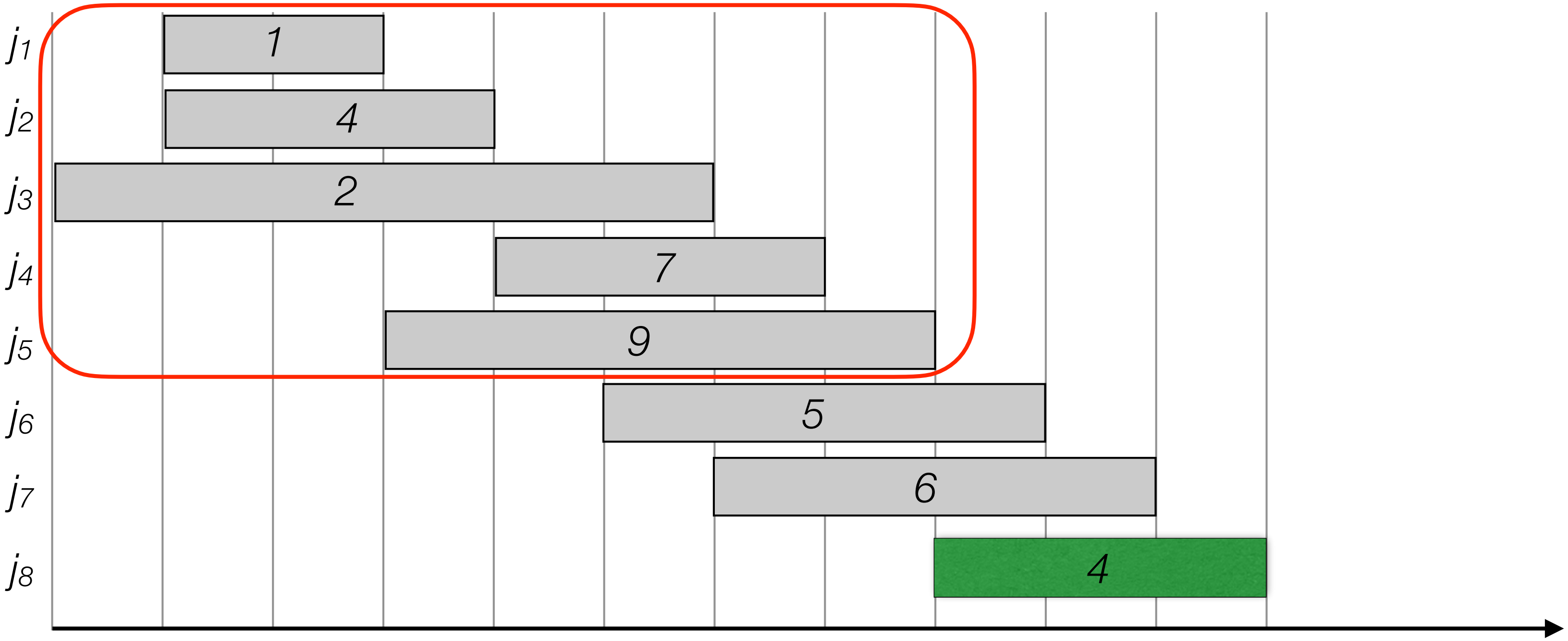
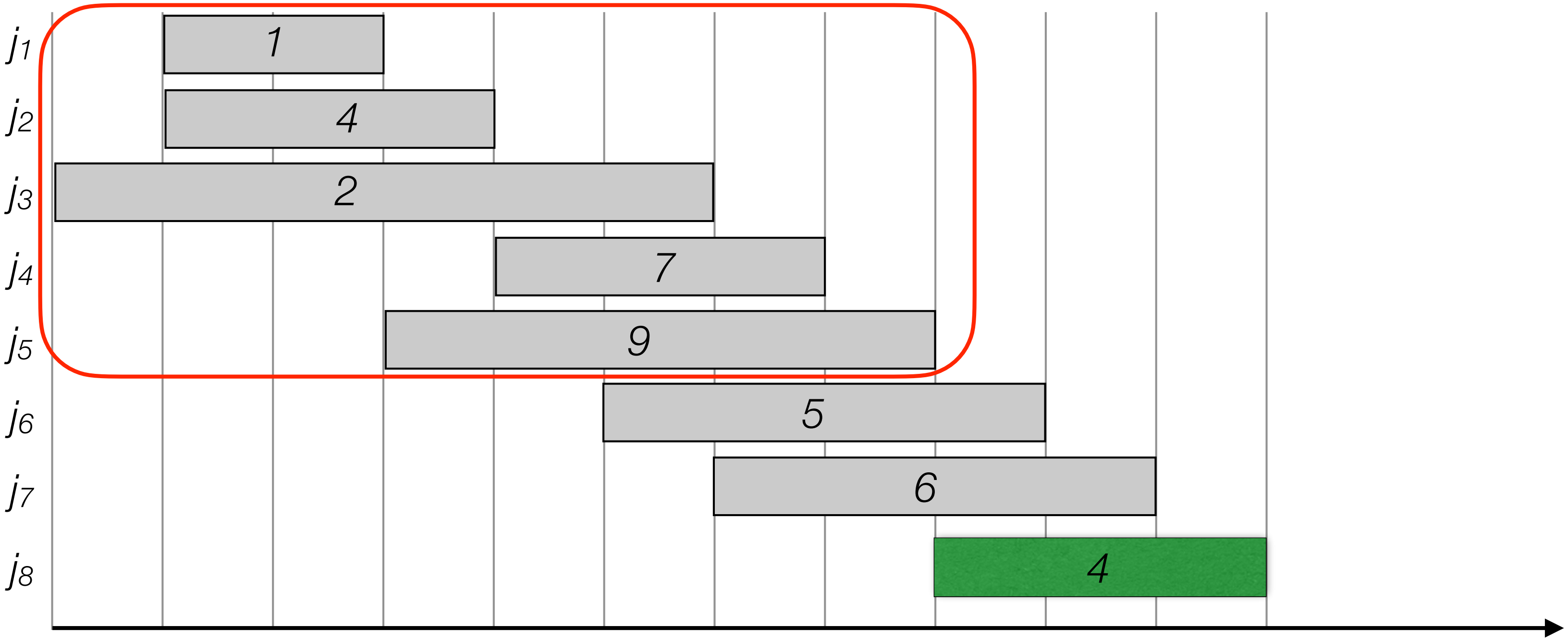


Weighted interval scheduling

- Label/sort jobs by finishing time: $f_1 \leq f_2 \leq \ldots \leq f_n$
- Optimal solution OPT:
  - Case 1. OPT selects last job

    OPT = v_n + optimal solution to subproblem on the subset of jobs ending before job n starts

  - Case 2. OPT does not select last job



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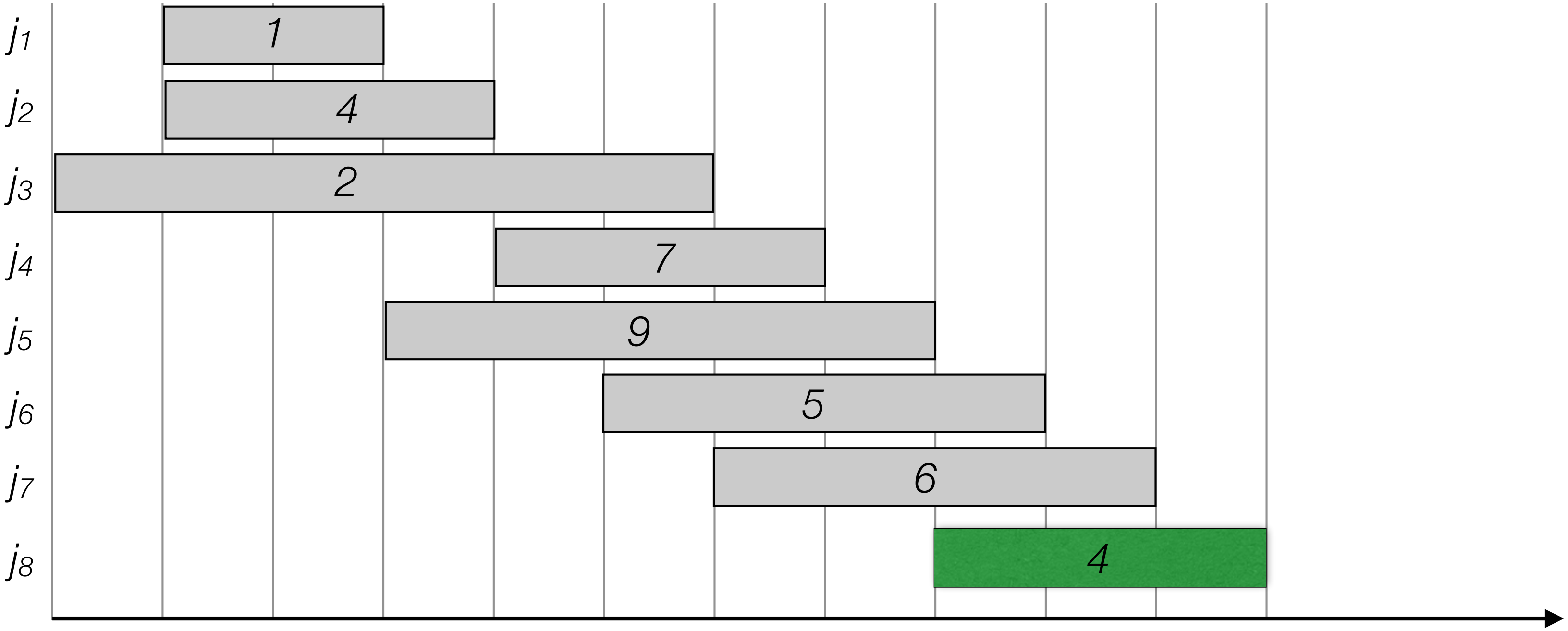

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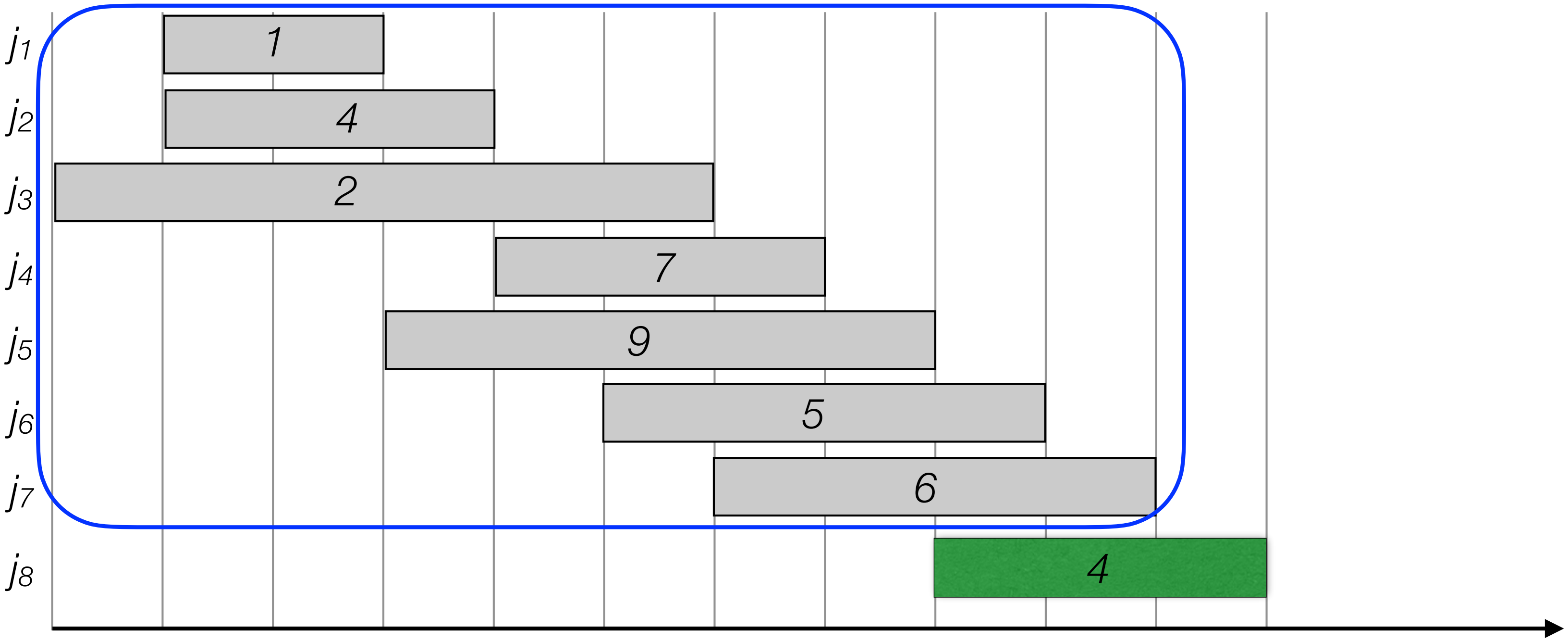


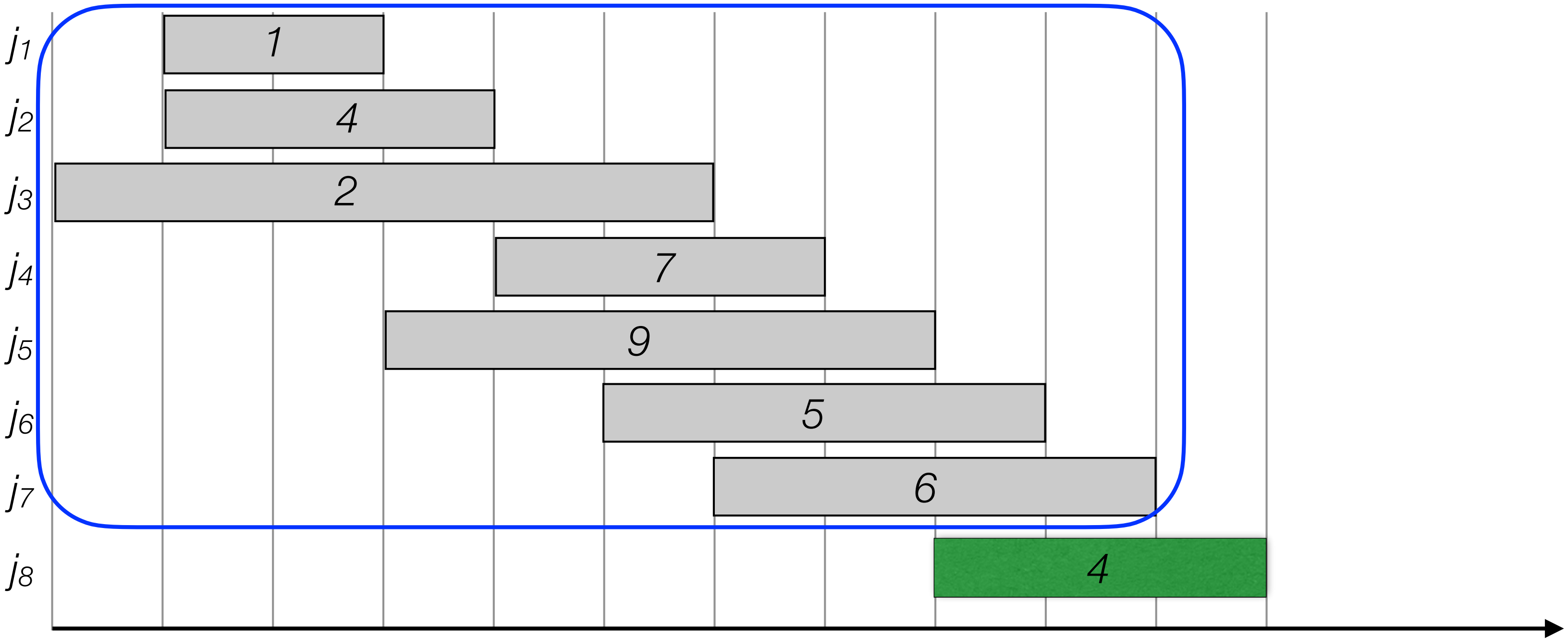
Weighted interval scheduling

- Label/sort jobs by finishing time: $f_1 \leq f_2 \leq \ldots \leq f_n$
- Optimal solution OPT:
  - Case 1. OPT selects last job

    OPT = v_n + *optimal solution to subproblem on the subset of jobs ending before job n starts*

  - Case 2. OPT does not select last job

    OPT = optimal solution to subproblem on 1,…,n-1



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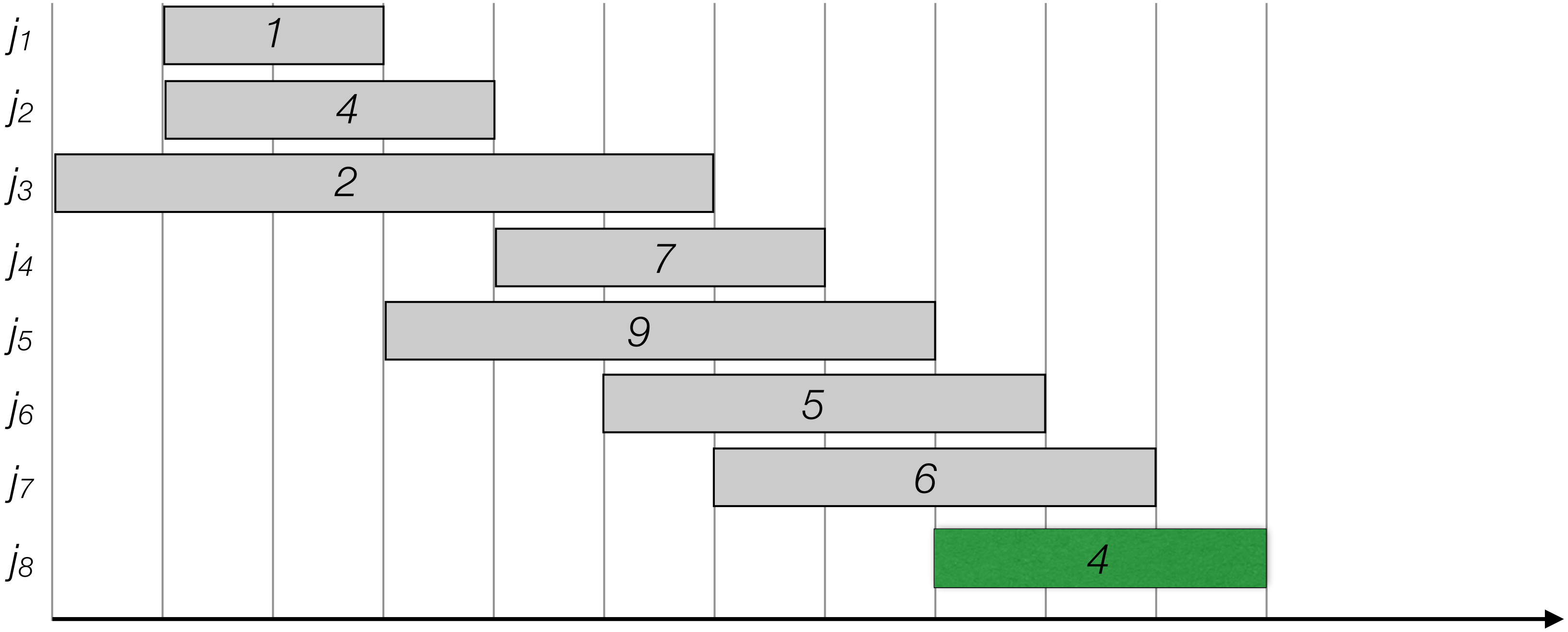

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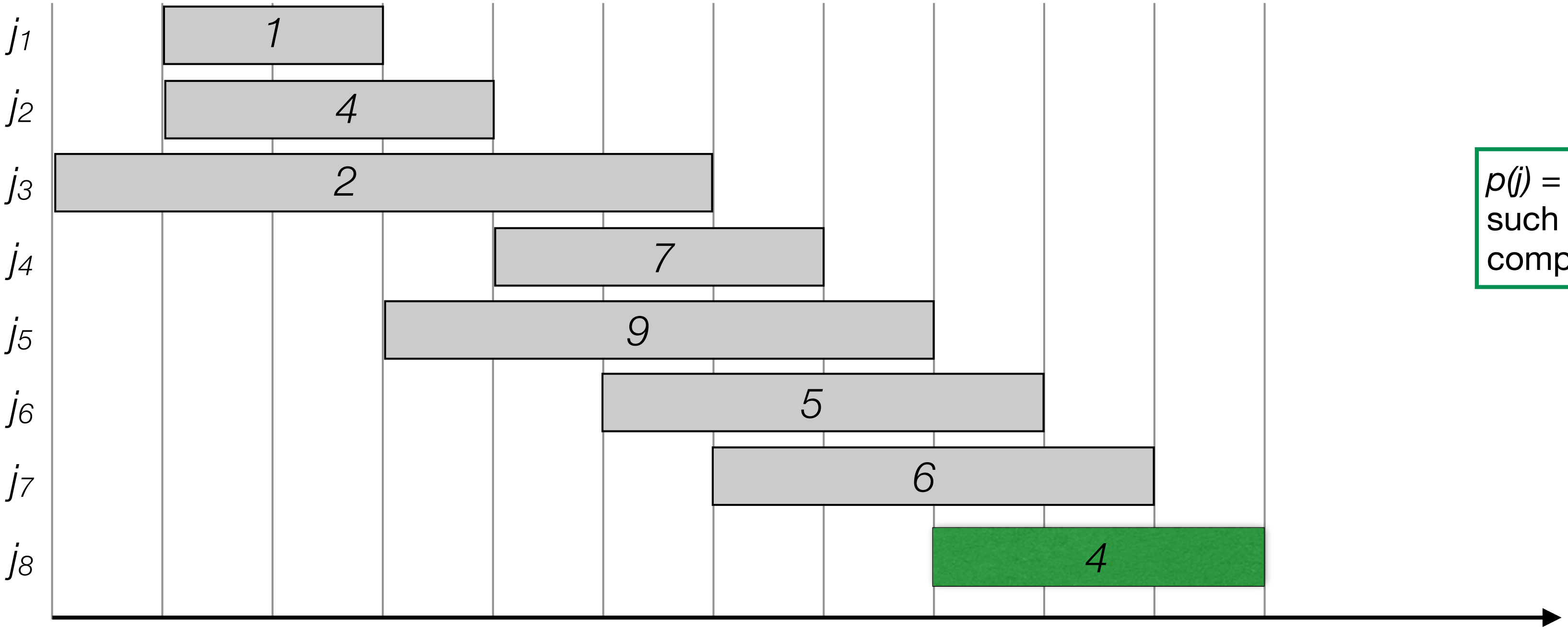

p(j) = largest index *i* < *j* such that job *i* is compatible with *j*.

Weighted interval scheduling

- Label/sort jobs by finishing time: $f_1 \leq f_2 \leq \ldots \leq f_n$
- Optimal solution OPT:
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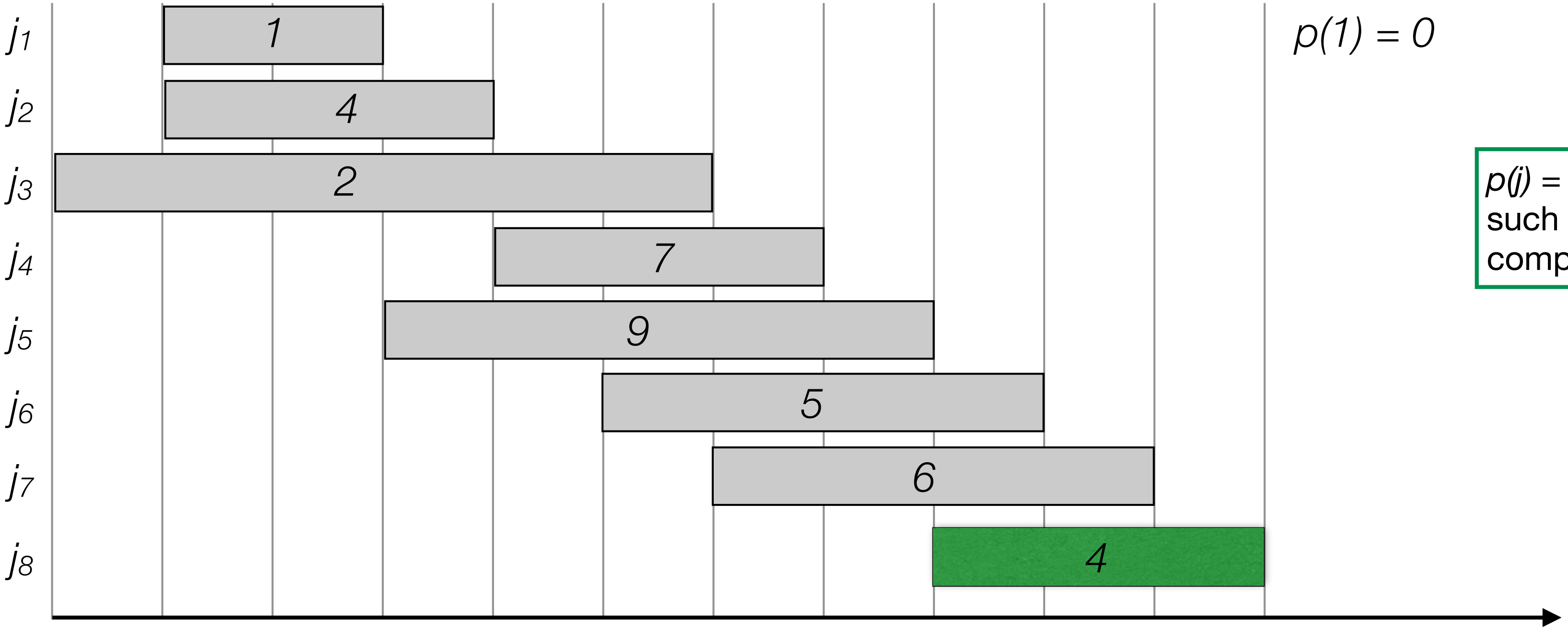

p(1) = 0

p(j) = largest index *i* < *j* such that job *i* is compatible with *j*.

Weighted interval scheduling

- Label/sort jobs by finishing time: $f_1 \leq f_2 \leq \ldots \leq f_n$
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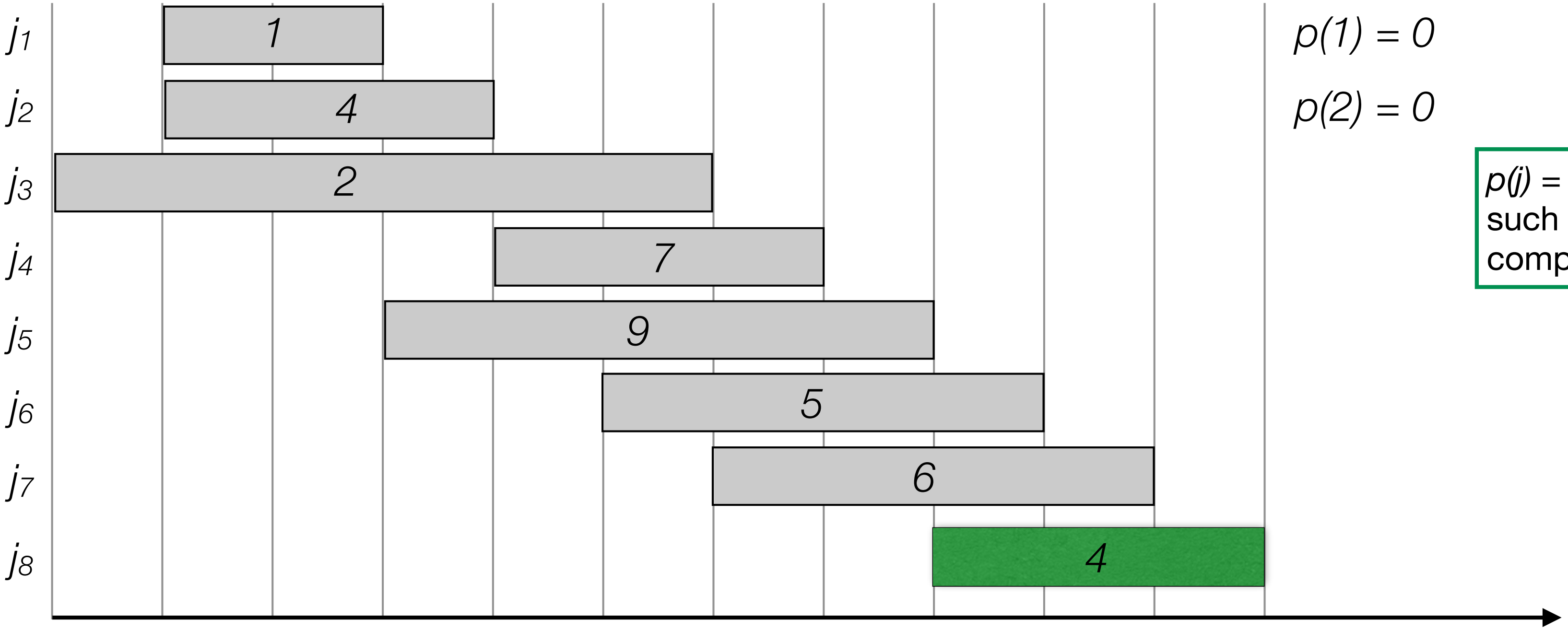

p(1) = 0

p(2) = 0

p(j) = largest index *i* < *j* such that job *i* is compatible with *j*.

Weighted interval scheduling

- Label/sort jobs by finishing time: $f_1 \leq f_2 \leq \ldots \leq f_n$
- Optimal solution OPT:
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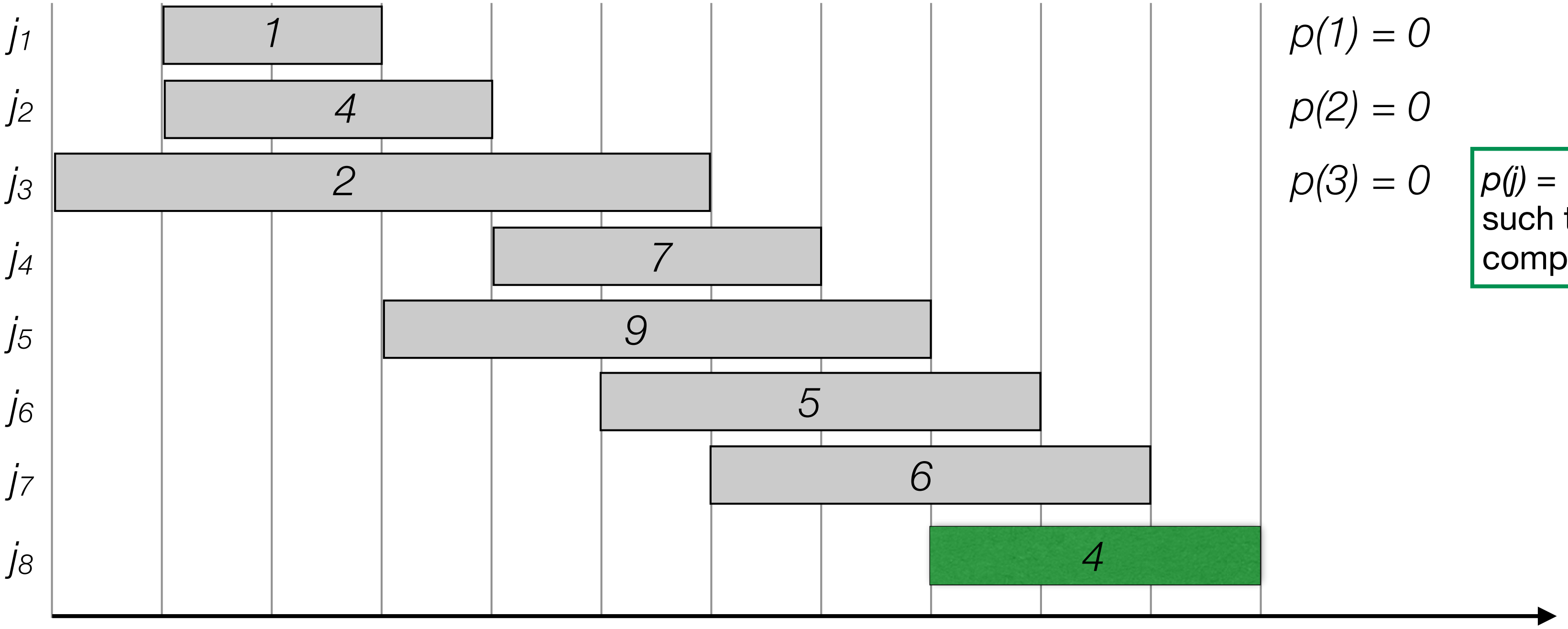


$p(1) = 0$

$p(2) = 0$

$p(3) = 0$

p(j) = largest index *i* < *j* such that job *i* is compatible with *j*.

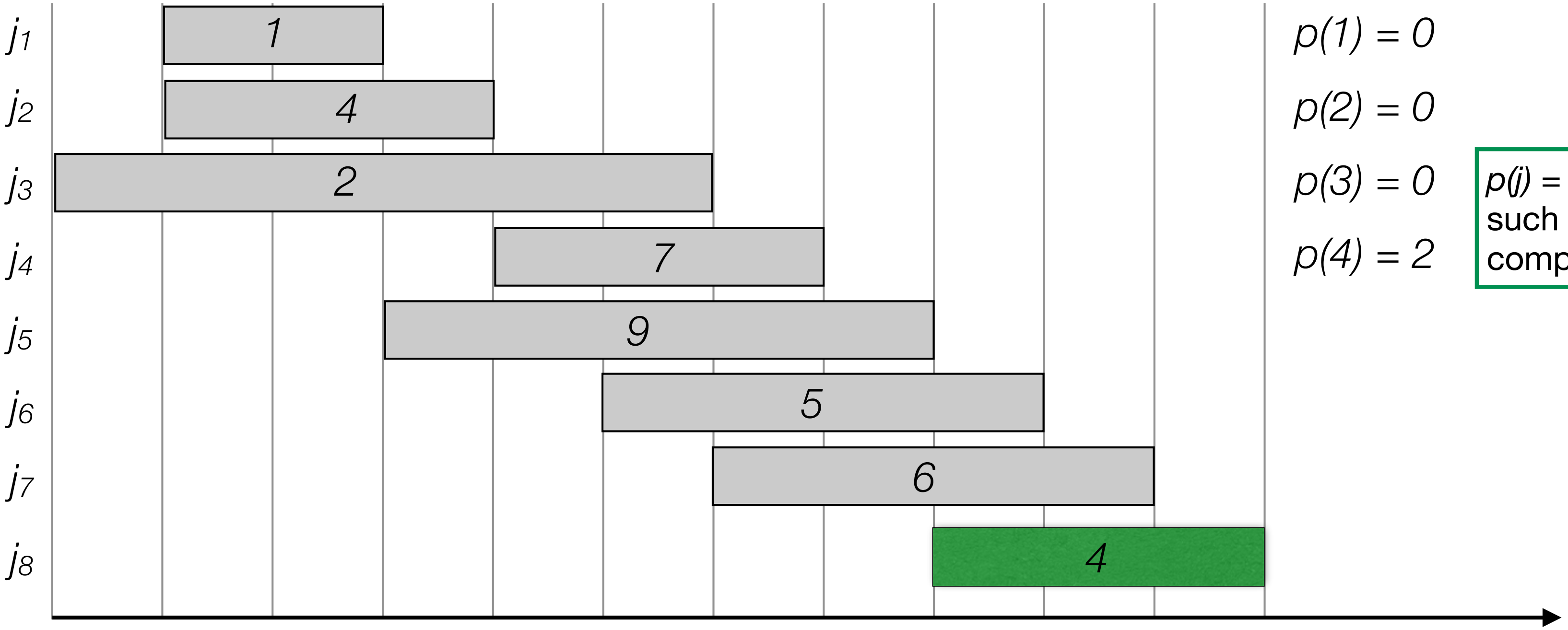
Weighted interval scheduling

- Label/sort jobs by finishing time: $f_1 \leq f_2 \leq \ldots \leq f_n$
- Optimal solution OPT:
  - Case 1. OPT selects last job

    $OPT = v_n +$ *optimal solution to subproblem on the subset of jobs ending before job n starts*

  - Case 2. OPT does not select last job

    $OPT =$ *optimal solution to subproblem on 1,…,n-1*



$p(1) = 0$

$p(2) = 0$

$p(3) = 0$

$p(4) = 2$

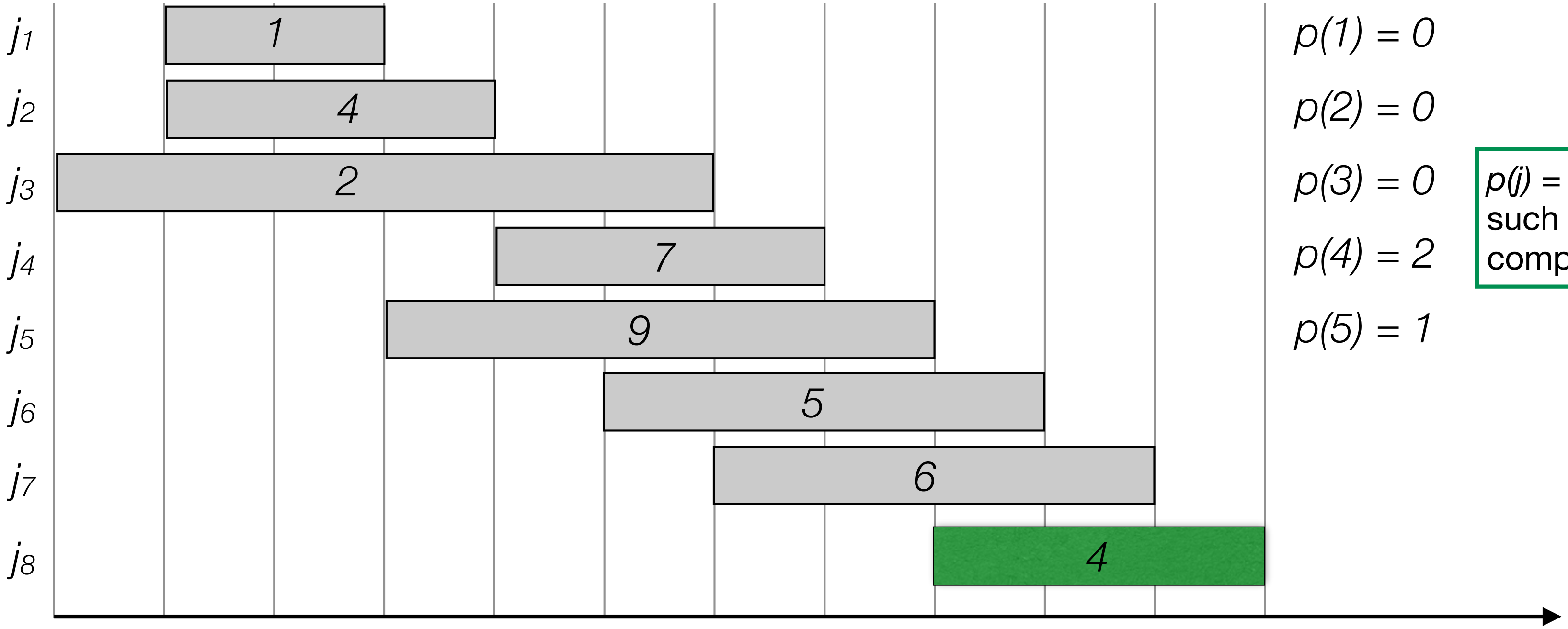
$p(j)$ = largest index $i < j$ such that job i is compatible with j.

Weighted interval scheduling

- Label/sort jobs by finishing time: $f_1 \leq f_2 \leq \ldots \leq f_n$
- Optimal solution OPT:
  - Case 1. OPT selects last job

    OPT = v_n + *optimal solution to subproblem on the subset of jobs ending before job n starts*
  - Case 2. OPT does not select last job

    OPT = optimal solution to subproblem on 1,…,n-1



$p(1) = 0$

$p(2) = 0$

$p(3) = 0$

$p(4) = 2$

$p(5) = 1$

p(j) = largest index *i* < *j* such that job *i* is compatible with *j*.

Weighted interval scheduling

- Label/sort jobs by finishing time: $f_1 \leq f_2 \leq \ldots \leq f_n$
- Optimal solution OPT:
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    OPT = v_n + optimal solution to subproblem on the subset of jobs ending before job n starts

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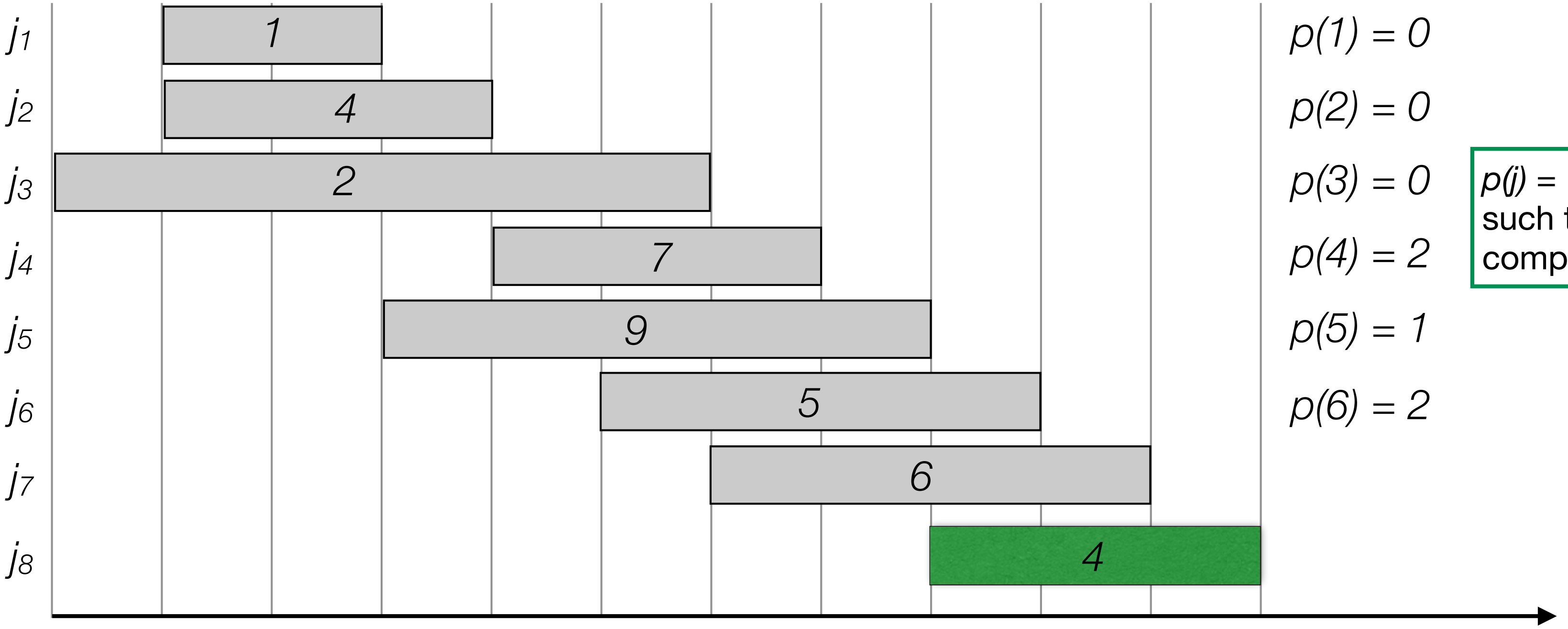


p(1) = 0

p(2) = 0

p(3) = 0

p(4) = 2

p(5) = 1

p(6) = 2

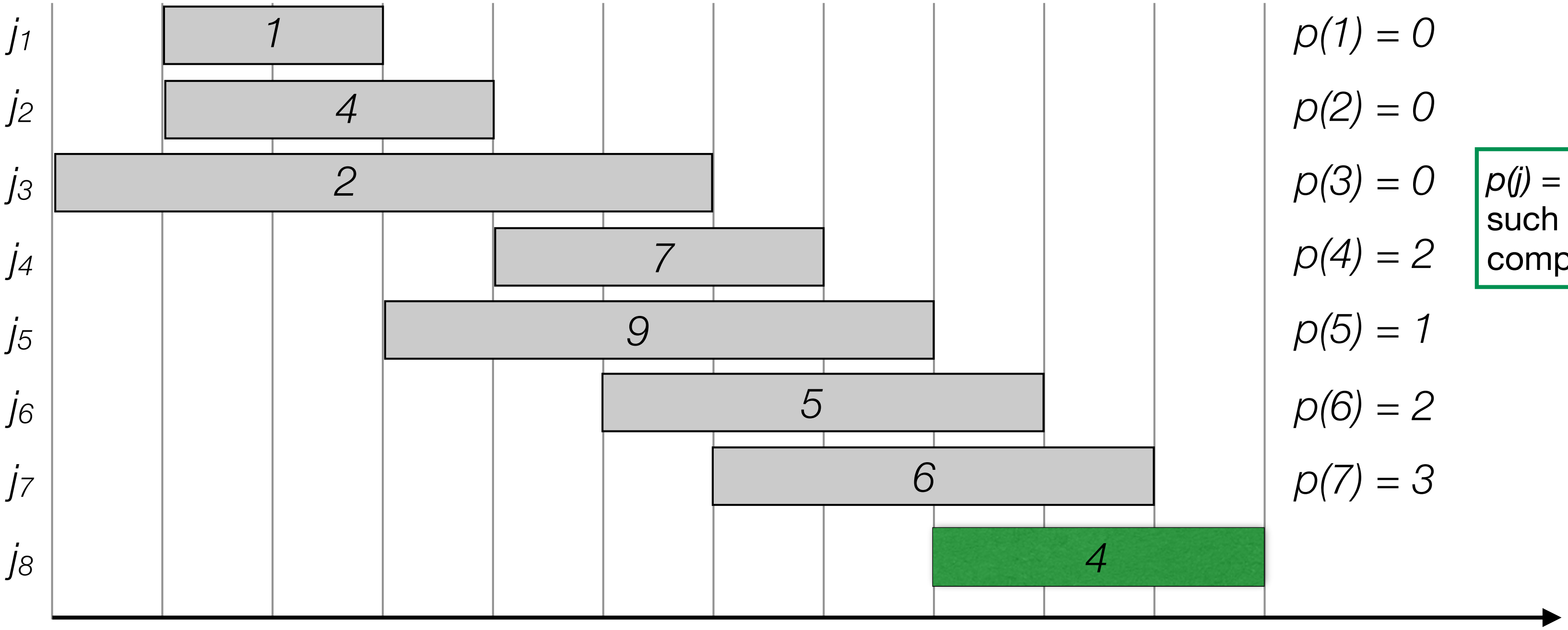
p(j) = largest index *i* < *j* such that job *i* is compatible with *j*.

Weighted interval scheduling

- Label/sort jobs by finishing time: $f_1 \leq f_2 \leq \ldots \leq f_n$
- Optimal solution OPT:
  - Case 1. OPT selects last job

    OPT = v_n + *optimal solution to subproblem on the subset of jobs ending before job n starts*
  - Case 2. OPT does not select last job

    OPT = optimal solution to subproblem on 1,…,n-1



$p(1) = 0$

$p(2) = 0$

$p(3) = 0$

$p(4) = 2$

$p(5) = 1$

$p(6) = 2$

$p(7) = 3$

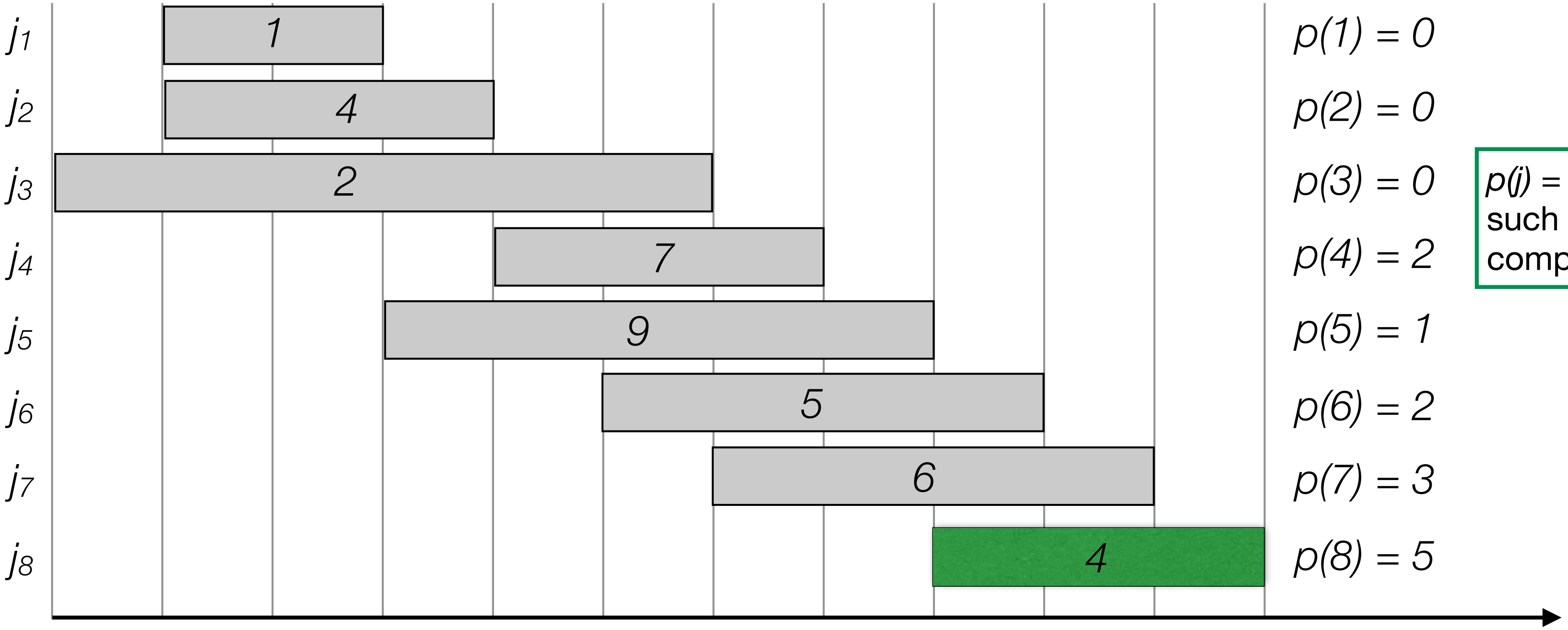
$p(j)$ = largest index $i < j$ such that job i is compatible with j.

Weighted interval scheduling

- Label/sort jobs by finishing time: $f_1 \leq f_2 \leq \ldots \leq f_n$
- Optimal solution OPT:
  - Case 1. OPT selects last job

    OPT = v_n + optimal solution to subproblem on the subset of jobs ending before job n starts
  - Case 2. OPT does not select last job

    OPT = optimal solution to subproblem on 1,…,n-1



p(1) = 0

p(2) = 0

p(3) = 0

p(4) = 2

p(5) = 1

p(6) = 2

p(7) = 3

p(8) = 5

p(j) = largest index *i* < *j* such that job *i* is compatible with *j*.

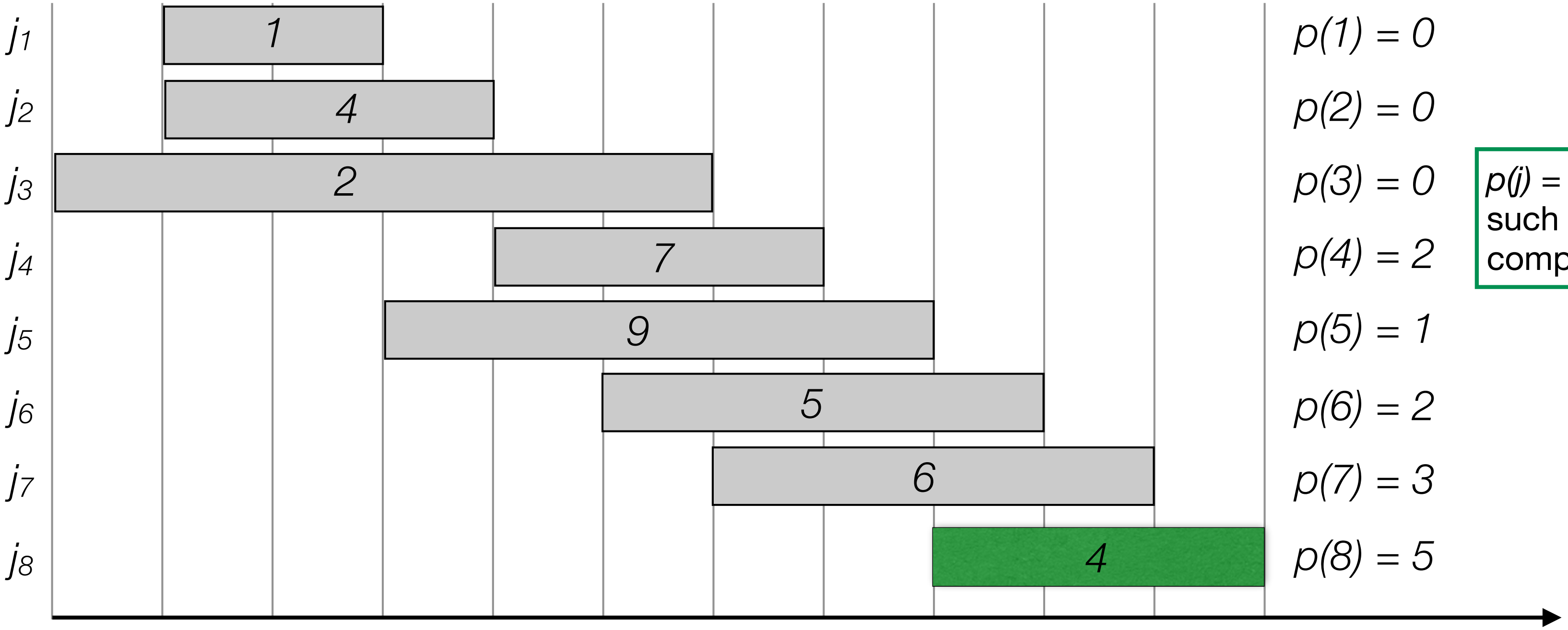
Weighted interval scheduling

- Label/sort jobs by finishing time: $f_1 \leq f_2 \leq \ldots \leq f_n$
- Optimal solution OPT:
  - Case 1. OPT selects last job

$$OPT = v_n + \text{optimal solution to subproblem on } 1,\ldots,p(n)$$

  - Case 2. OPT does not select last job

$$OPT = \text{optimal solution to subproblem on } 1,\ldots,n\text{-}1$$



$p(1) = 0$

$p(2) = 0$

$p(3) = 0$

$p(4) = 2$

$p(5) = 1$

$p(6) = 2$

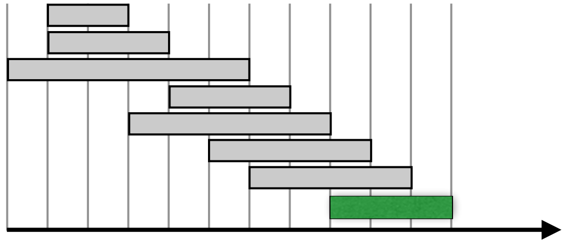
$p(7) = 3$

$p(8) = 5$

$p(j)$ = largest index $i < j$ such that job i is compatible with j.

Weighted interval scheduling

- OPT(j) = value of optimal solution to the problem consisting job requests 1,2,..,*j*.



Weighted interval scheduling

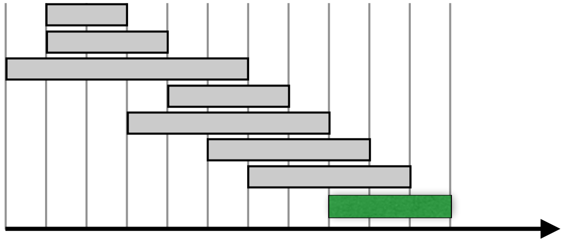
- OPT(j) = value of optimal solution to the problem consisting job requests 1,2,..,*j*.

  - Case 1. OPT(*j*) selects job j

    OPT(j) = v_j + optimal solution to subproblem on 1,…,p(j)

  - Case 2. OPT(*j*) does not select job j

    OPT = optimal solution to subproblem 1,…j-1



Weighted interval scheduling

- OPT(j) = value of optimal solution to the problem consisting job requests 1,2,..,*j*.

  - Case 1. OPT(*j*) selects job j

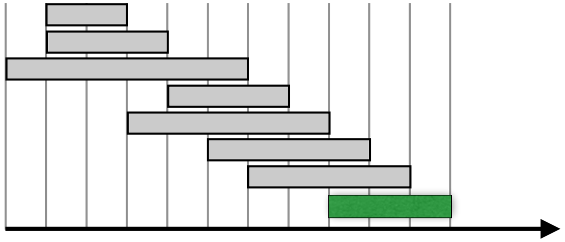
    OPT(j) = v_j + optimal solution to subproblem on 1,…,p(j)

  - Case 2. OPT(*j*) does not select job j

    OPT = optimal solution to subproblem 1,…j-1

- Recurrence:

$$OPT(j) = \begin{cases} 0 & \texttt{if } j = 0 \\ \max\{v_j + OPT(p(j)), OPT(j-1)\} & \texttt{otherwise} \end{cases}$$



Weighted interval scheduling: brute force

$$OPT(j) = \begin{cases} 0 & \texttt{if } j = 0 \\ \max\{v_j + OPT(p(j)), OPT(j-1)\} & \texttt{otherwise} \end{cases}$$

Weighted interval scheduling: brute force

$$OPT(j) = \begin{cases} 0 & \texttt{if } j = 0 \\ \max\{v_j + OPT(p(j)), OPT(j-1)\} & \texttt{otherwise} \end{cases}$$

```
Input: n, s[1..n], f[1..n], v[1..n]

Sort jobs by finish time so that f[1] ≤ f[2]≤ … ≤ f[n]
Compute p[1], p[2], …, p[n]
Compute-BruteForce–Opt(n)

Compute-Brute-Force-Opt(j)
if j = 0
  return 0
else
  return max(v[j] + Compute-Brute-Force-Opt(p[j]),
          Compute-Brute-Force-Opt(j-1))
```

Weighted interval scheduling: brute force

$$OPT(j) = \begin{cases} 0 & \texttt{if } j = 0 \\ \max\{v_j + OPT(p(j)), OPT(j-1)\} & \texttt{otherwise} \end{cases}$$

```
Input: n, s[1..n], f[1..n], v[1..n]

Sort jobs by finish time so that f[1] ≤ f[2]≤ … ≤ f[n]
Compute p[1], p[2], …, p[n]
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if j = 0
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Weighted interval scheduling: brute force

$$OPT(j) = \begin{cases} 0 & \texttt{if } j = 0 \\ \max\{v_j + OPT(p(j)), OPT(j-1)\} & \texttt{otherwise} \end{cases}$$

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Input: n, s[1..n], f[1..n], v[1..n]

Sort jobs by finish time so that f[1] ≤ f[2]≤ … ≤ f[n]
Compute p[1], p[2], …, p[n]
Compute-BruteForce—Opt(n)

Compute-Brute-Force-Opt(j)
if j = 0
  return 0
else
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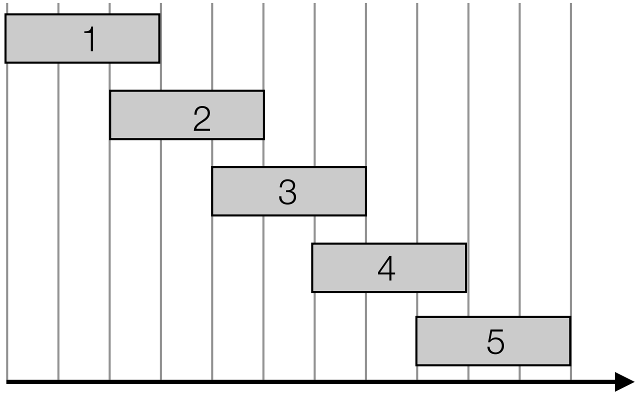

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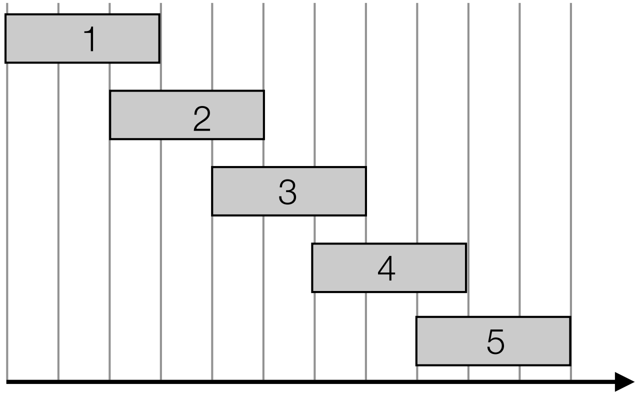

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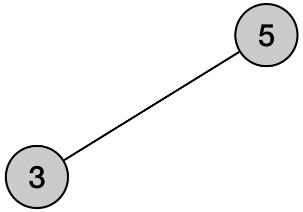

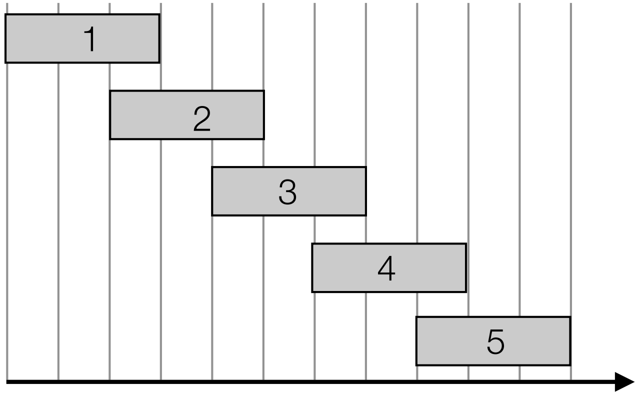

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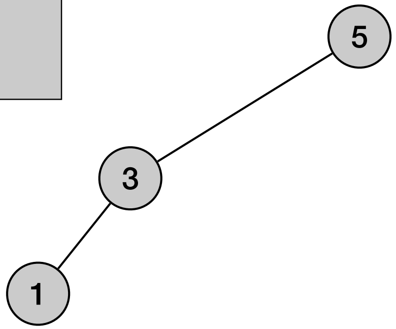

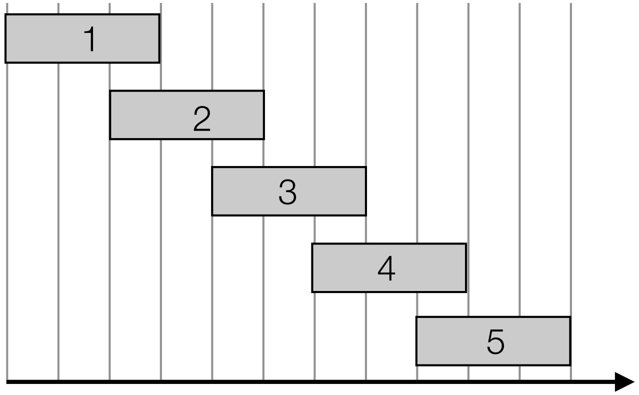

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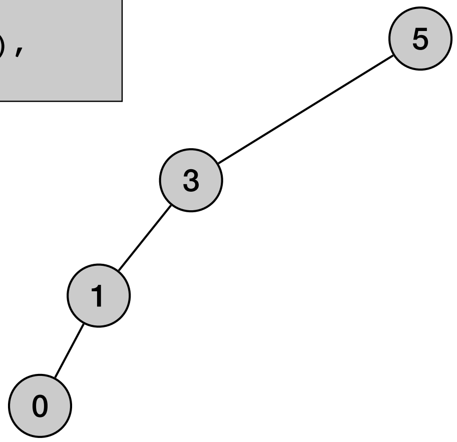

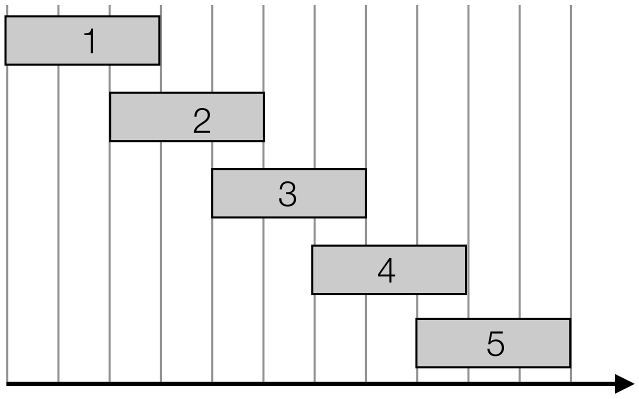

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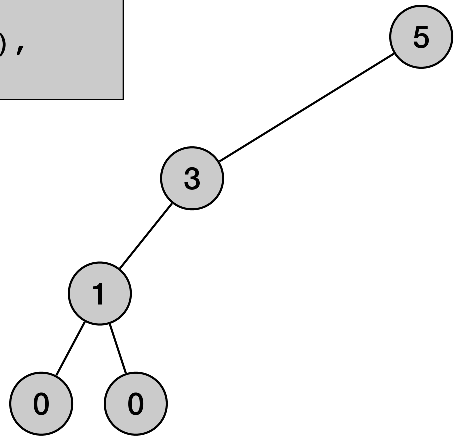

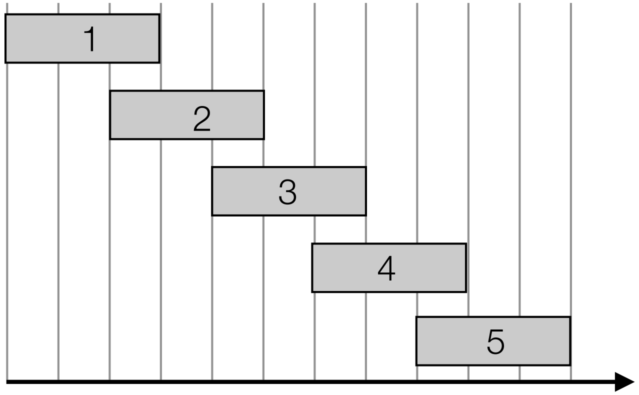

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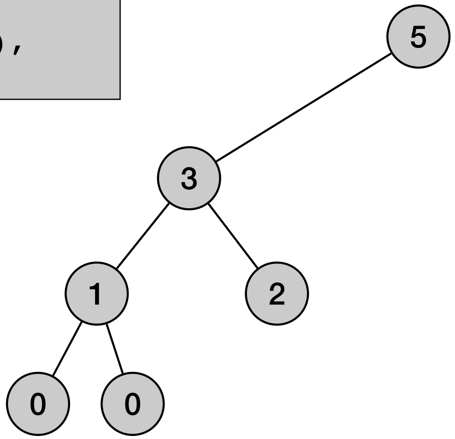

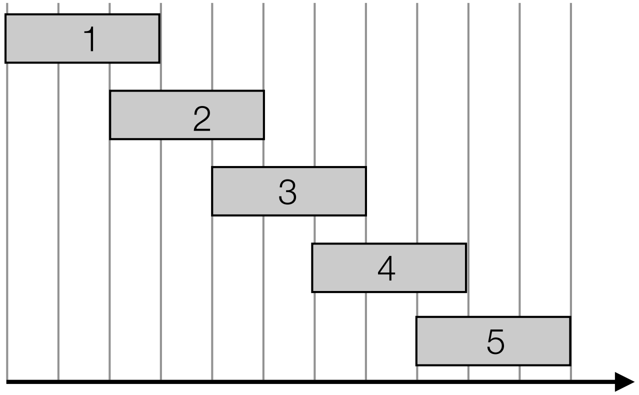

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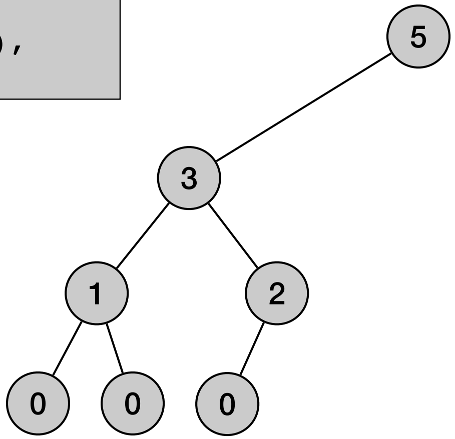

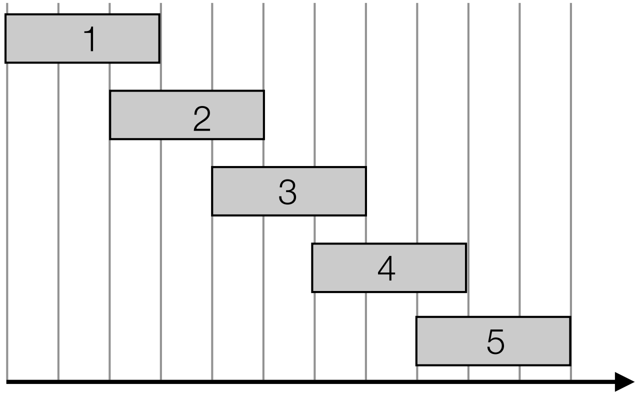

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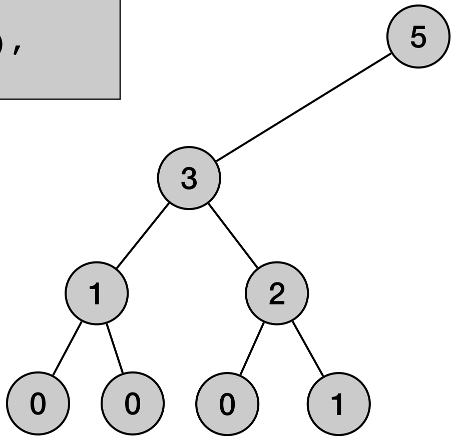

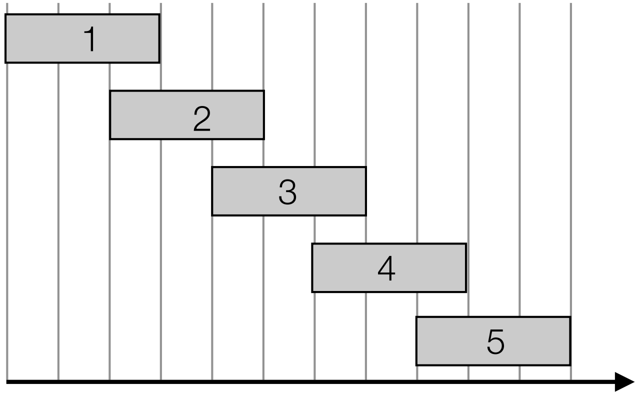

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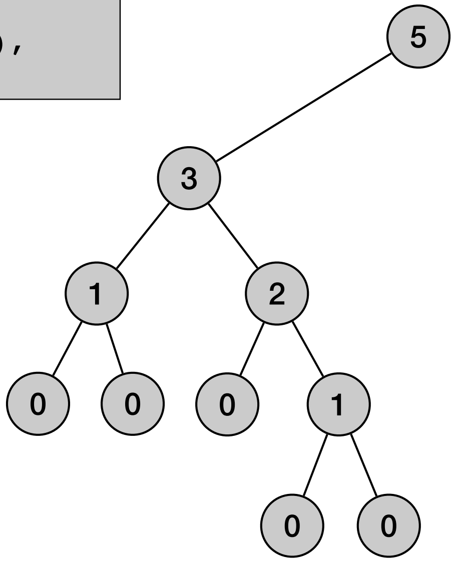

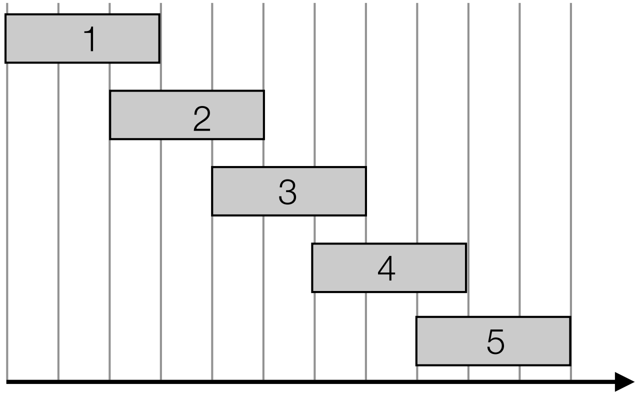

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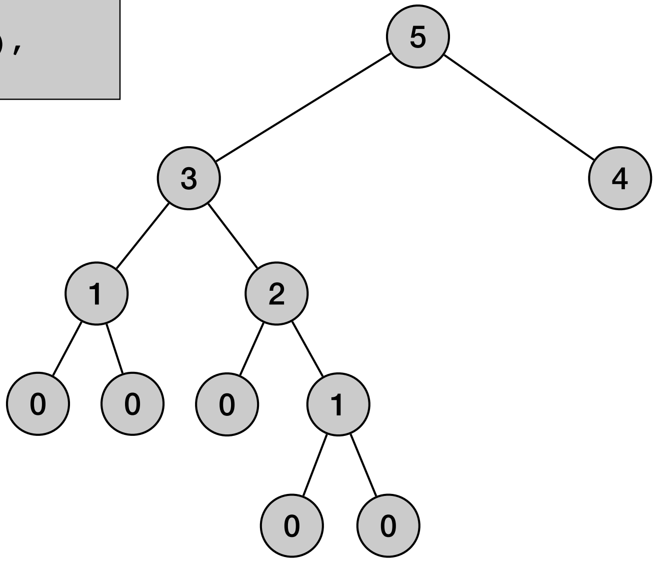

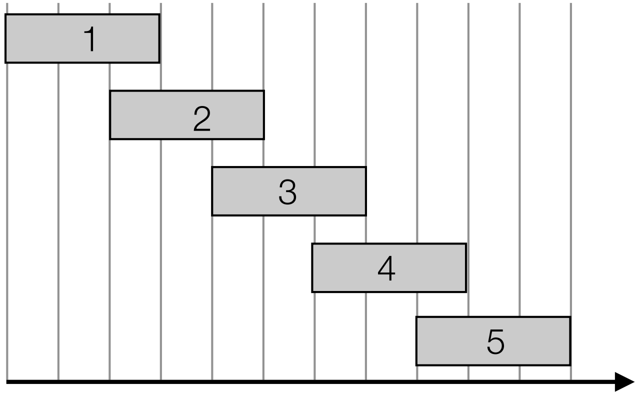

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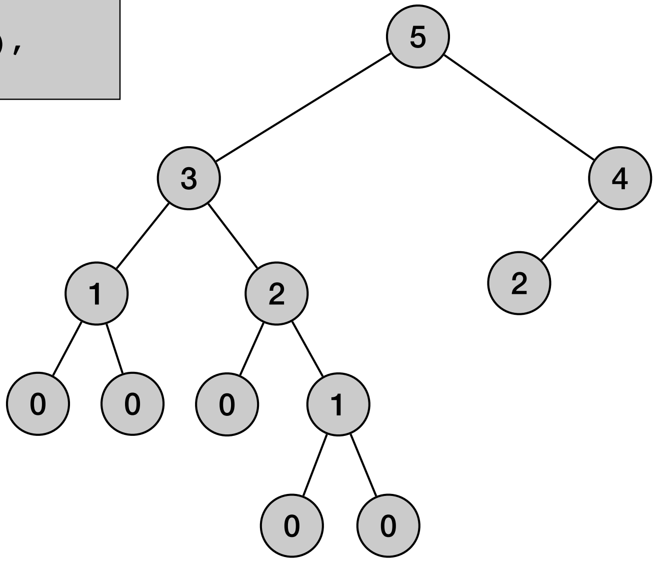

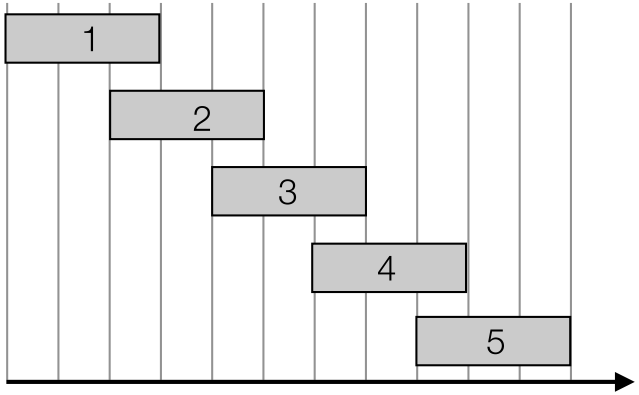

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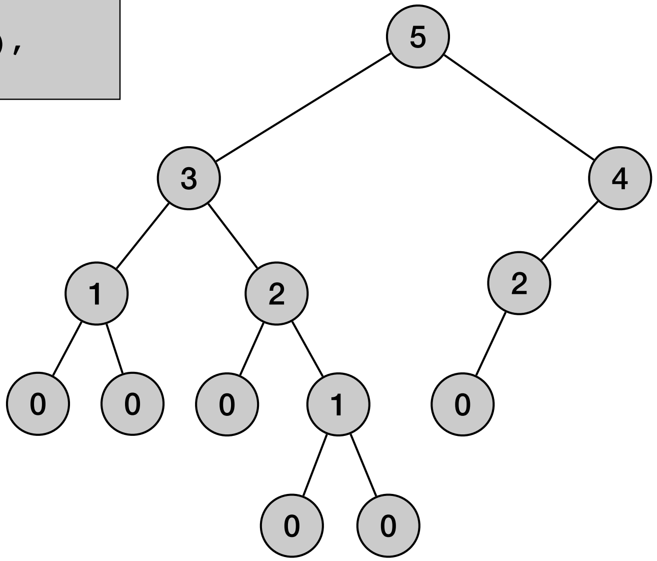

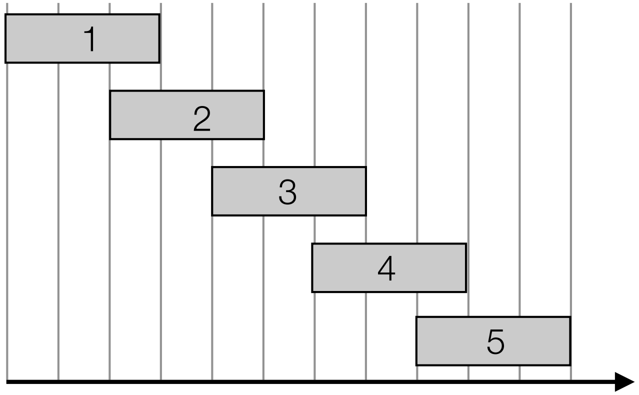

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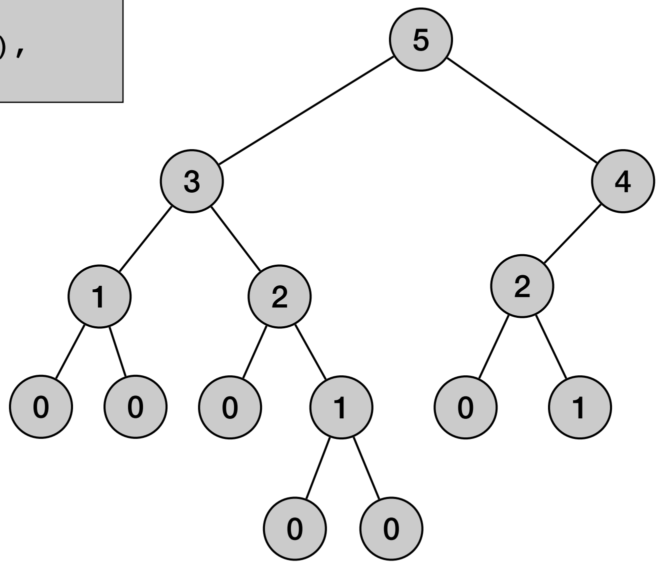

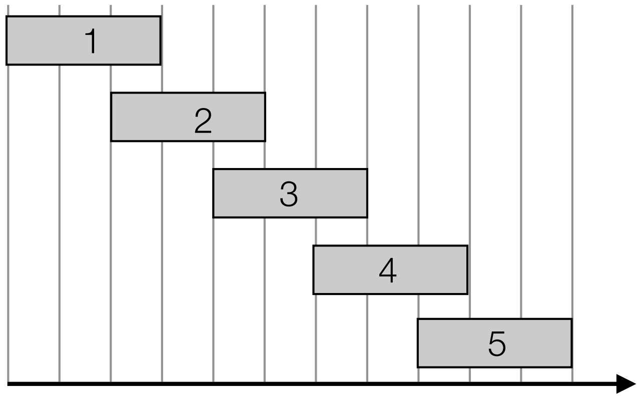

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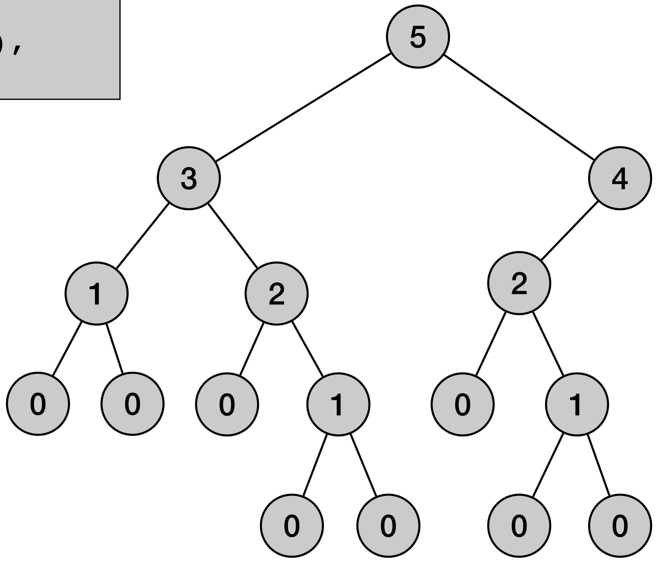

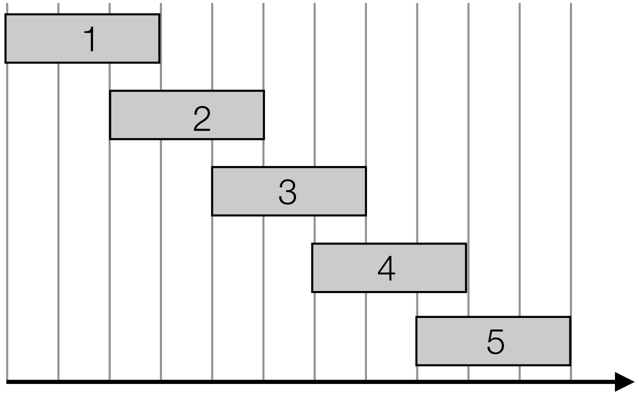

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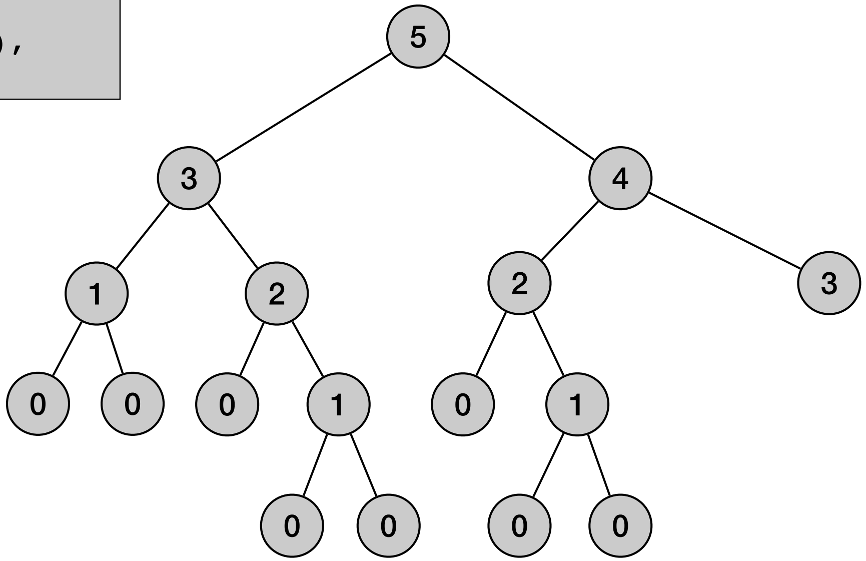

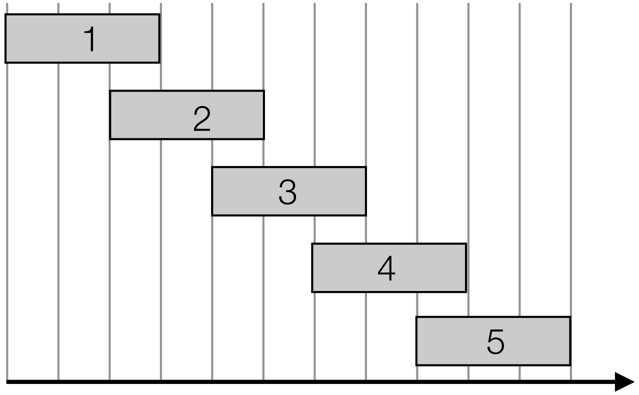

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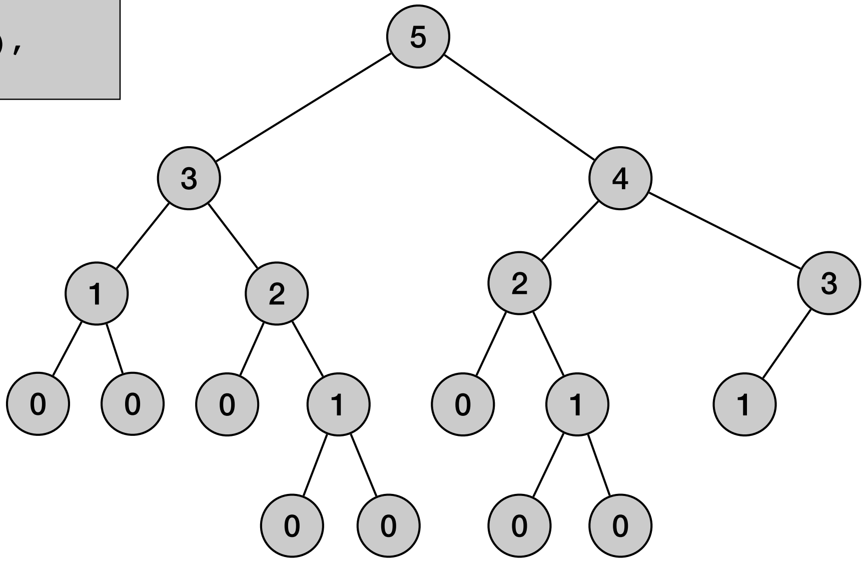

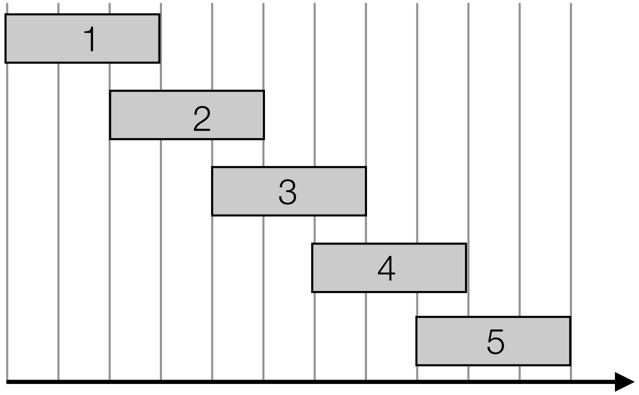

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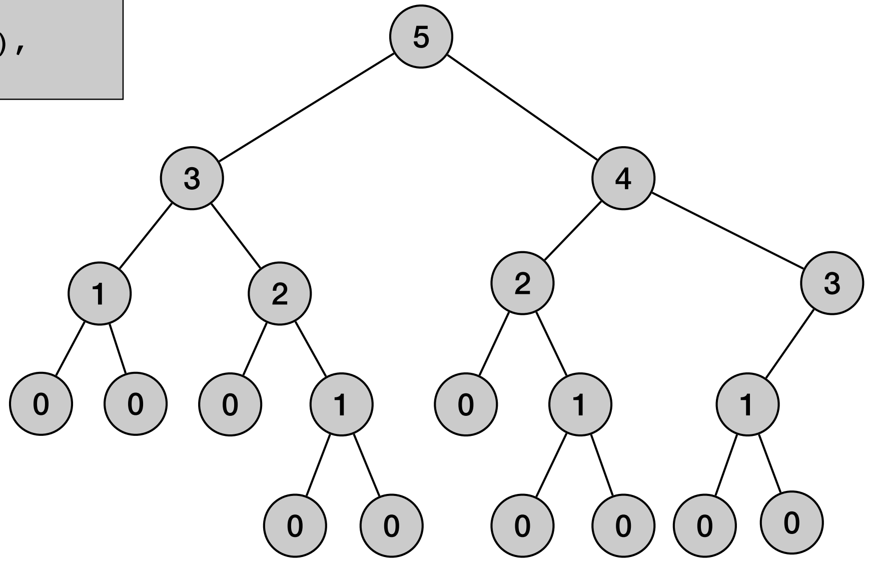

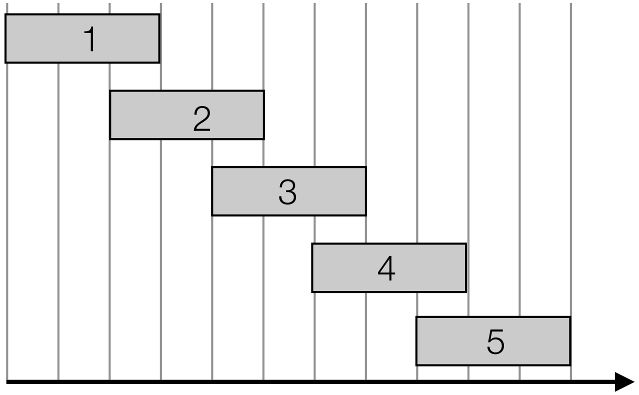

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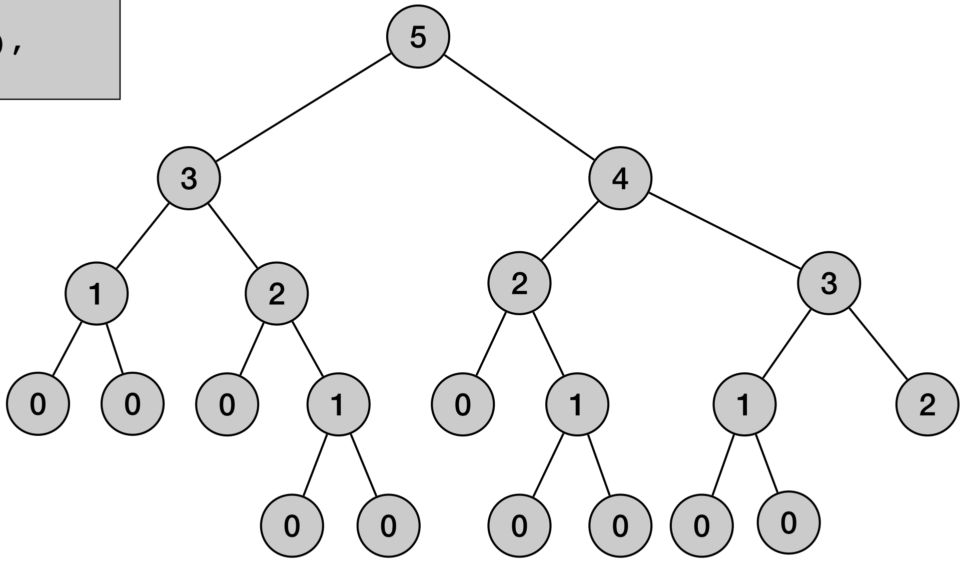

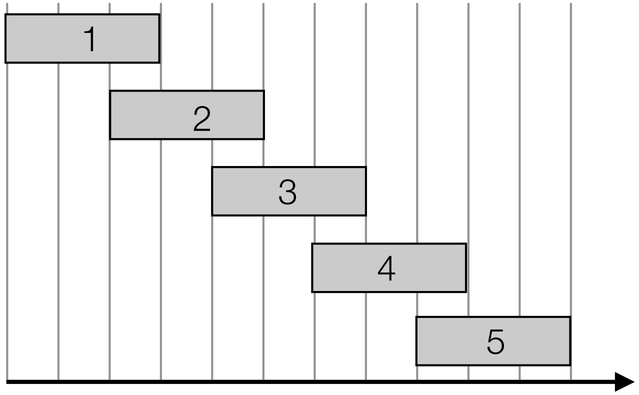

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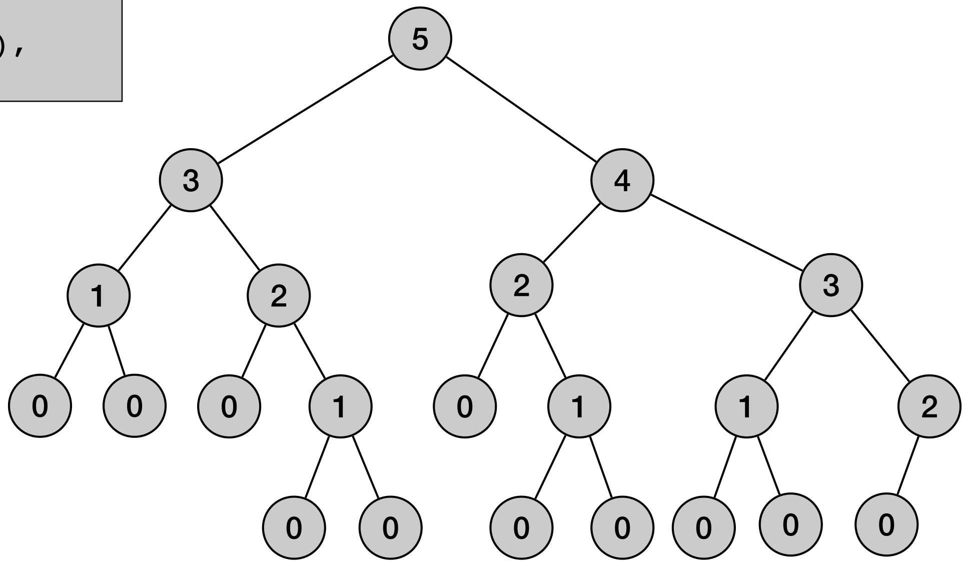

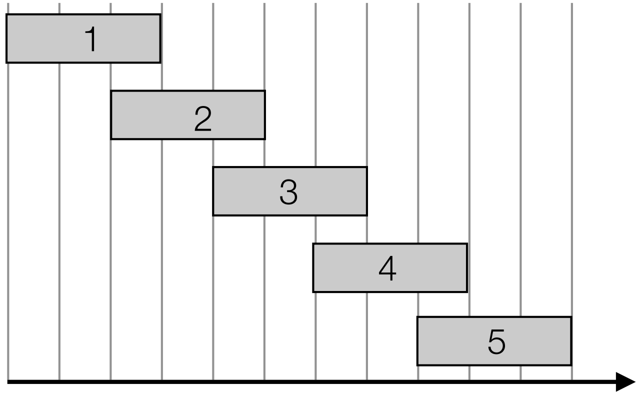

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Input: n, s[1..n], f[1..n], v[1..n]

Sort jobs by finish time so that f[1] ≤ f[2]≤ … ≤ f[n]
Compute p[1], p[2], …, p[n]
Compute-BruteForce—Opt(n)

Compute-Brute-Force-Opt(j)
if j = 0
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else
  return max(v[j] + Compute-Brute-Force-Opt(p[j]),
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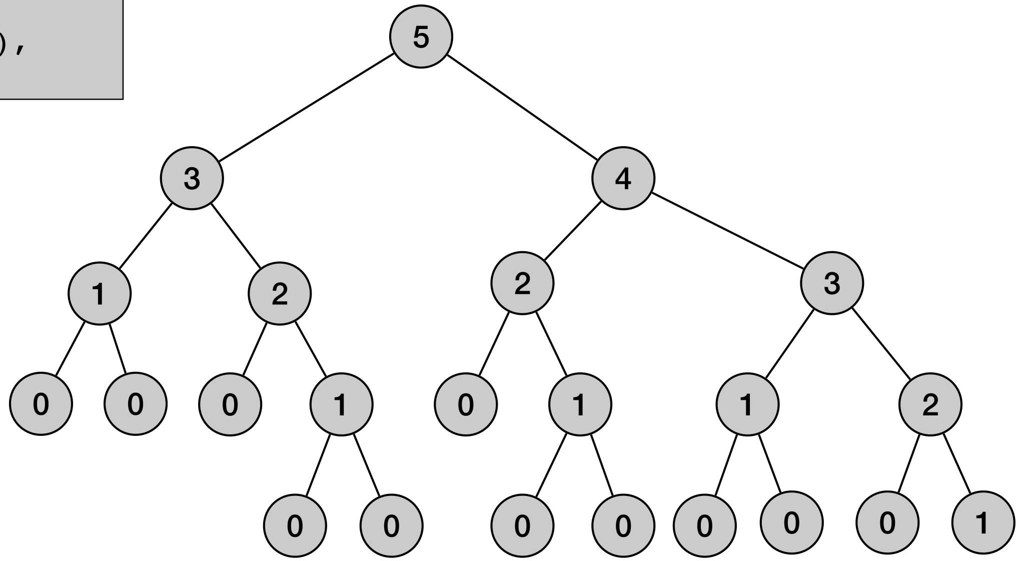

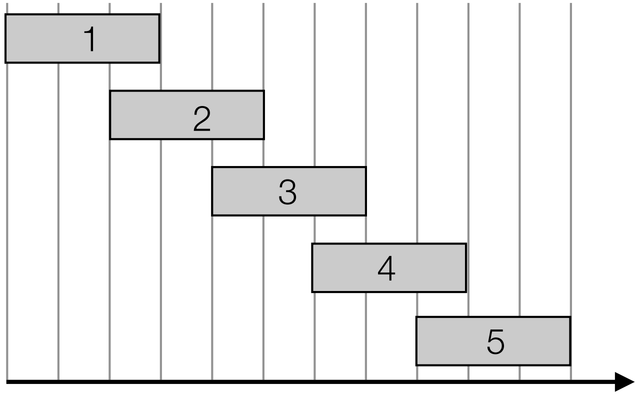

Weighted interval scheduling: brute force

$$OPT(j) = \begin{cases} 0 & \texttt{if } j = 0 \\ \max\{v_j + OPT(p(j)), OPT(j-1)\} & \texttt{otherwise} \end{cases}$$

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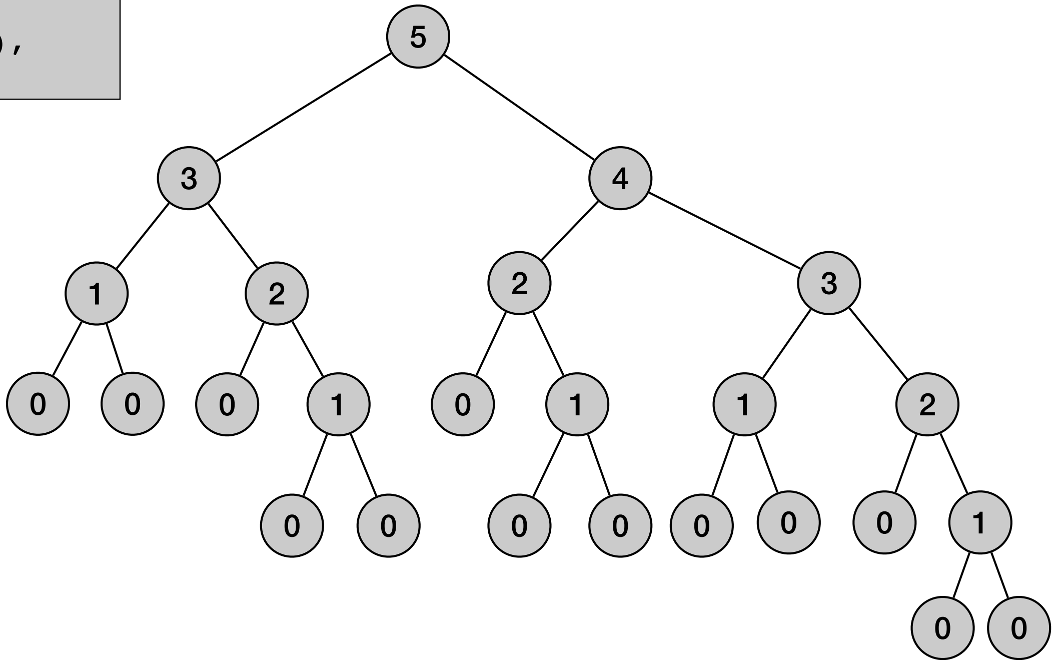

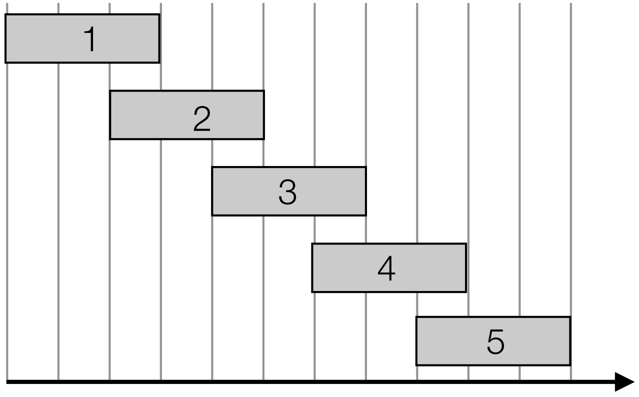

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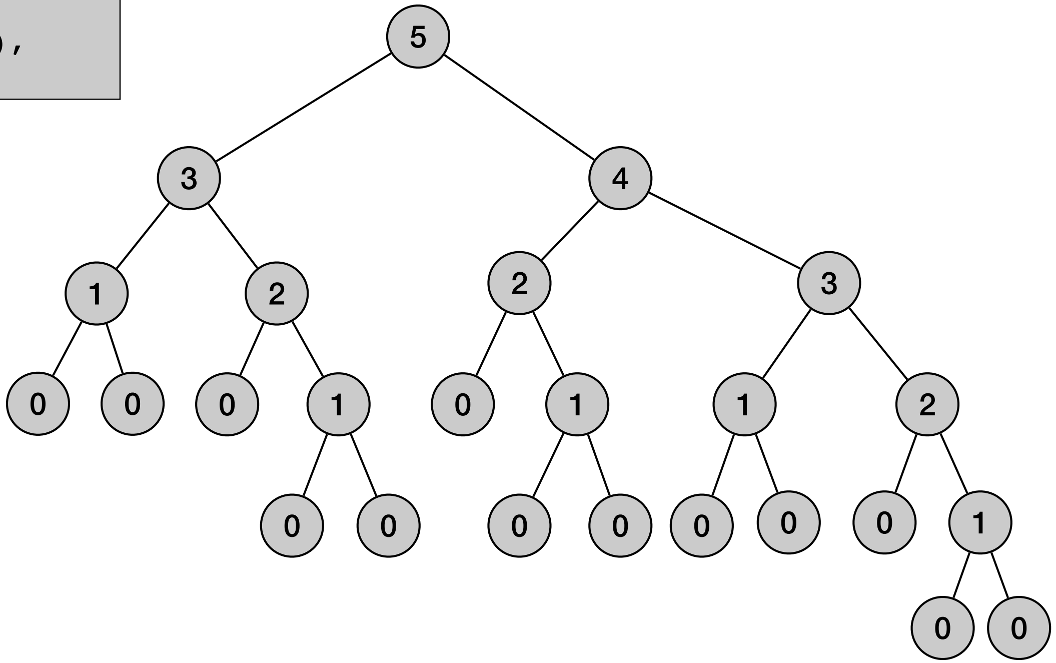
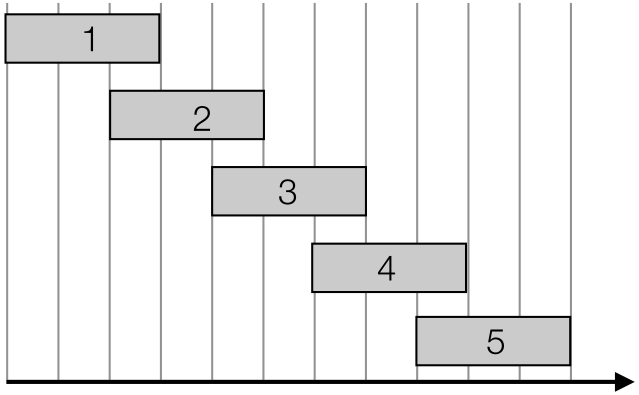
time $\Theta(2^n)$





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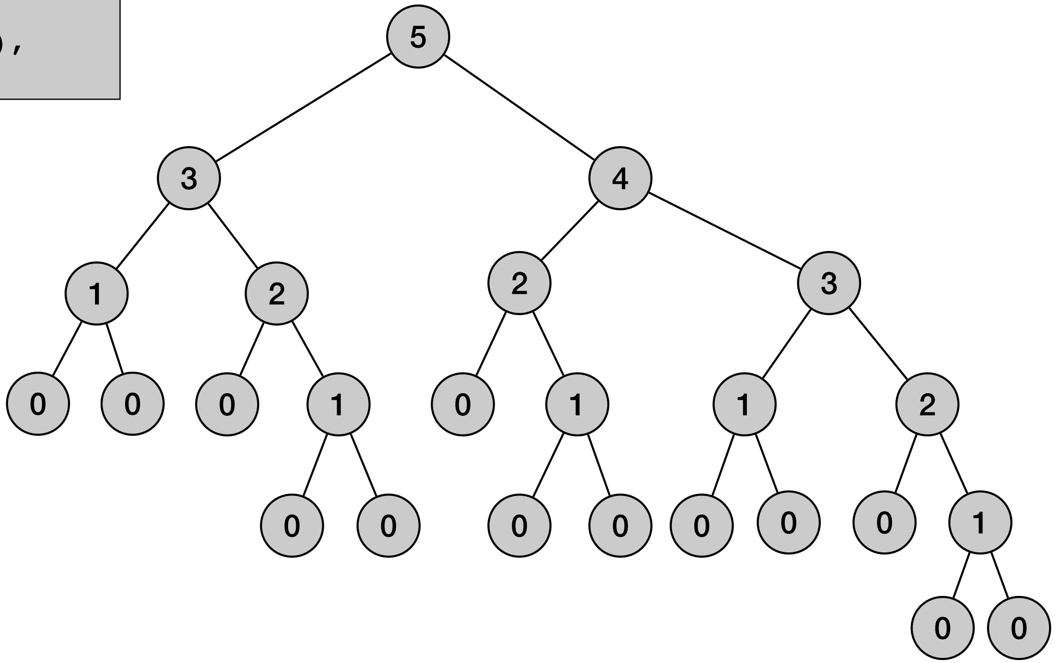

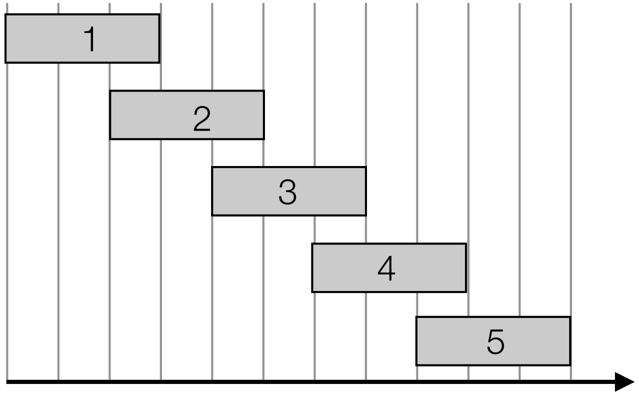

Weighted interval scheduling: memoization

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Input: n, s[1..n], f[1..n], v[1..n]

Sort jobs by finish time so that f[1] ≤ f[2]≤ … ≤ f[n]
Compute p[1], p[2], …, p[n]

for j=1 to n
  M[j] = null
M[0] = 0.
Compute-Memoized-Opt(n)

Compute-Memoized-Opt(j)
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Weighted interval scheduling: memoization

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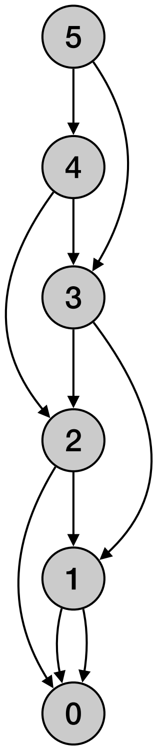

Weighted interval scheduling: memoization

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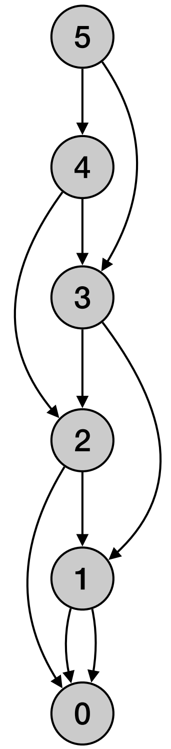

- Running time O(n log n):

Weighted interval scheduling: memoization

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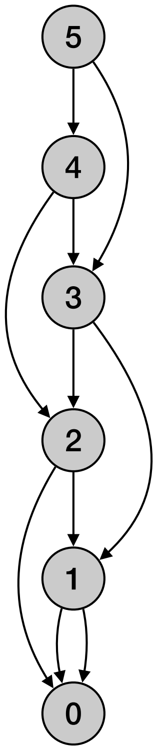

- Running time O(n log n):
  - Sorting takes O(n log n) time.

Weighted interval scheduling: memoization

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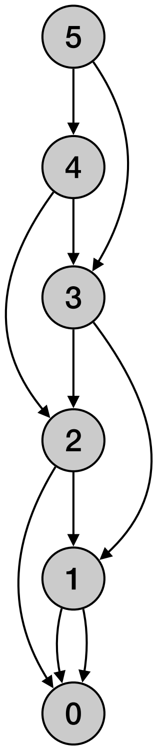

- Running time O(n log n):
  - Sorting takes O(n log n) time.
  - Computing p(n): O(n log n) - use log n time to find each p(i).

Weighted interval scheduling: memoization

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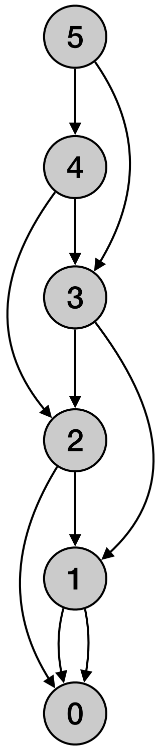

- Running time O(n log n):
  - Sorting takes O(n log n) time.
  - Computing p(n): O(n log n) - use log n time to find each p(i).
  - Each subproblem solved once.

Weighted interval scheduling: memoization

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Input: n, s[1..n], f[1..n], v[1..n]

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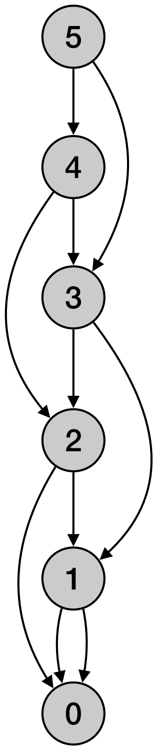

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  - Each subproblem solved once.
  - Time to solve a subproblem constant.

Weighted interval scheduling: memoization

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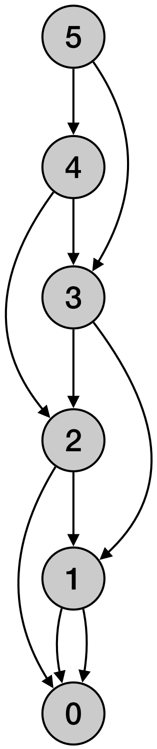

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  - Sorting takes O(n log n) time.
  - Computing p(n): O(n log n) - use log n time to find each p(i).
  - Each subproblem solved once.
  - Time to solve a subproblem constant.
- Space O(n)

Weighted interval scheduling: memoization

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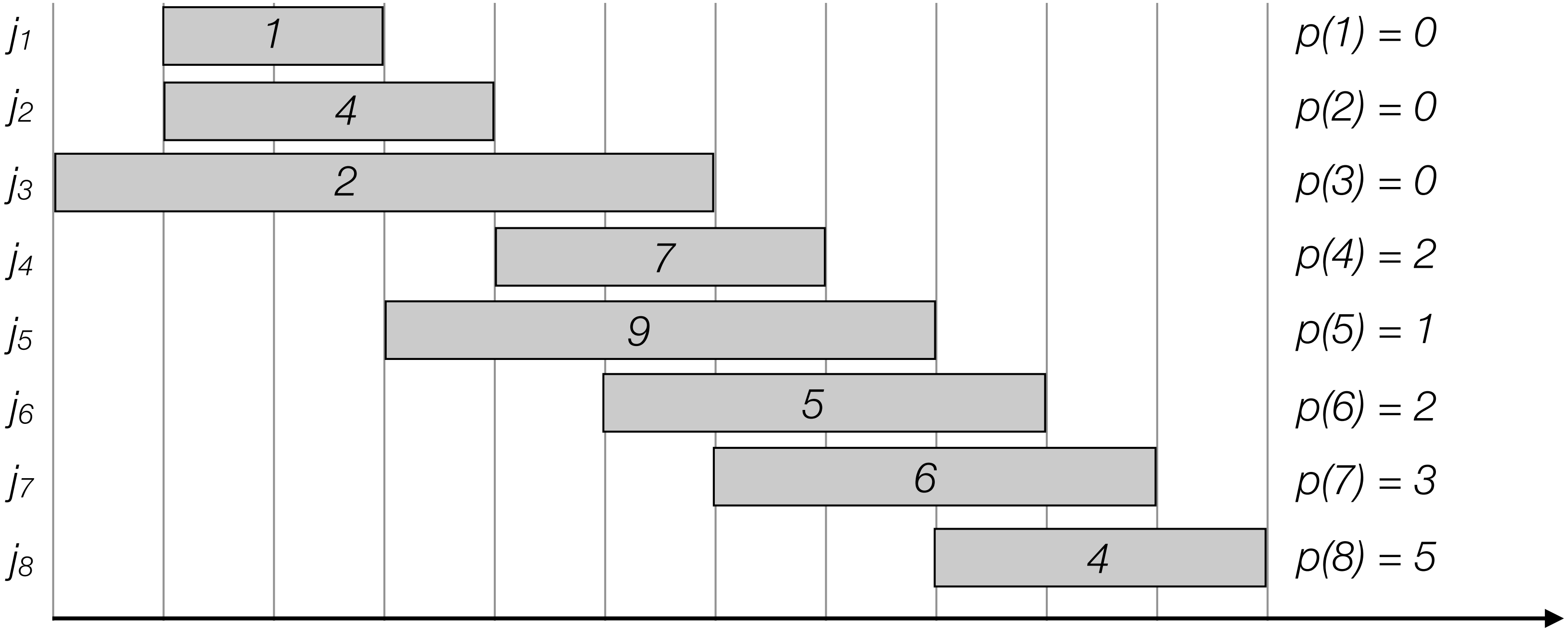

$p(1) = 0$

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$p(7) = 3$

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i	M[i]
0	0
1	
2	
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8	

Weighted interval scheduling: memoization

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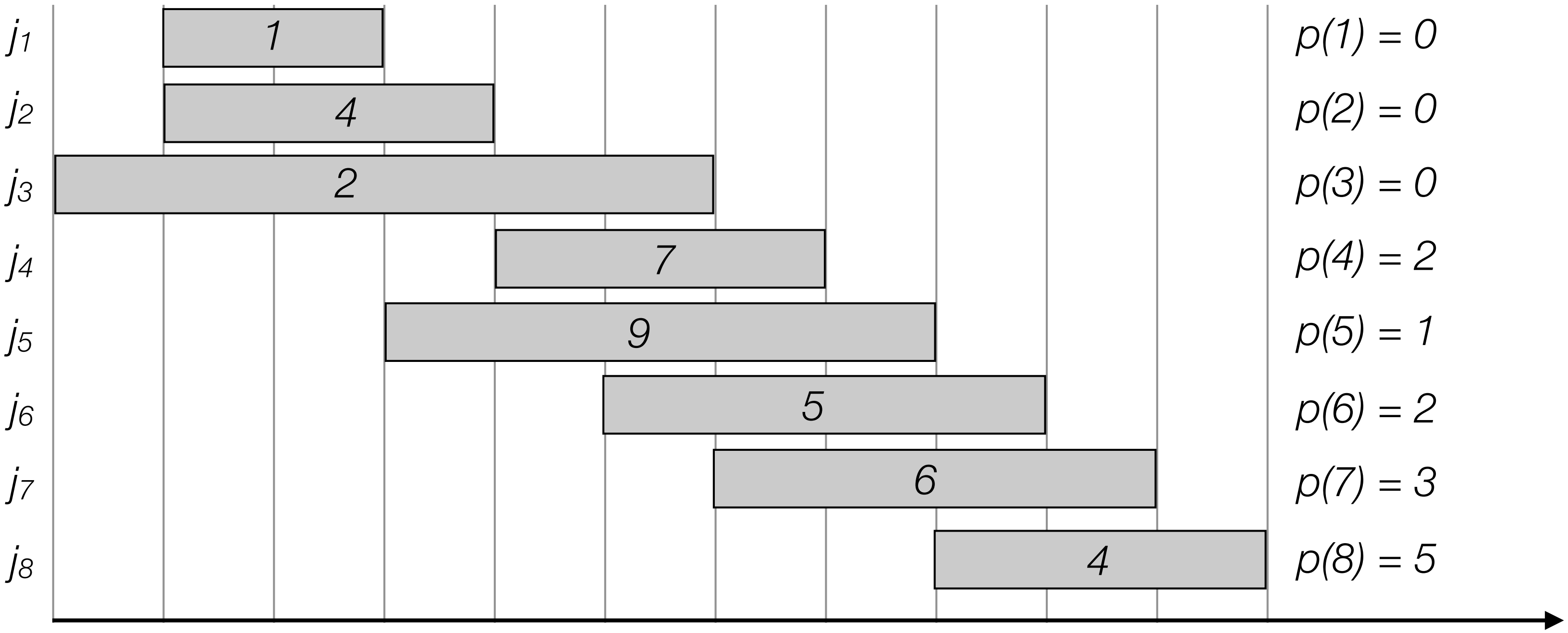

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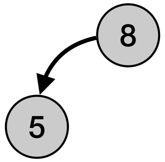

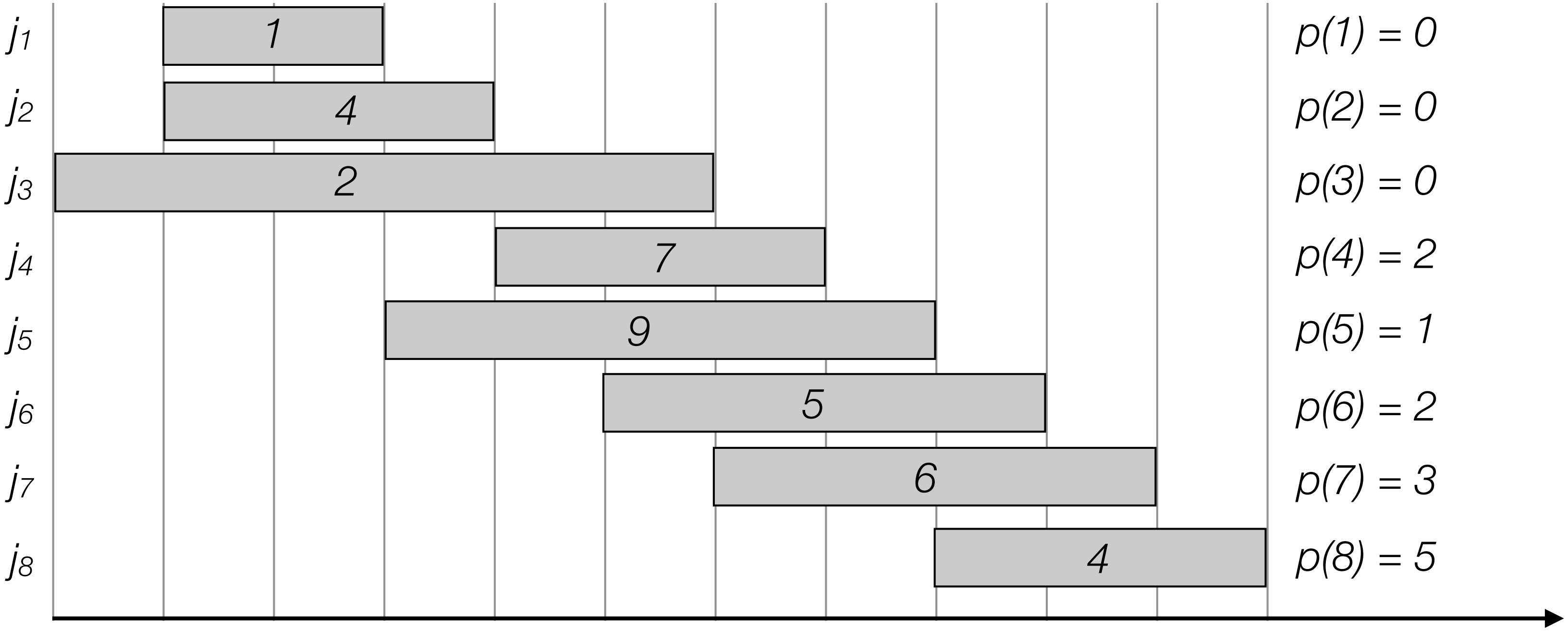

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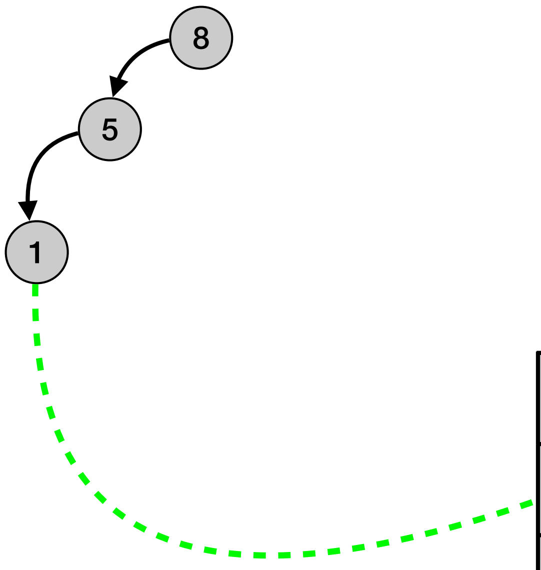

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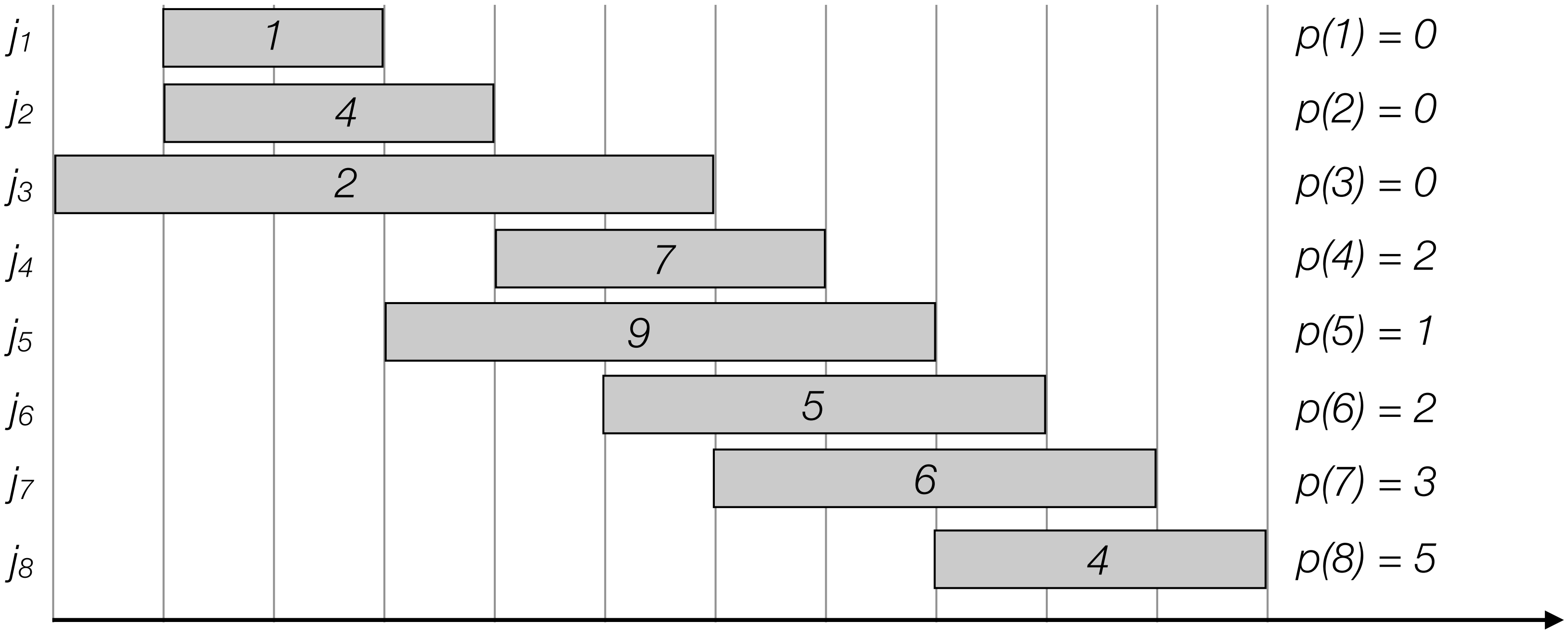

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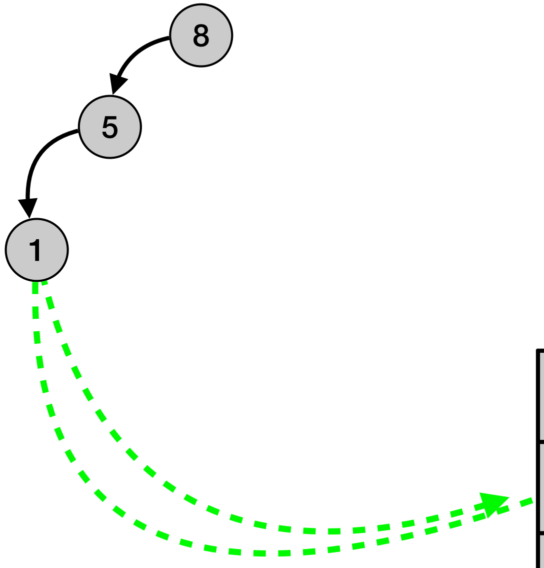

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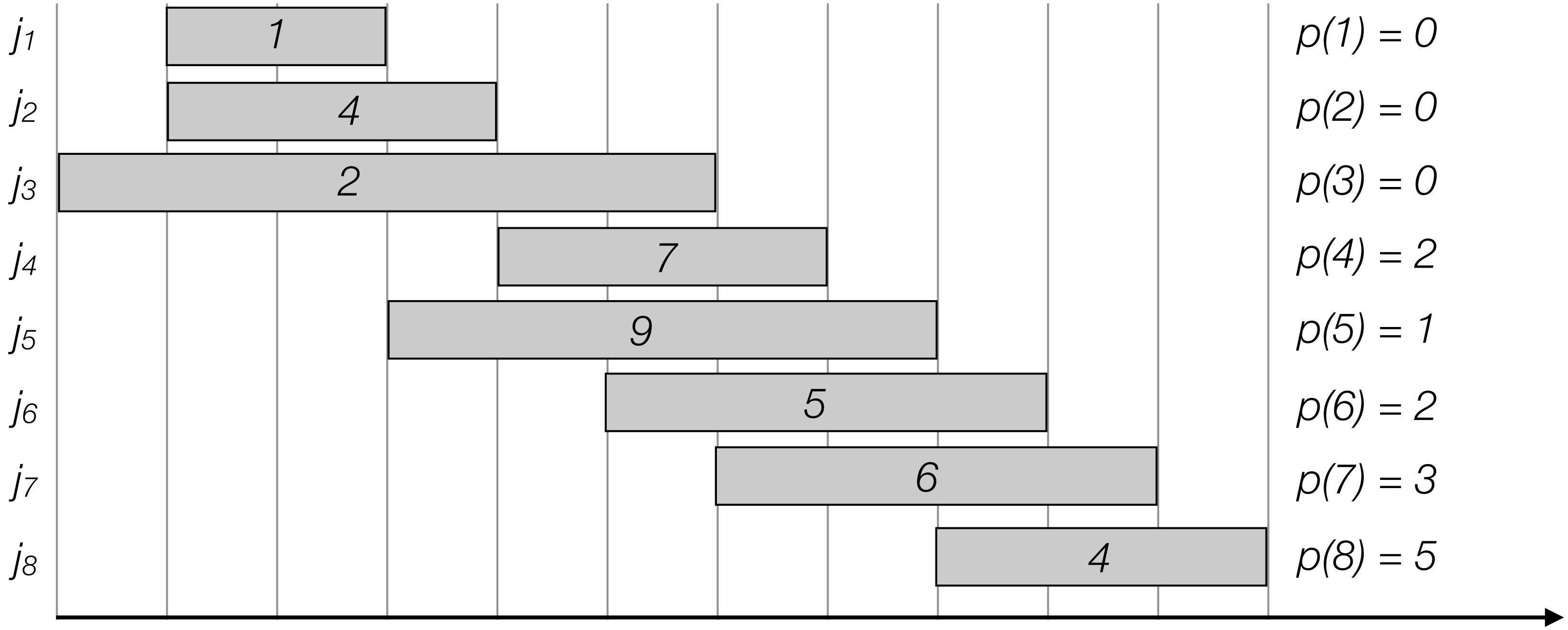

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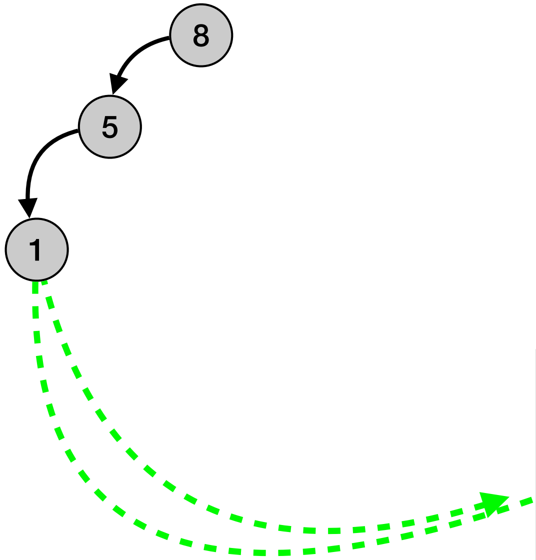

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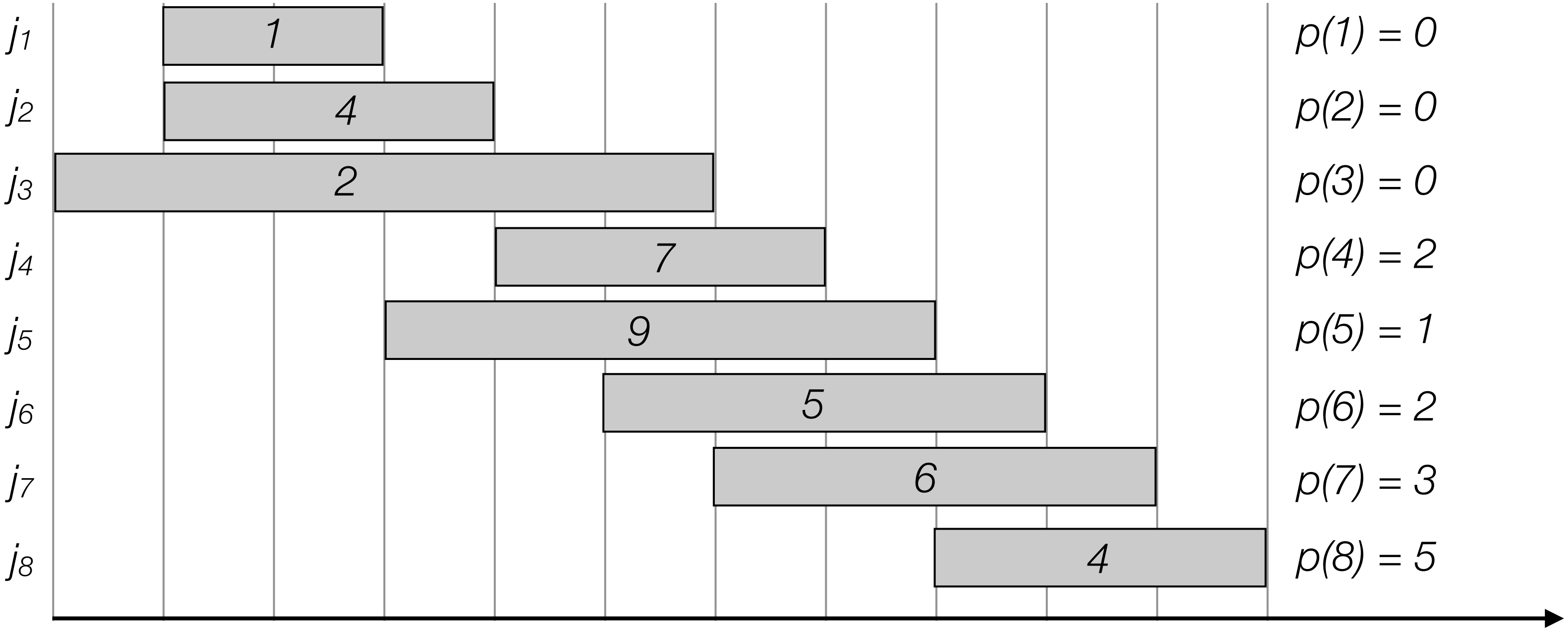

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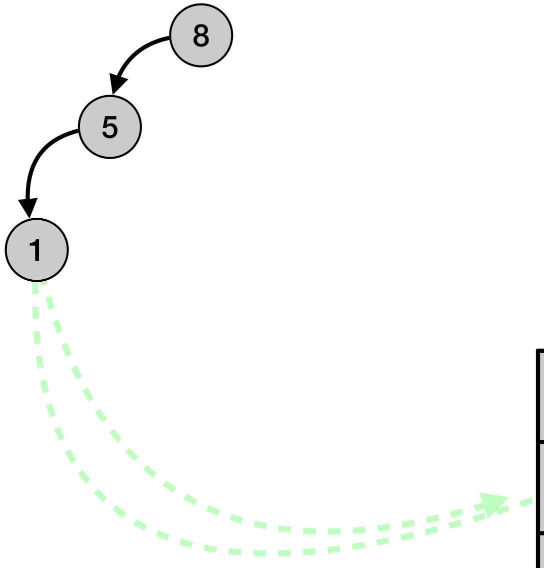

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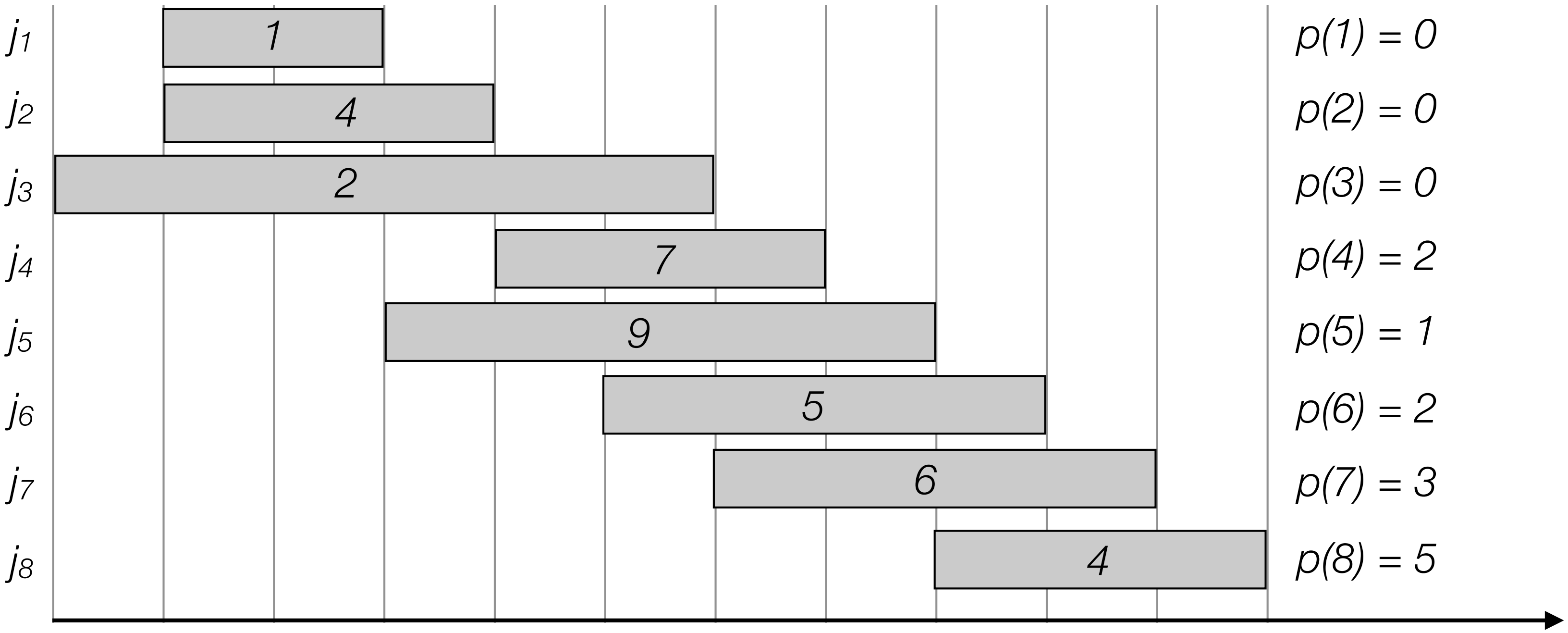

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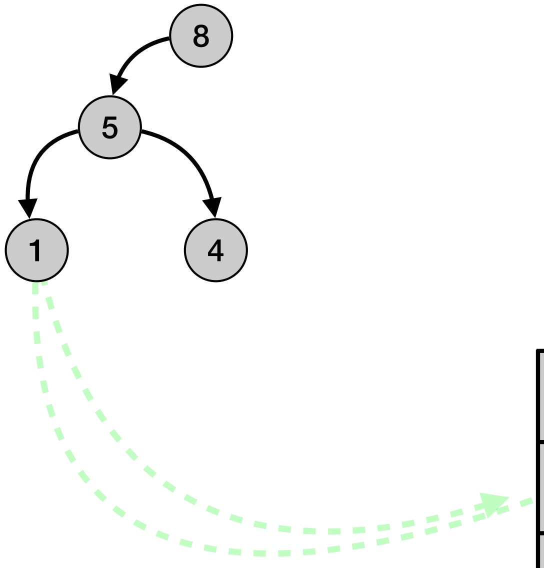

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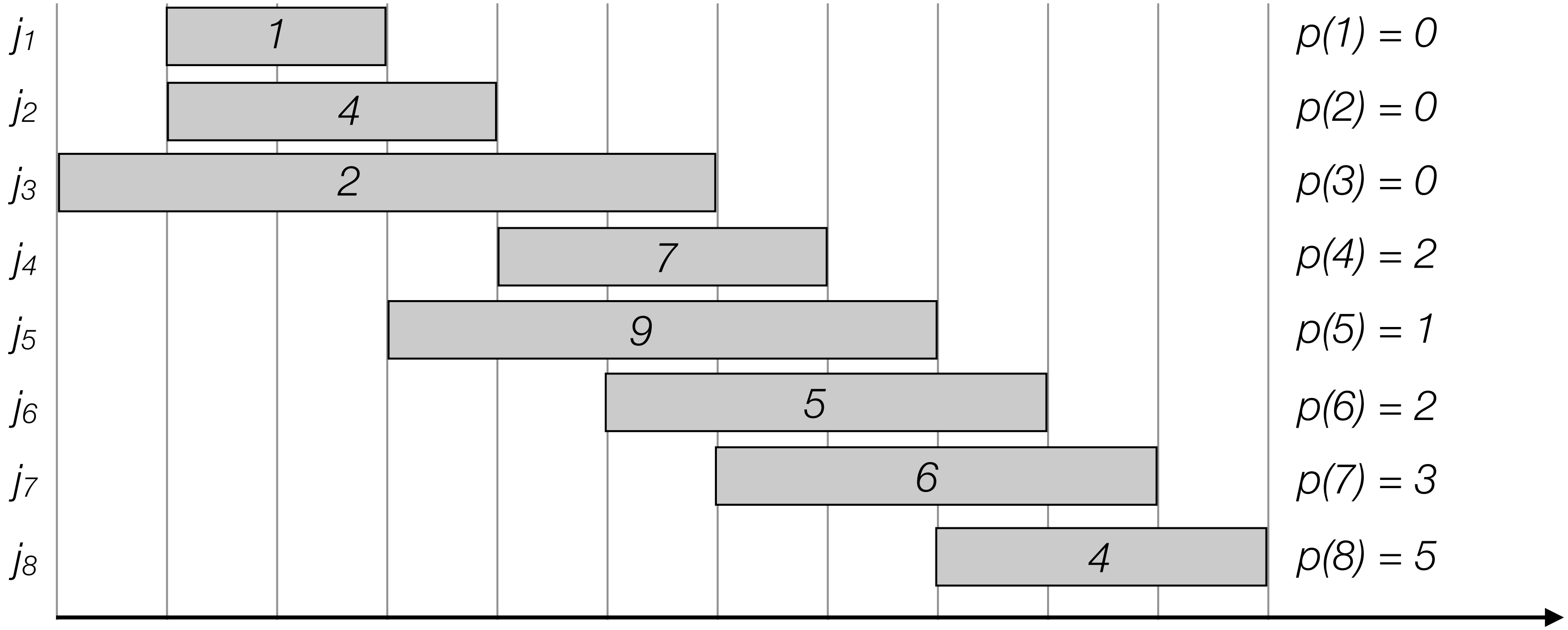
8	



$p(1) = 0$

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$p(3) = 0$

$p(4) = 2$

$p(5) = 1$

$p(6) = 2$

$p(7) = 3$

$p(8) = 5$

Weighted interval scheduling: memoization

```
Input: n, s[1..n], f[1..n], v[1..n]

Sort jobs by finish time so that f[1] ≤ f[2]≤ … ≤ f[n]
Compute p[1], p[2], …, p[n]

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Compute-Memoized-Opt(j)
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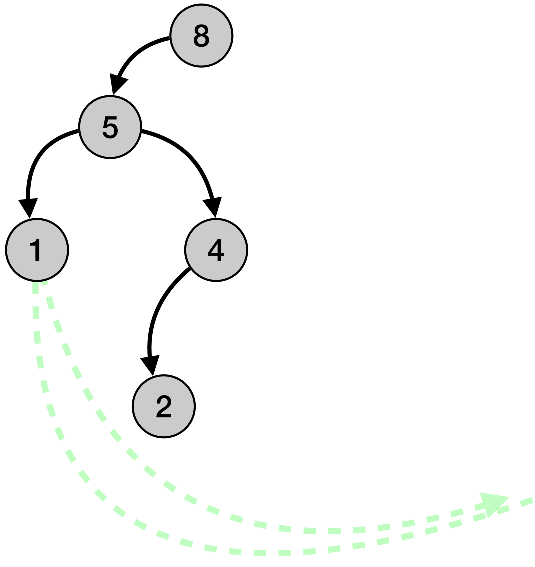
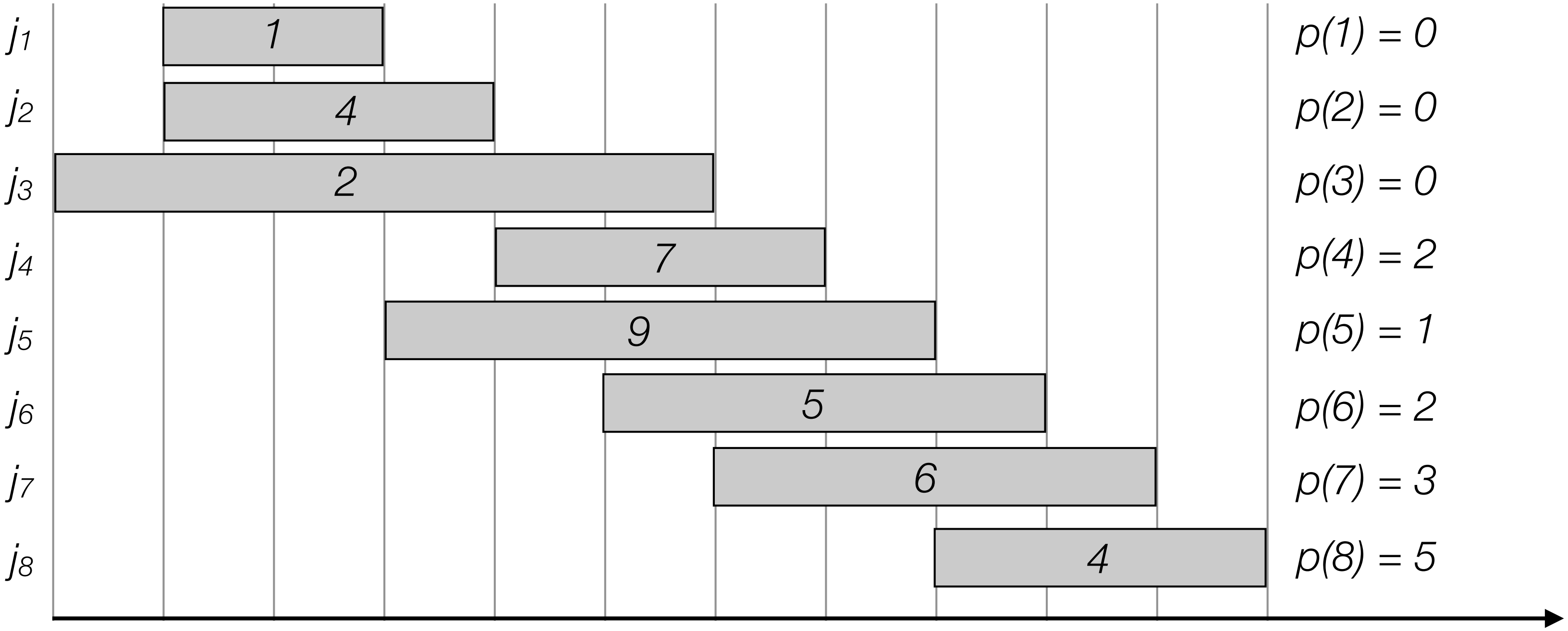
return M[j]
```



i	M[i]
0	0
1	1
2	
3	
4	
5	
6	
7	
8	



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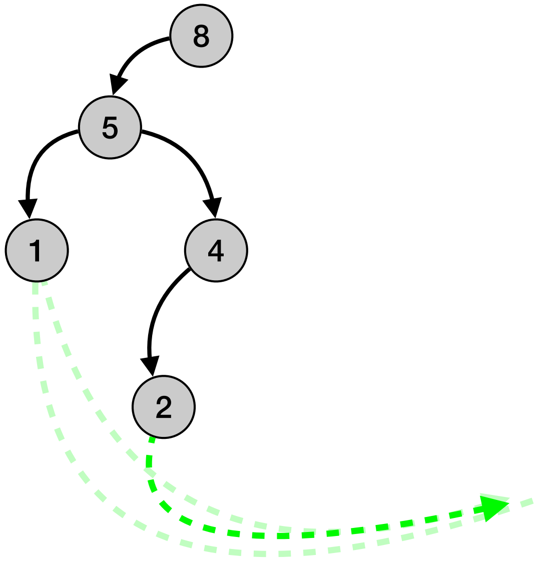
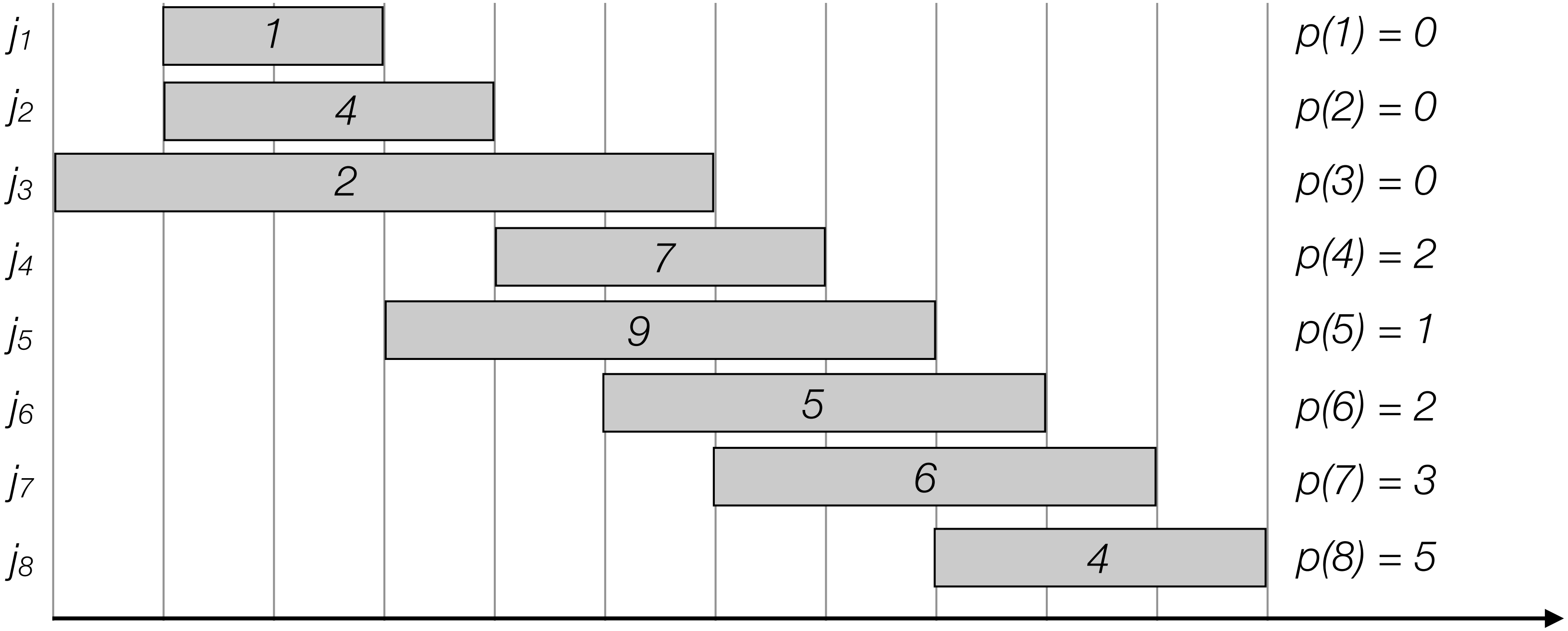


i	M[i]
0	0
1	1
2	
3	
4	
5	
6	
7	
8	



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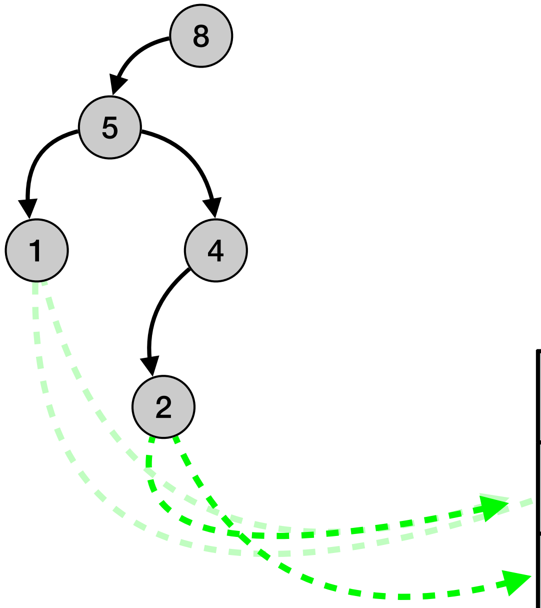
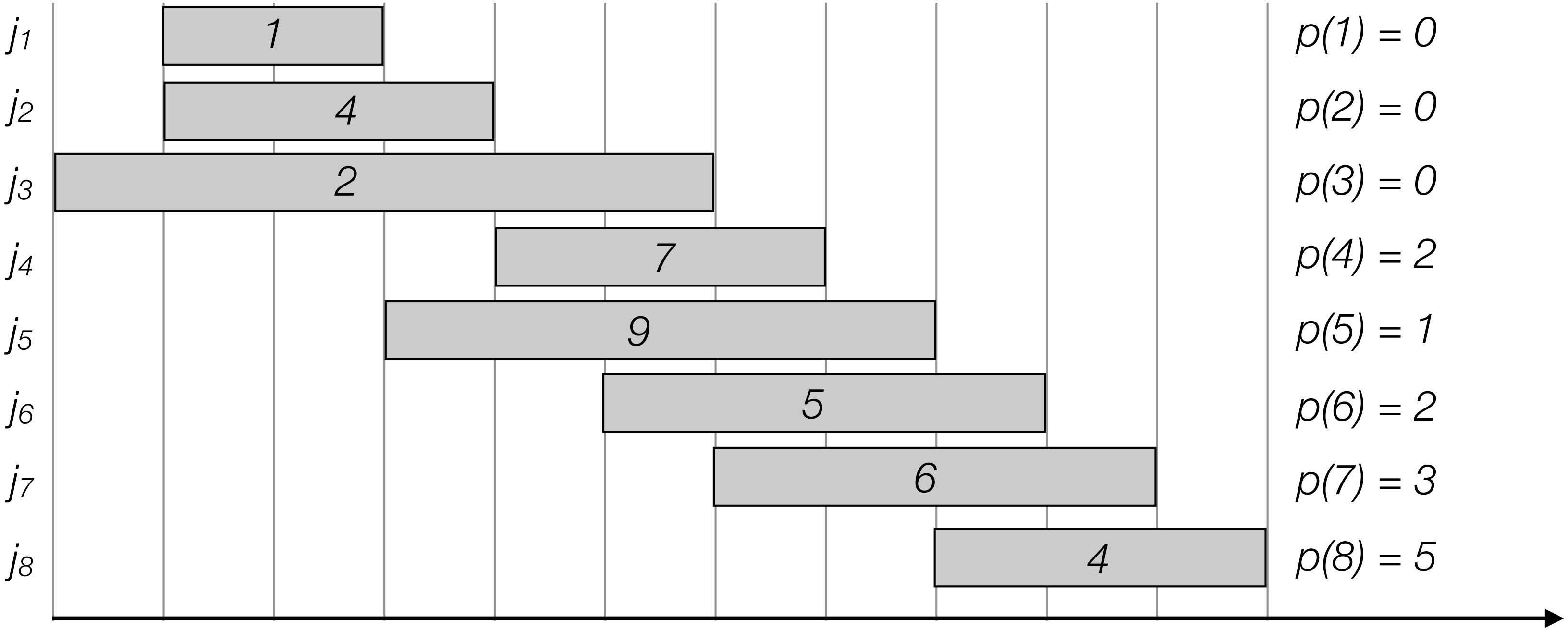


i	M[i]
0	0
1	1
2	
3	
4	
5	
6	
7	
8	



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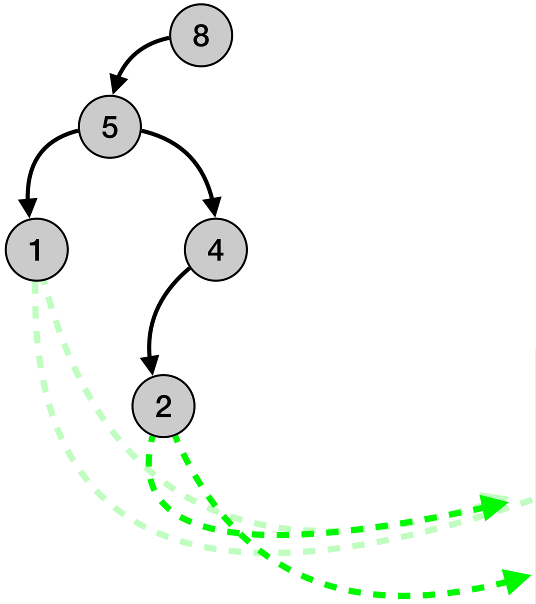
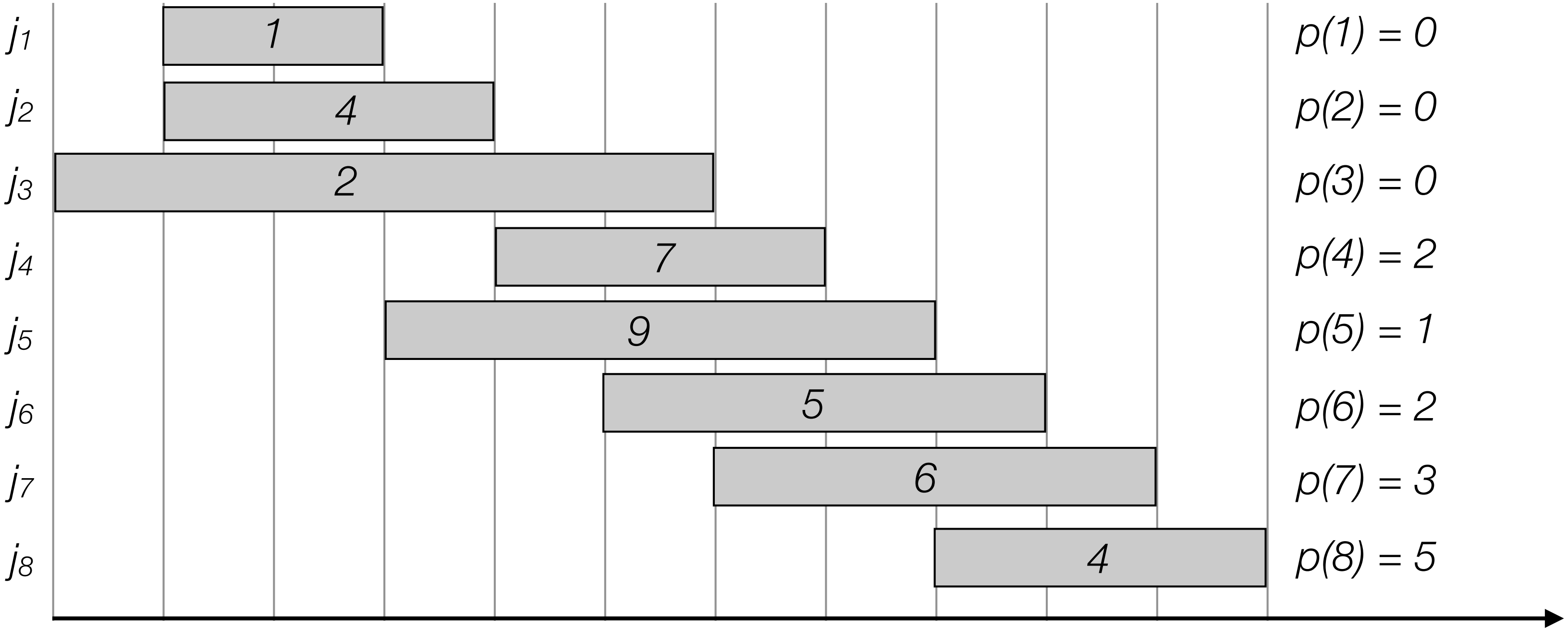


i	M[i]
0	0
1	1
2	4
3	
4	
5	
6	
7	
8	



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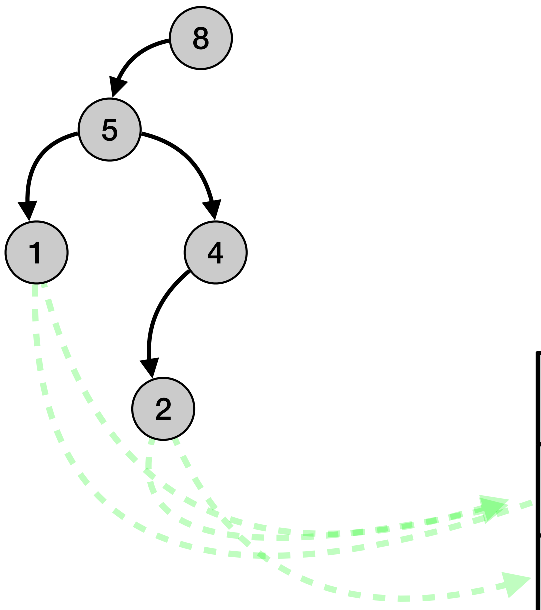
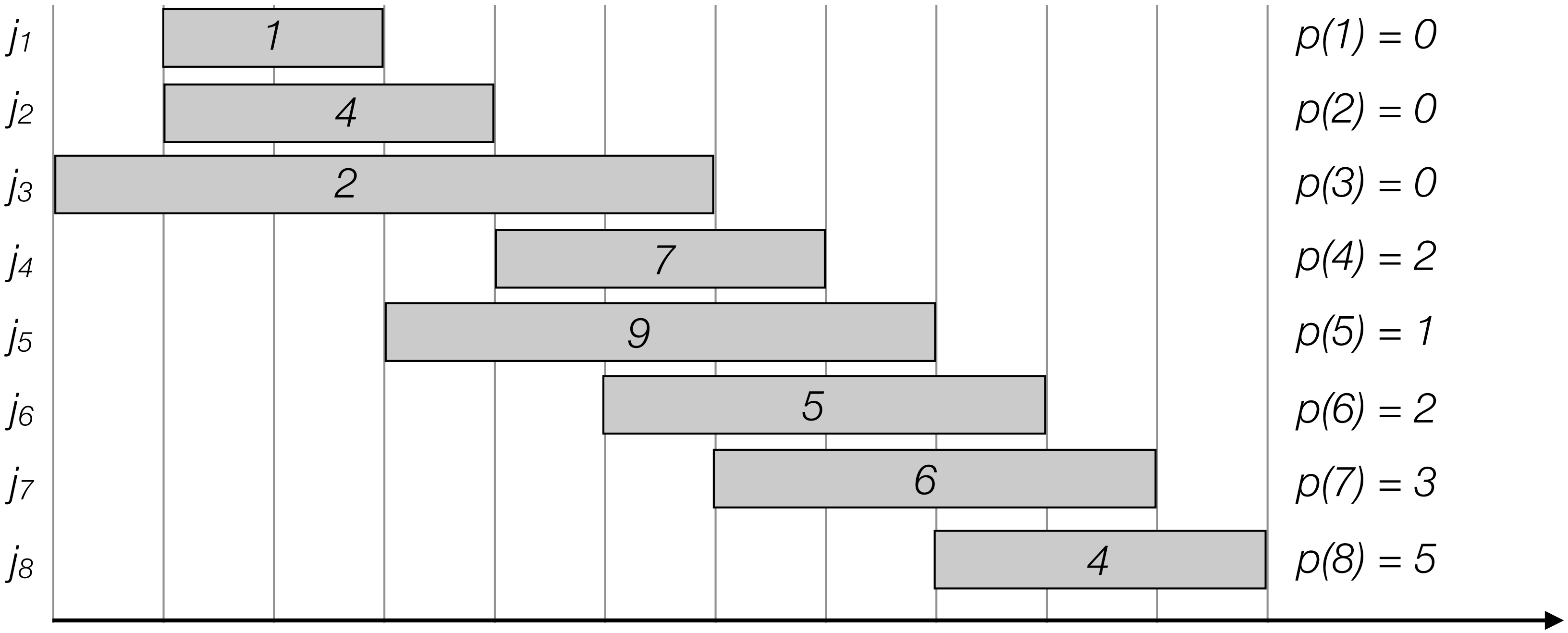


i	M[i]
0	0
1	1
2	4
3	
4	
5	
6	
7	
8	



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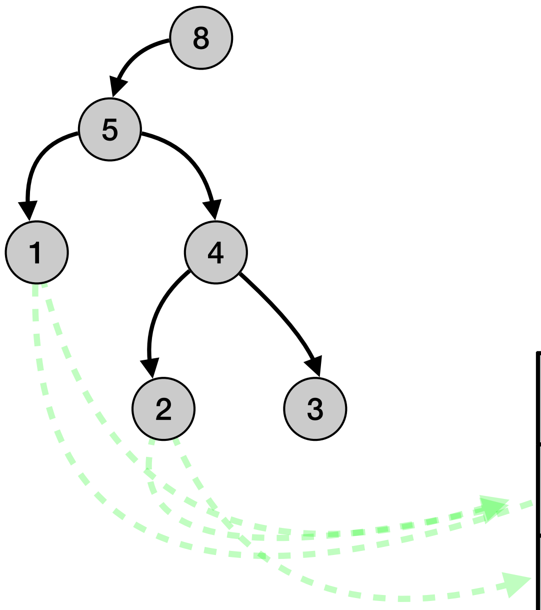
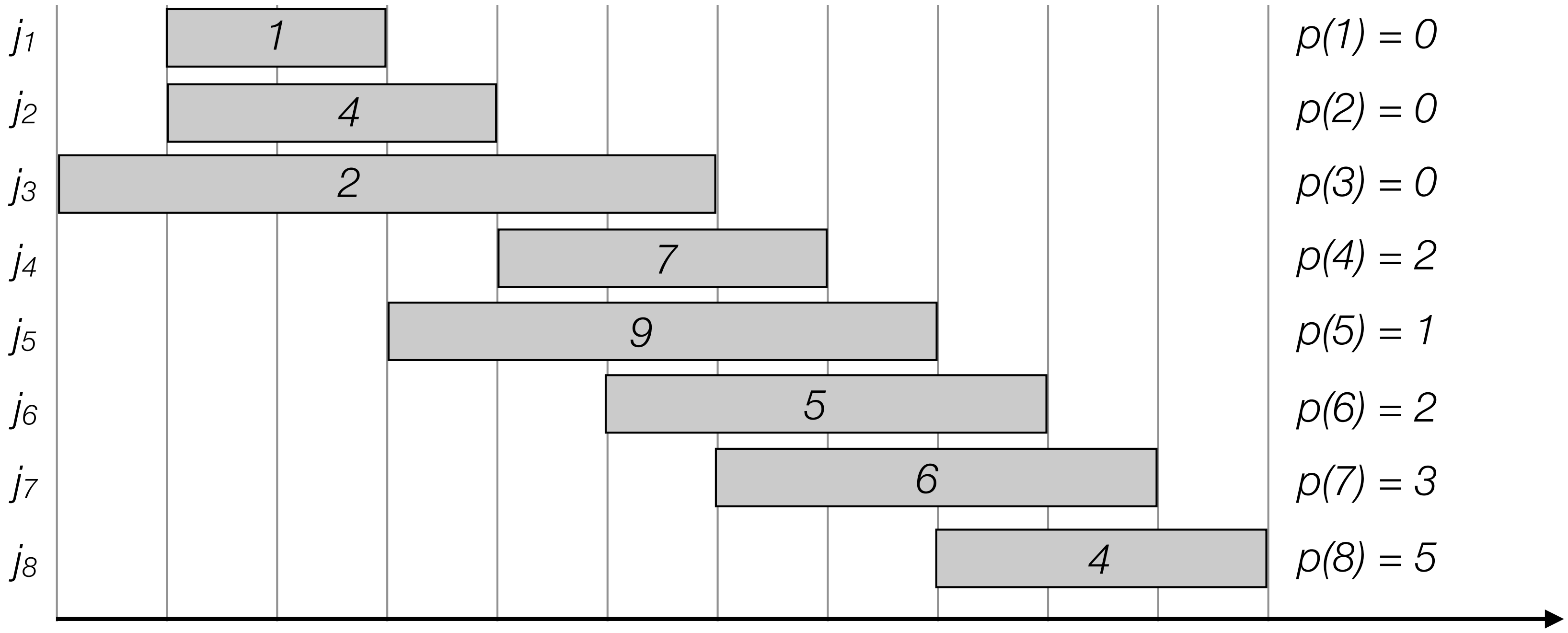


i	M[i]
0	0
1	1
2	4
3	
4	
5	
6	
7	
8	



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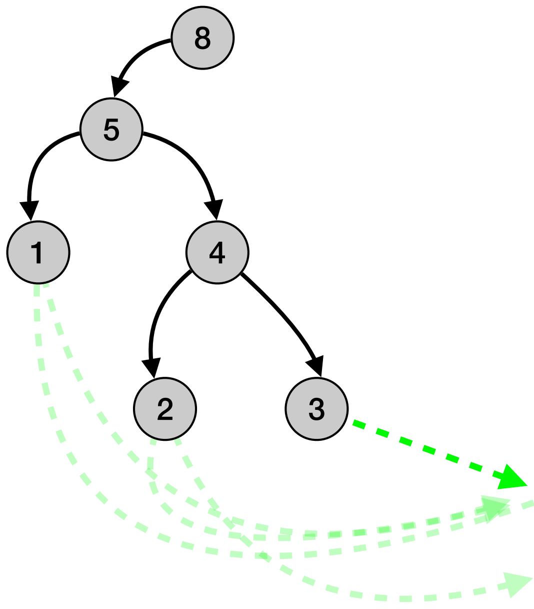
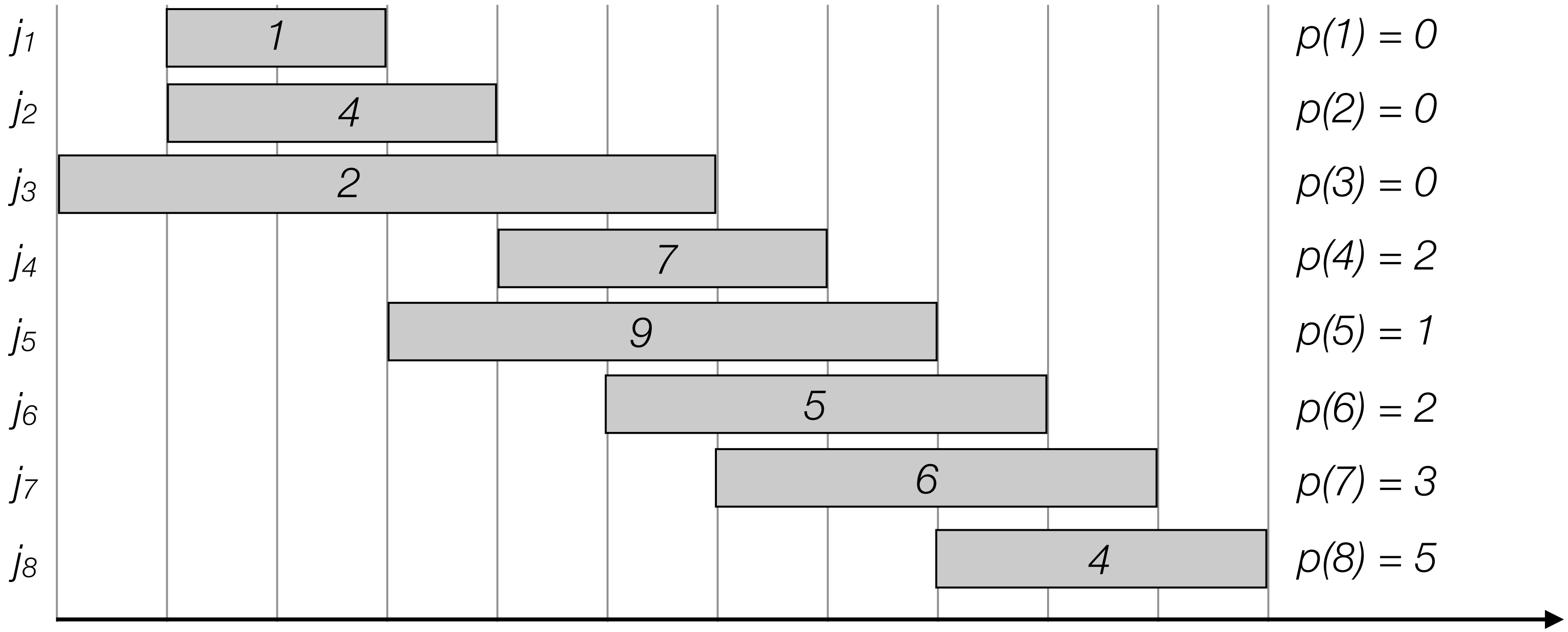


i	M[i]
0	0
1	1
2	4
3	
4	
5	
6	
7	
8	



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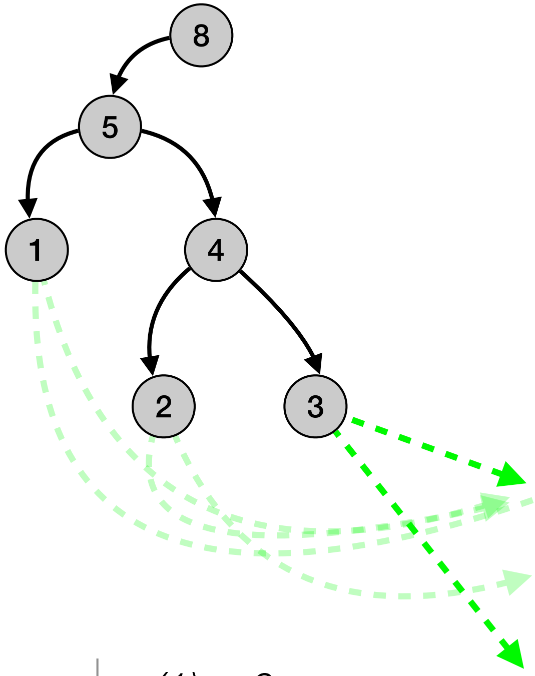

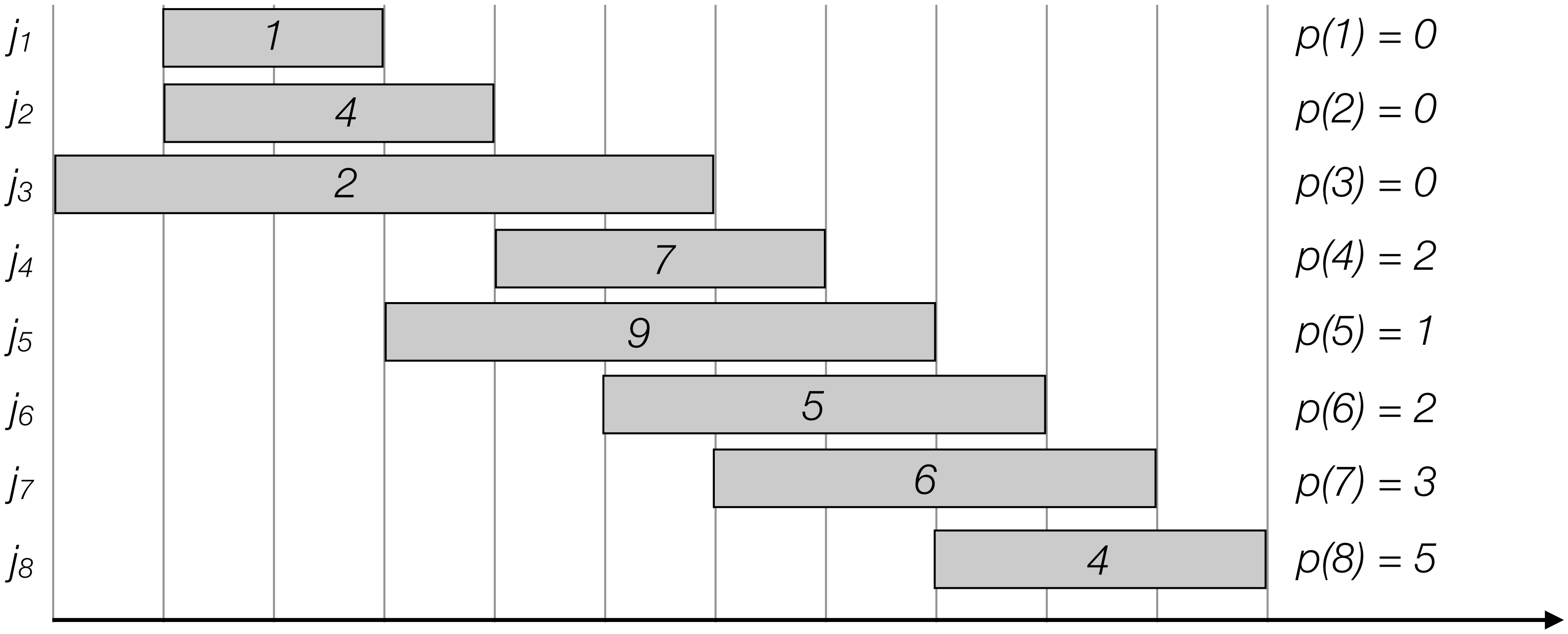

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i	M[i]
0	0
1	1
2	4
3	
4	
5	
6	
7	
8	

Weighted interval scheduling: memoization

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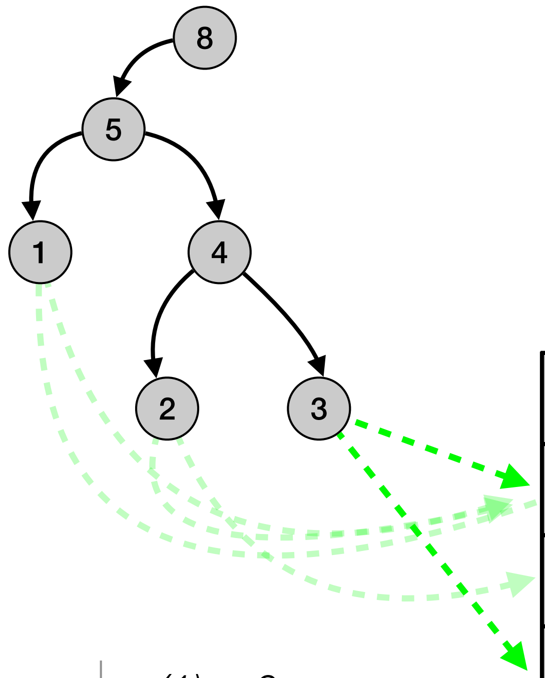
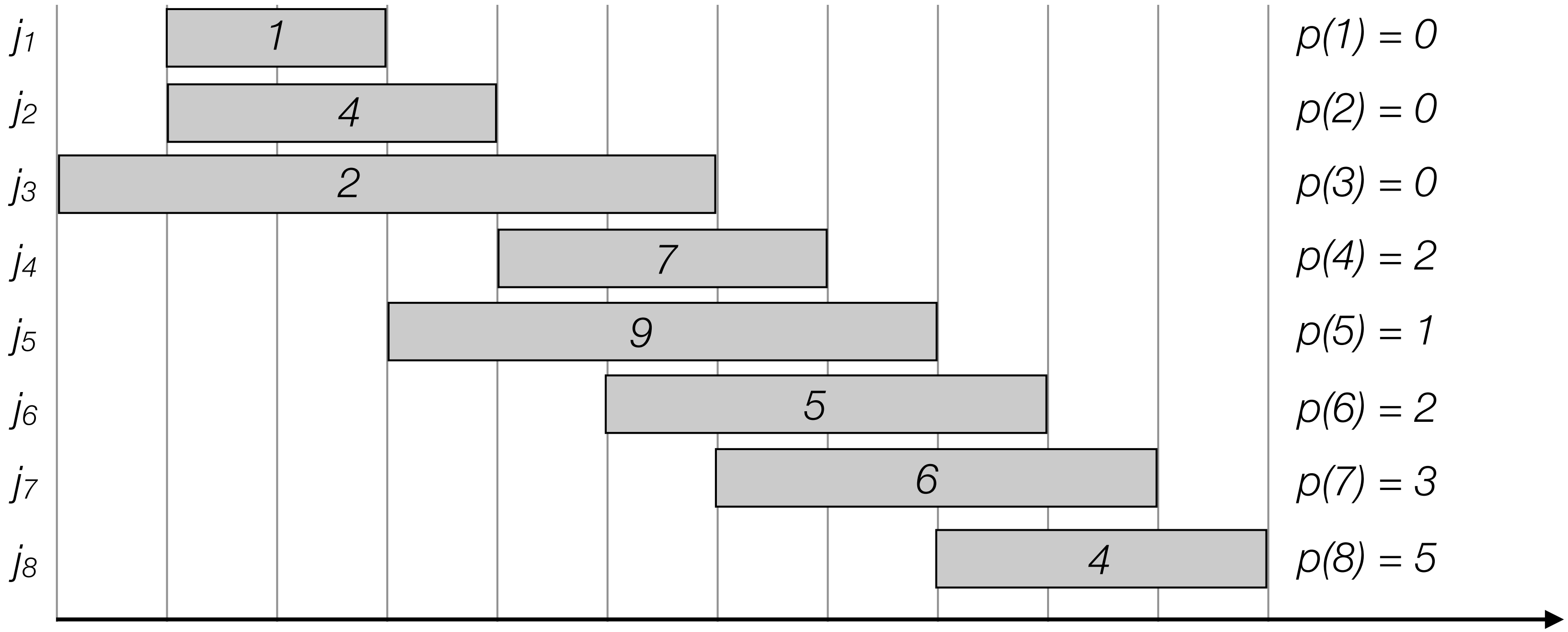


i	M[i]
0	0
1	1
2	4
3	4
4	
5	
6	
7	
8	



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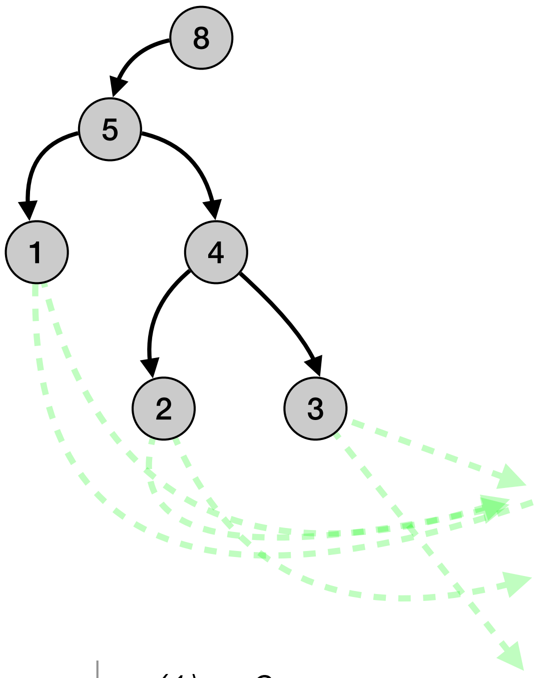
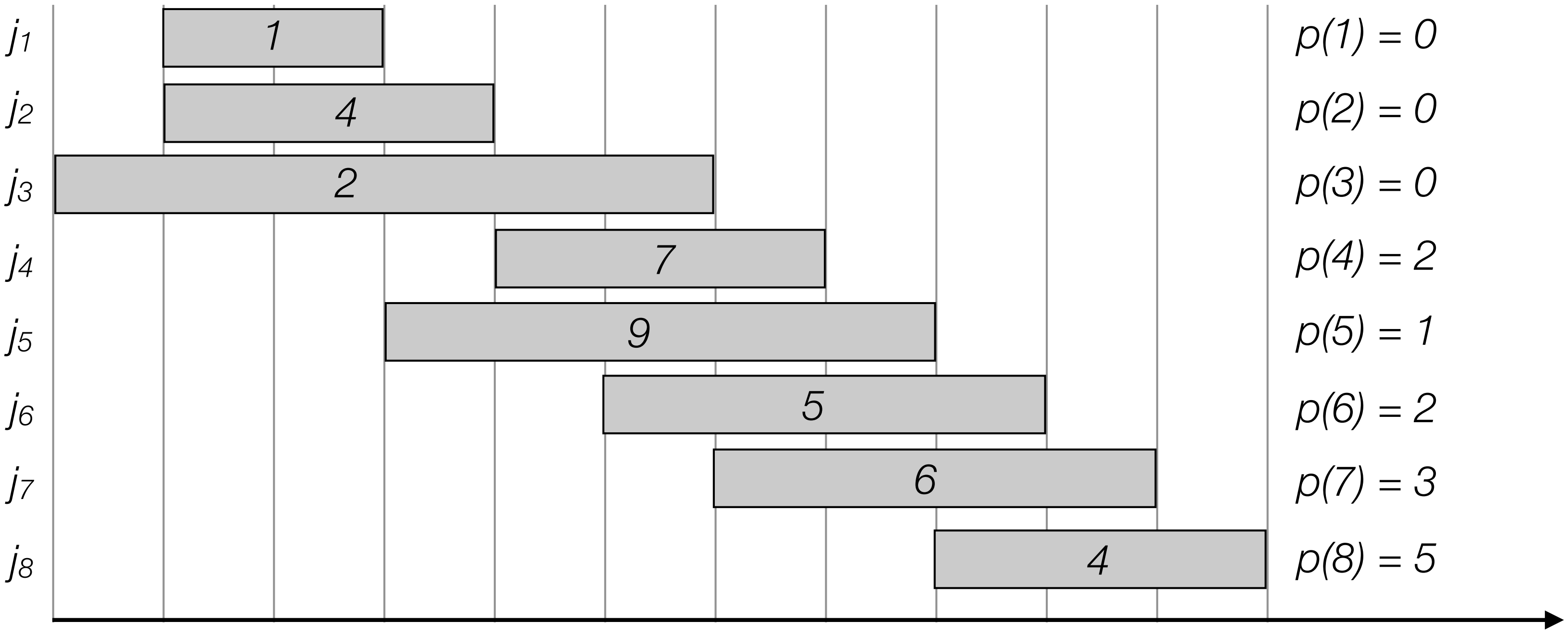


i	M[i]
0	0
1	1
2	4
3	4
4	
5	
6	
7	
8	



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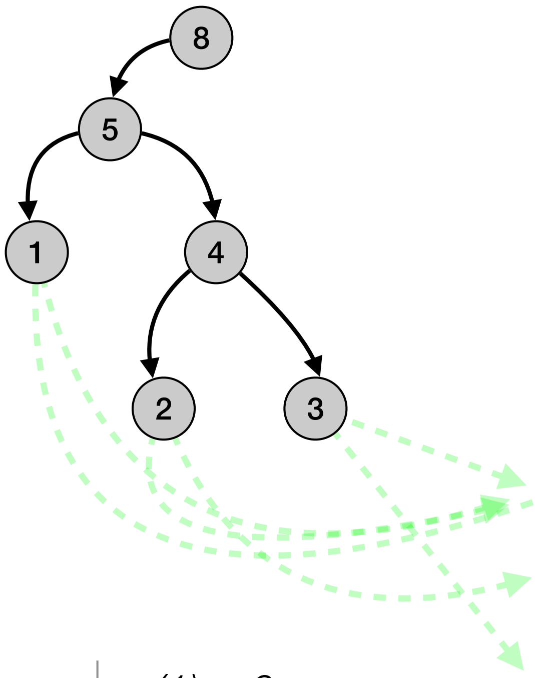
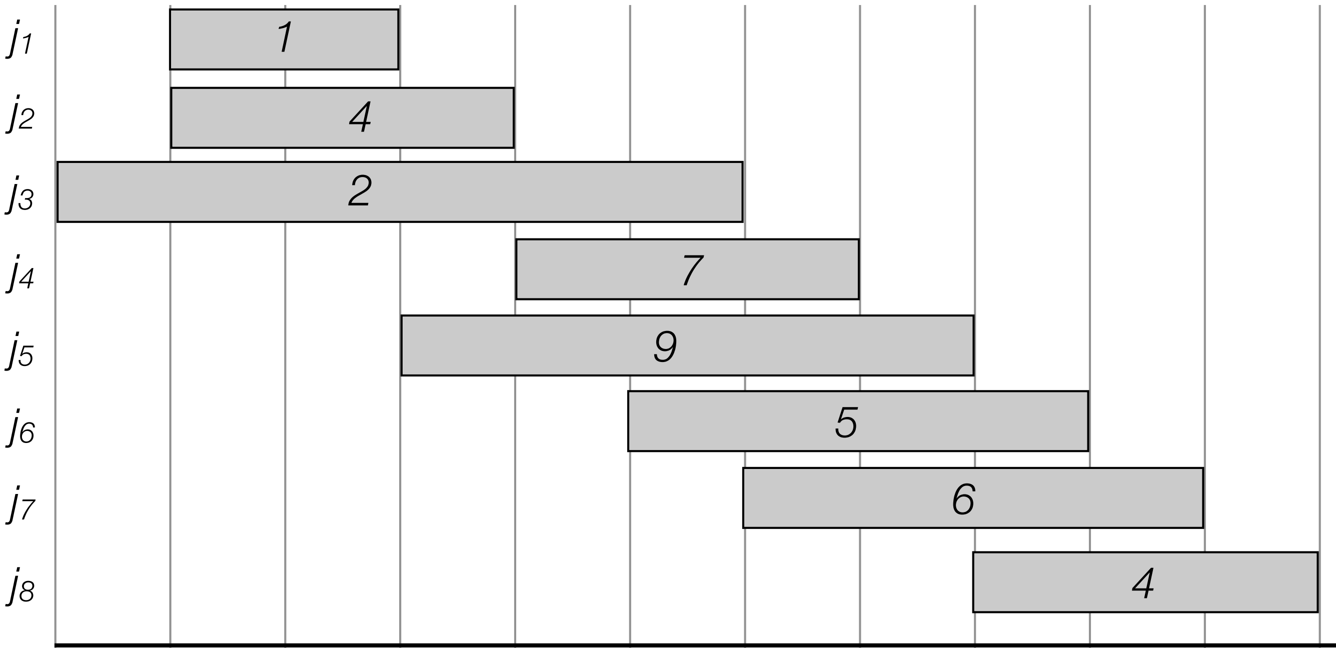


i	M[i]
0	0
1	1
2	4
3	4
4	11
5	
6	
7	
8	



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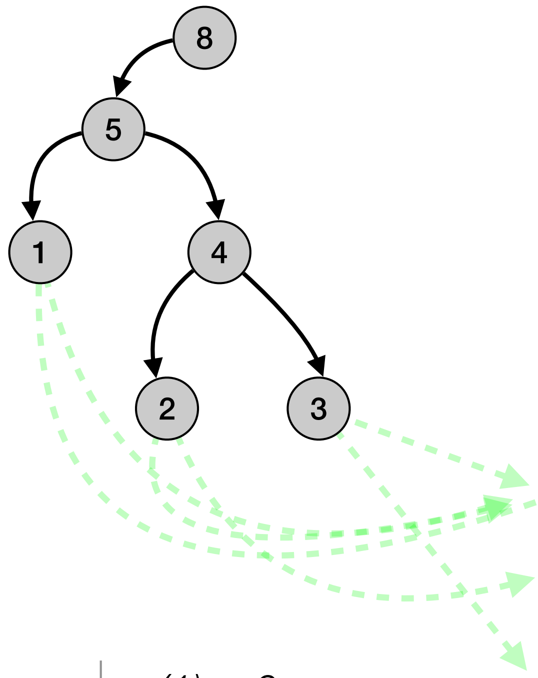
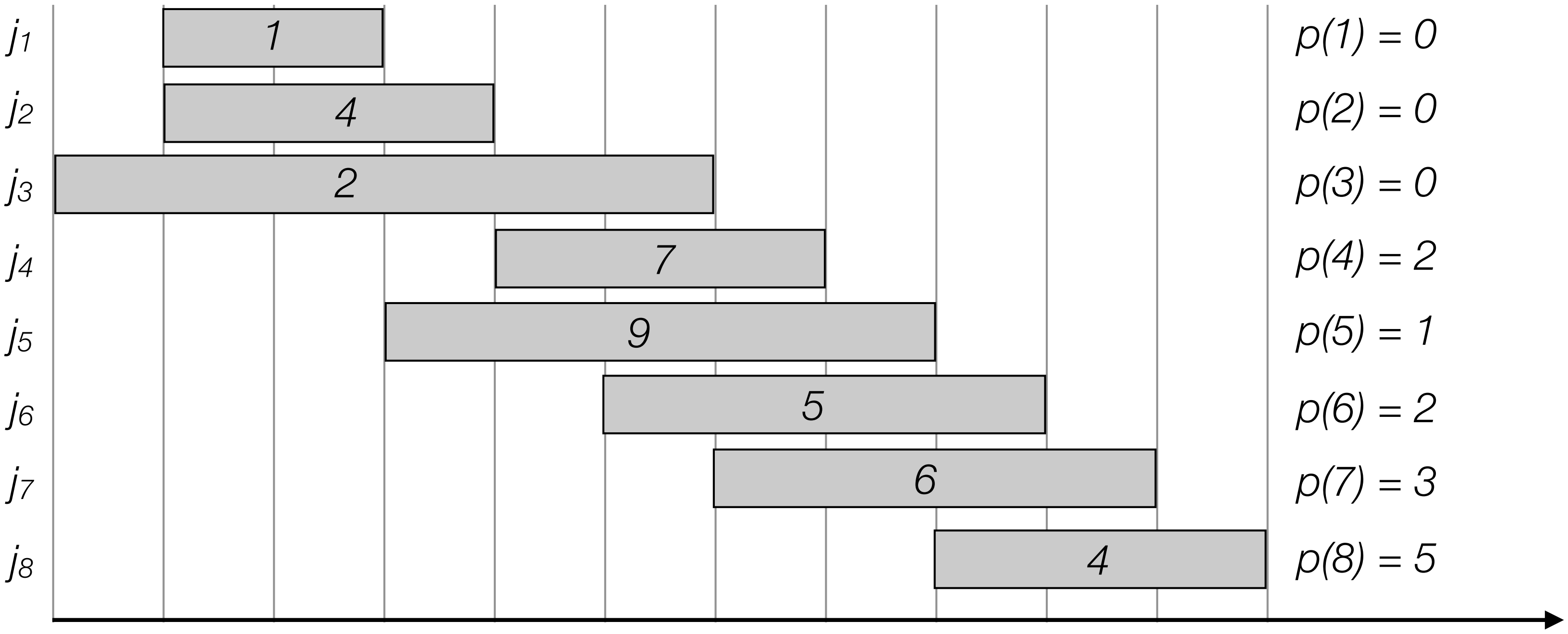


i	M[i]
0	0
1	1
2	4
3	4
4	11
5	11
6	
7	
8	



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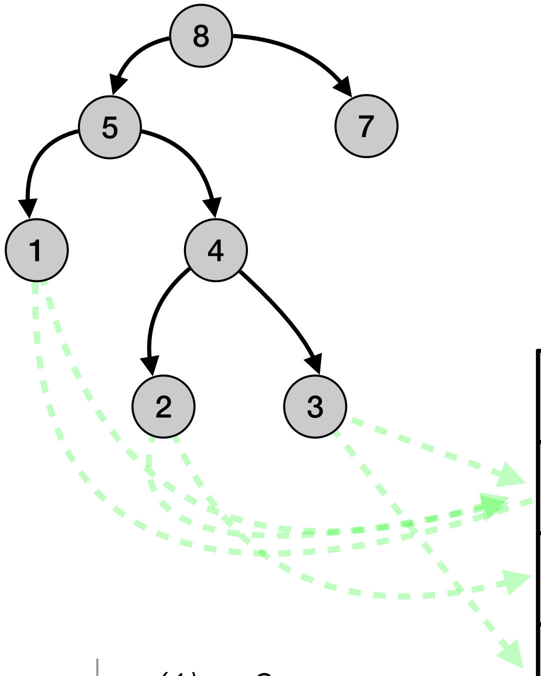
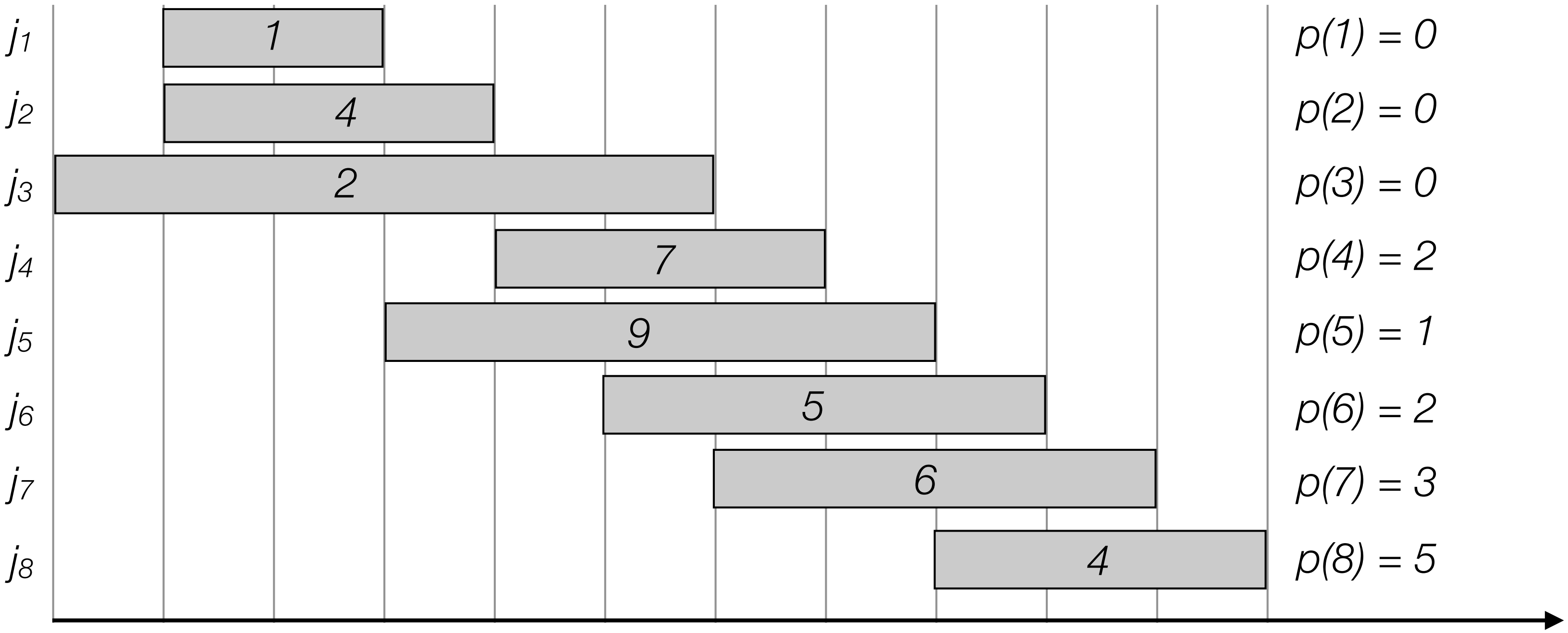


i	M[i]
0	0
1	1
2	4
3	4
4	11
5	11
6	
7	
8	



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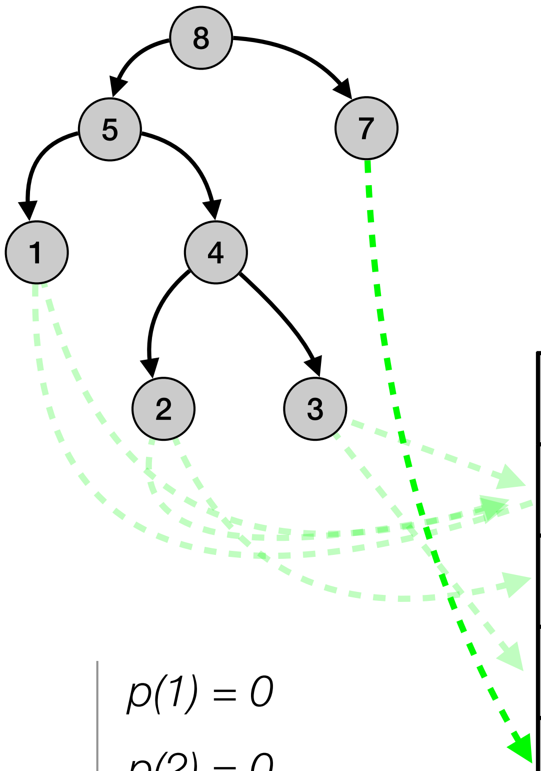
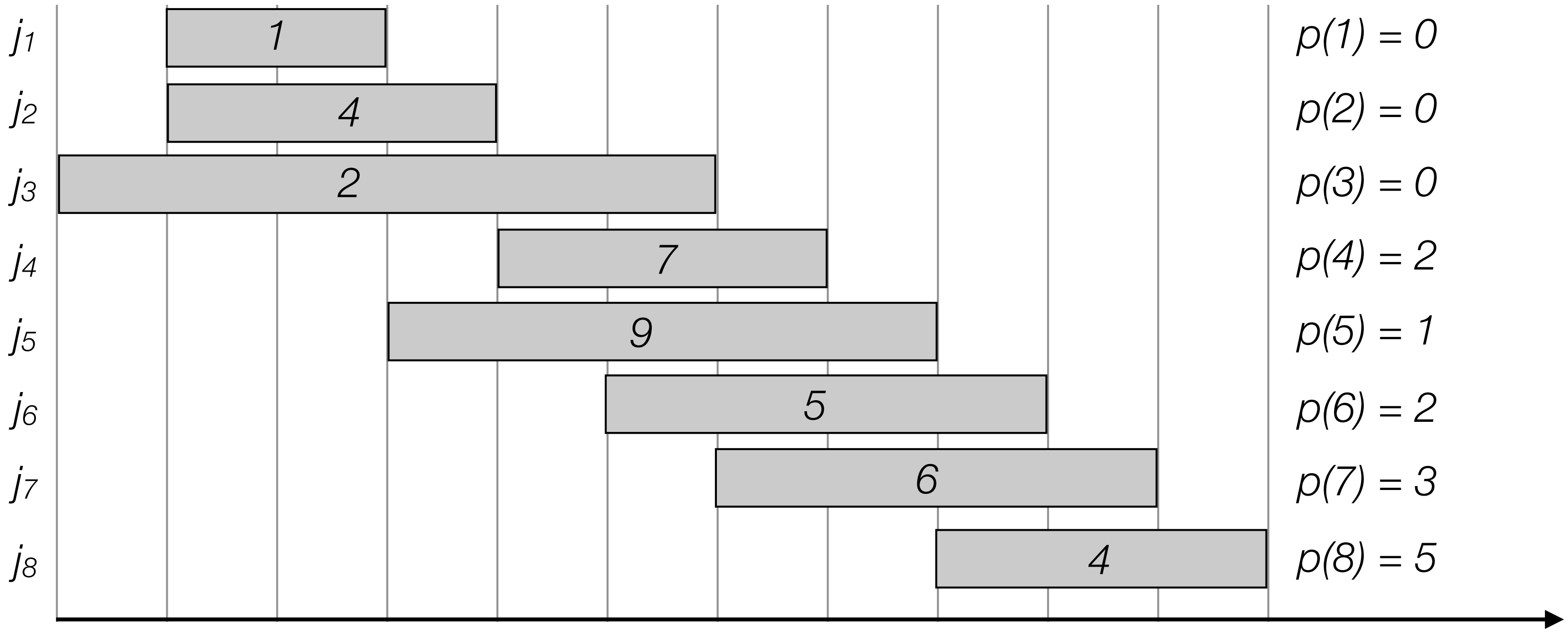


i	M[i]
0	0
1	1
2	4
3	4
4	11
5	11
6	
7	
8	



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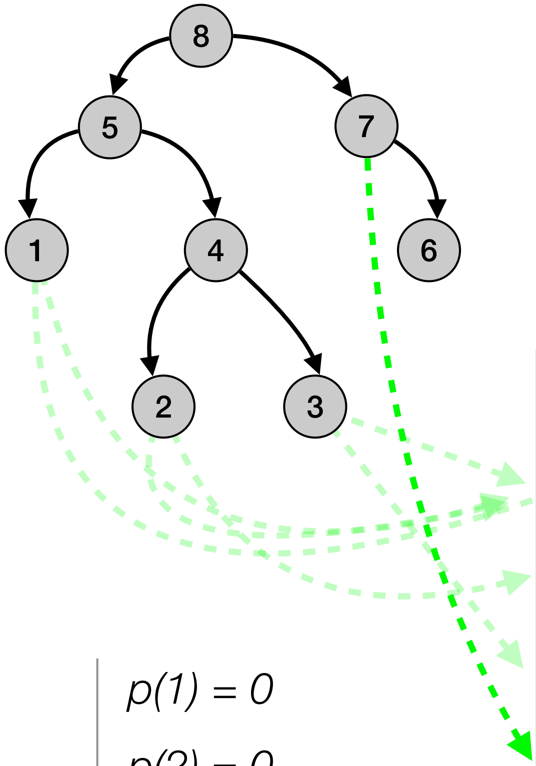
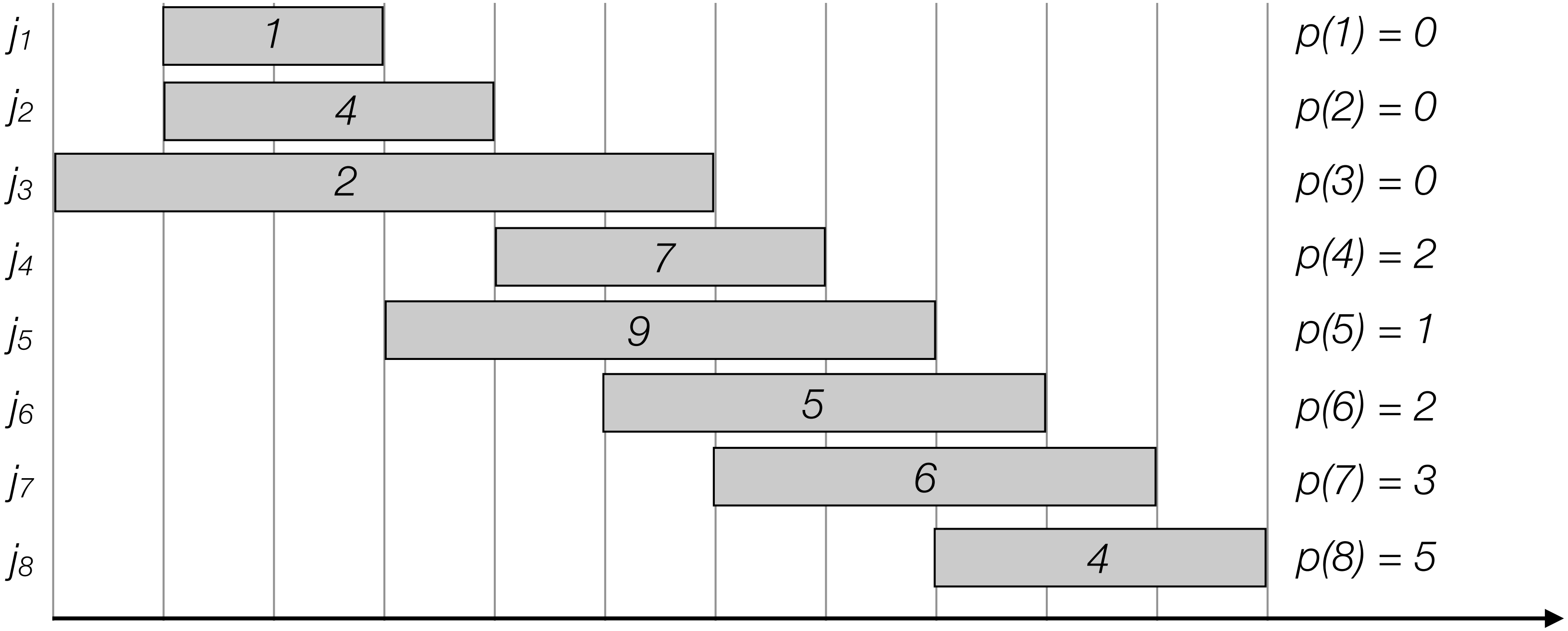


i	M[i]
0	0
1	1
2	4
3	4
4	11
5	11
6	
7	
8	



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```
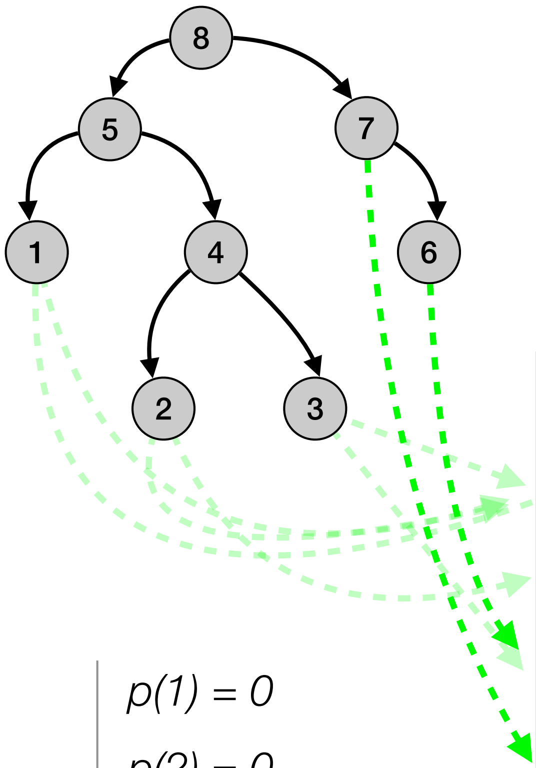
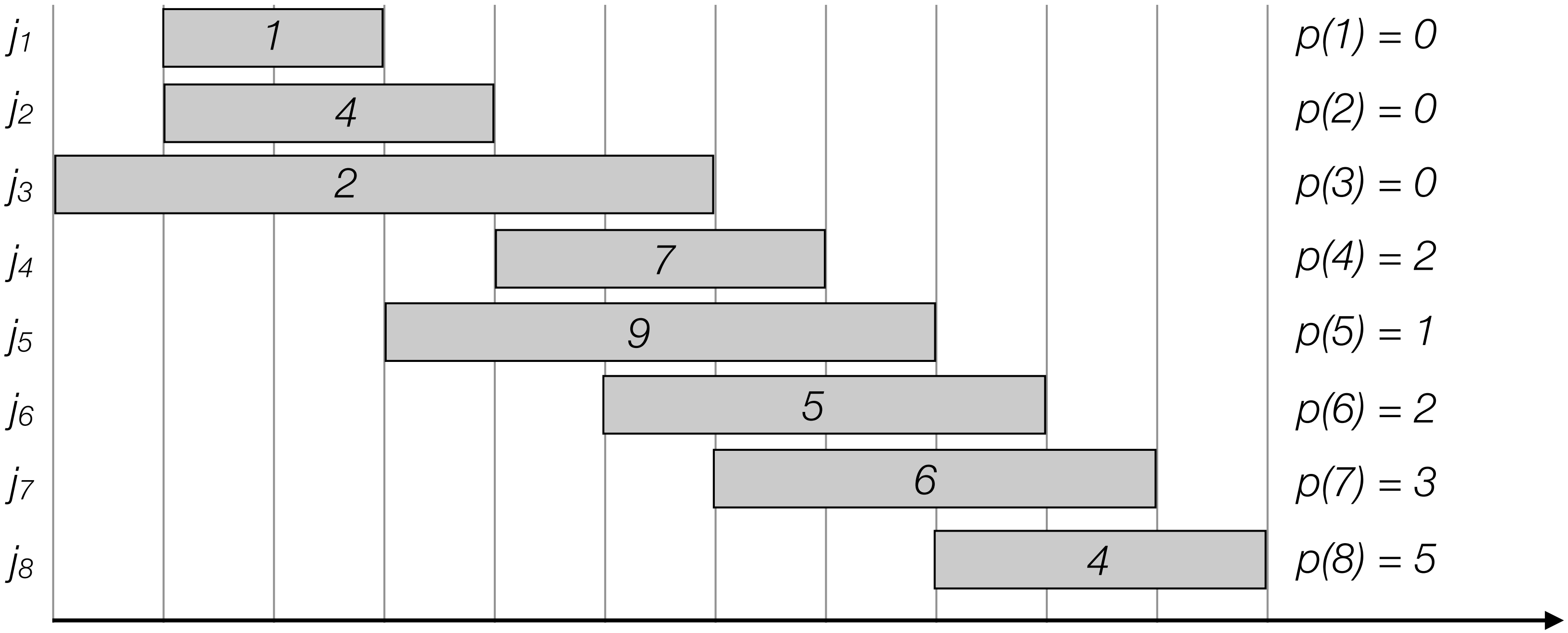



i	M[i]
0	0
1	1
2	4
3	4
4	11
5	11
6	
7	
8	



p(1) = 0

p(2) = 0

p(3) = 0

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p(5) = 1

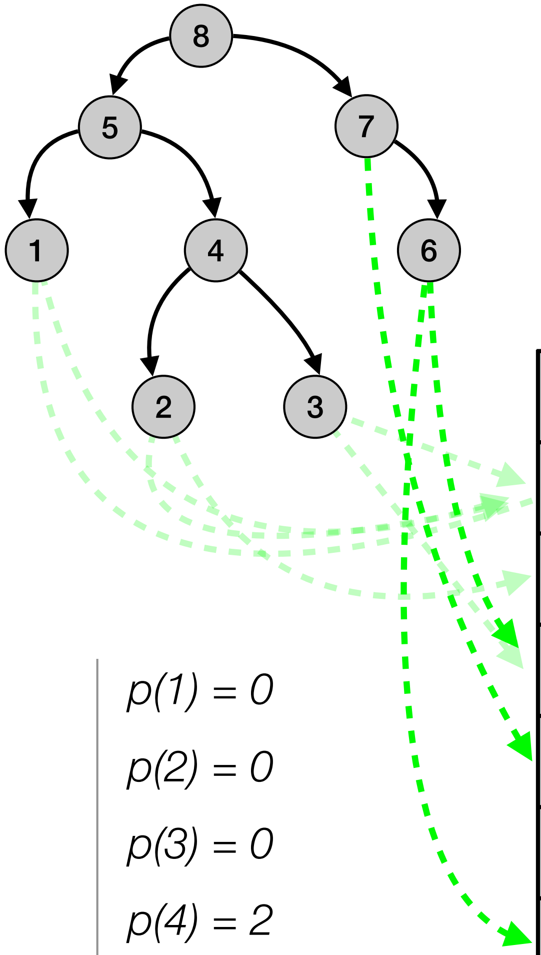
p(6) = 2

p(7) = 3

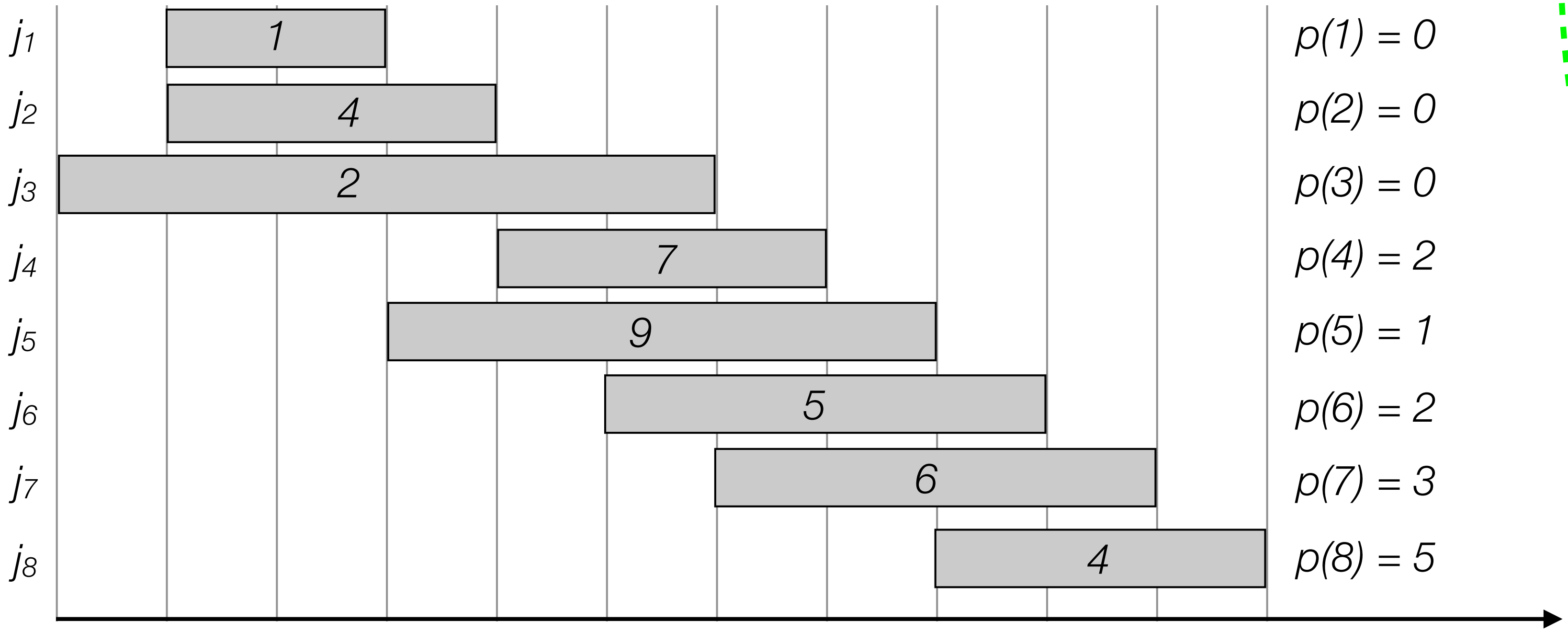
p(8) = 5

Weighted interval scheduling: memoization

```
Input: n, s[1..n], f[1..n], v[1..n]

Sort jobs by finish time so that f[1] ≤ f[2]≤ … ≤ f[n]
Compute p[1], p[2], …, p[n]

for j=1 to n
   M[j] = empty
M[0] = 0.
Compute-Memoized-Opt(n)

Compute-Memoized-Opt(j)
if M[j] is empty
   M[j] = max(v[j] + Compute-Memoized-Opt(p[j]),
              Compute-Memoized-Opt(j-1))
return M[j]
```



i	M[i]
0	0
1	1
2	4
3	4
4	11
5	11
6	
7	
8	



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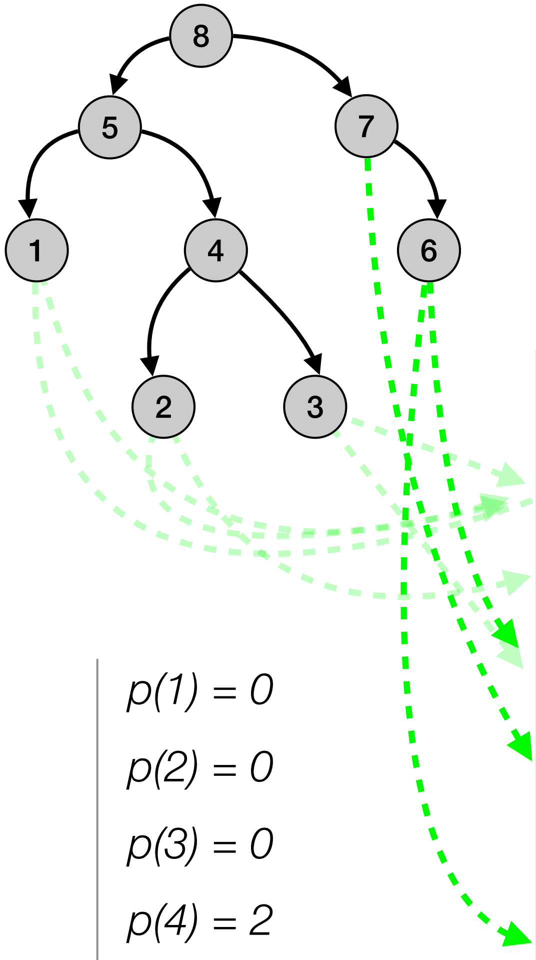
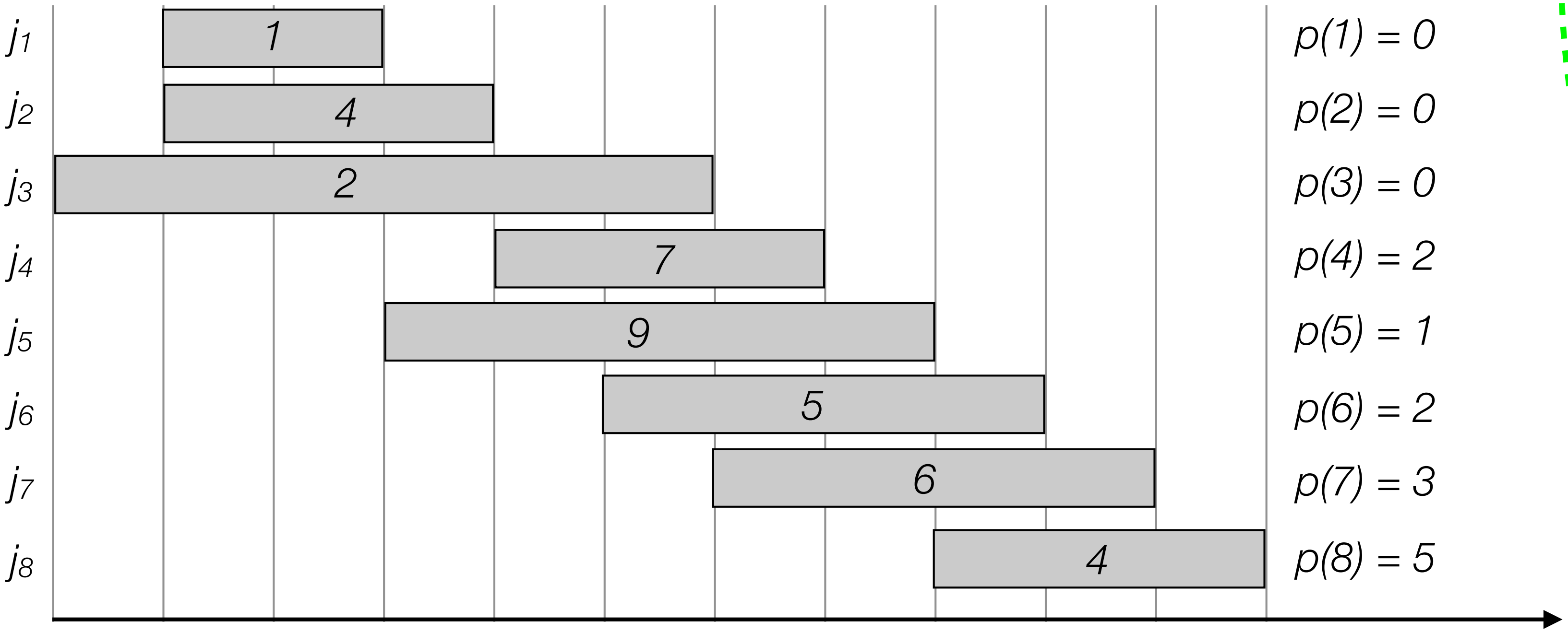
```



i	M[i]
0	0
1	1
2	4
3	4
4	11
5	11
6	11
7	
8	



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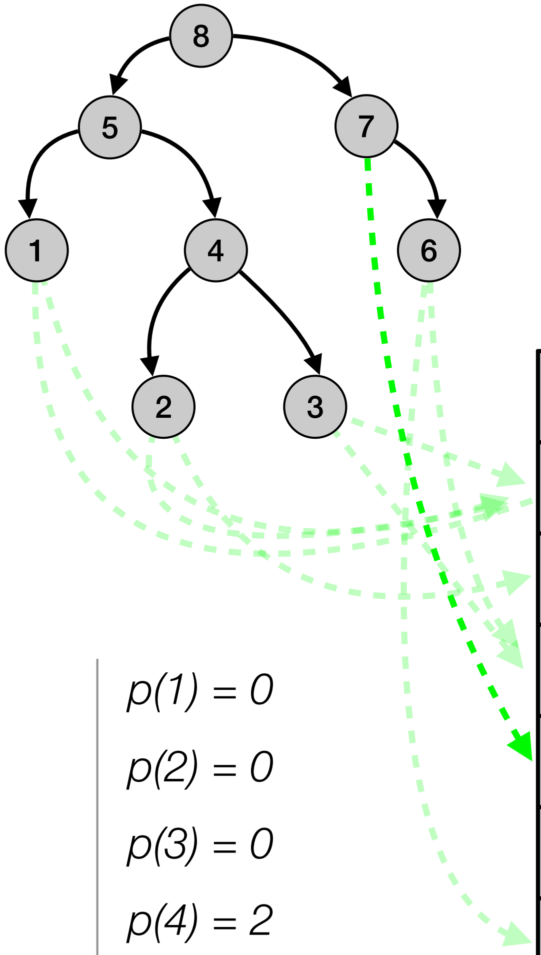

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0	0
1	1
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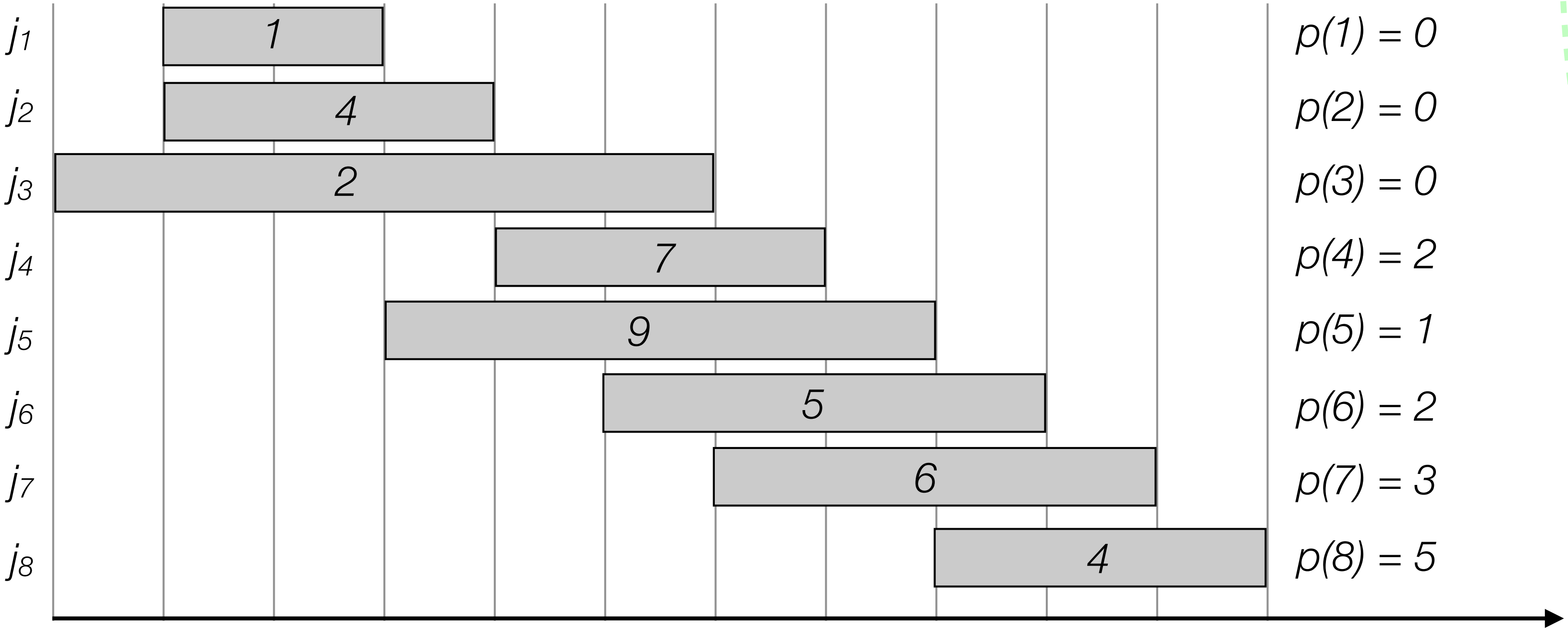

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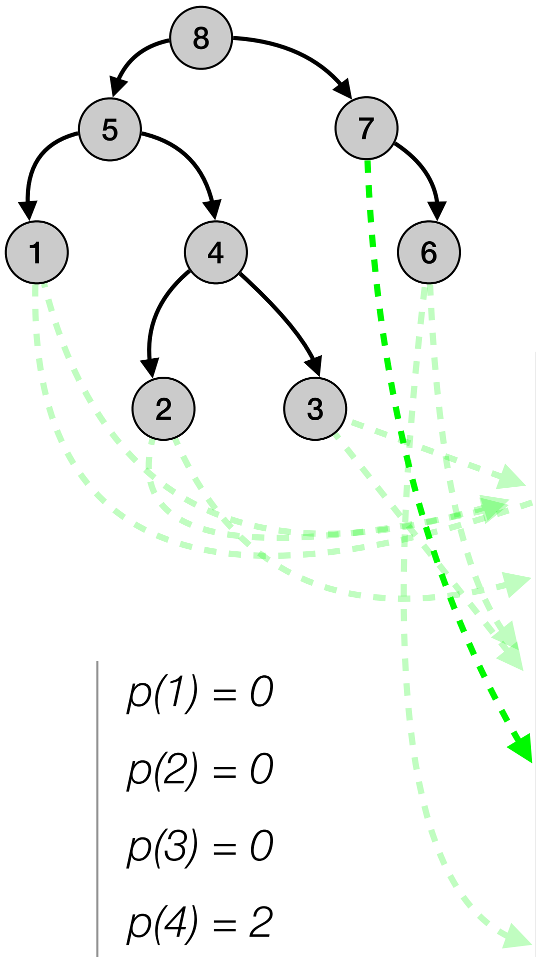

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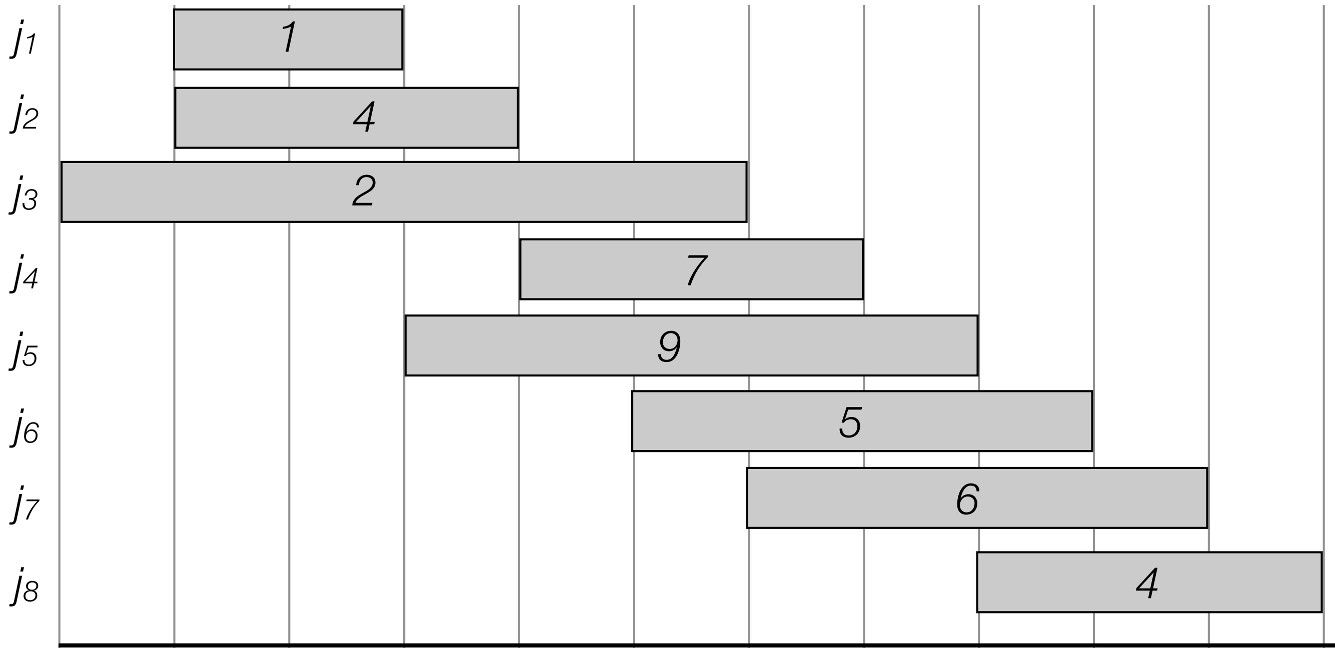

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```

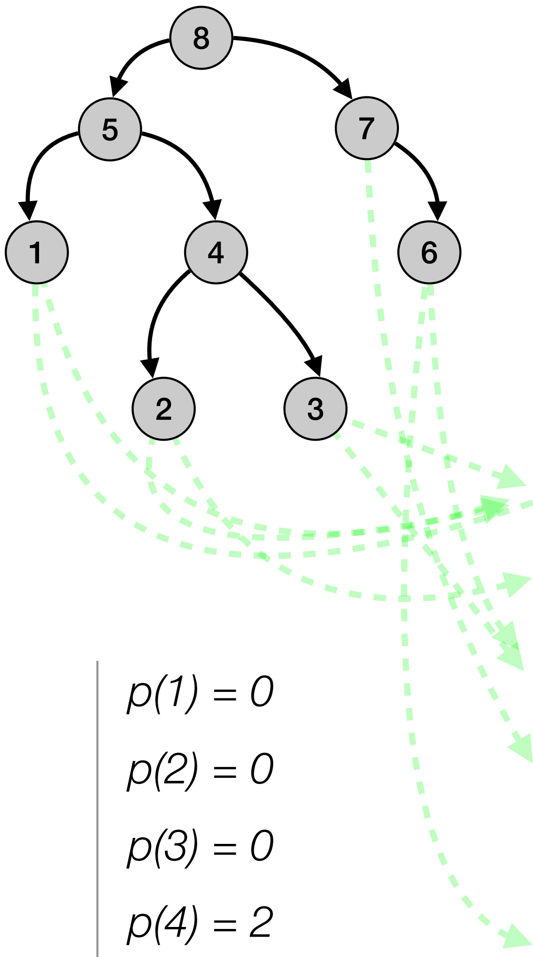

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2	4
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7	11
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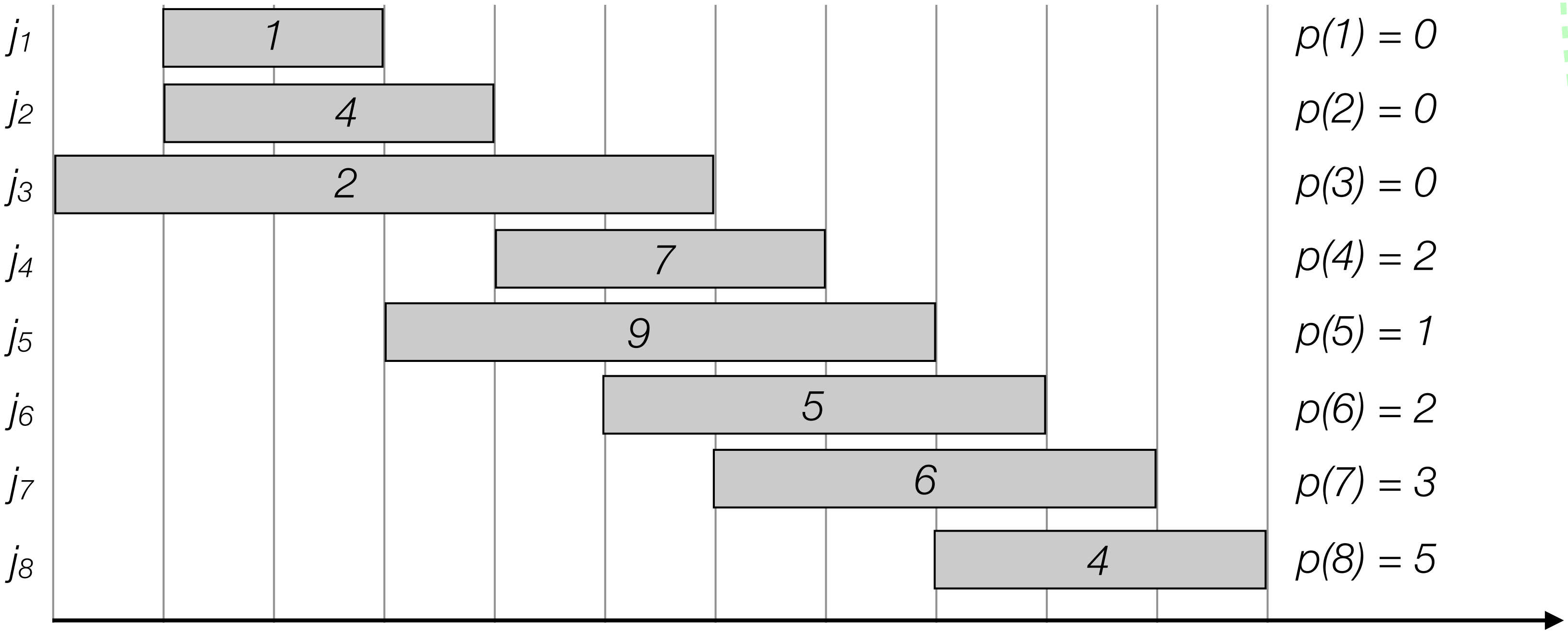

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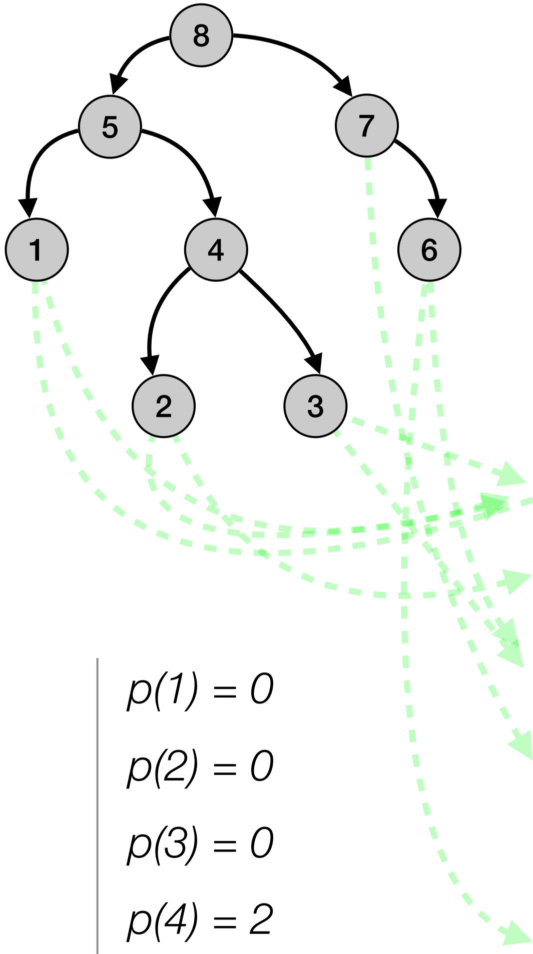
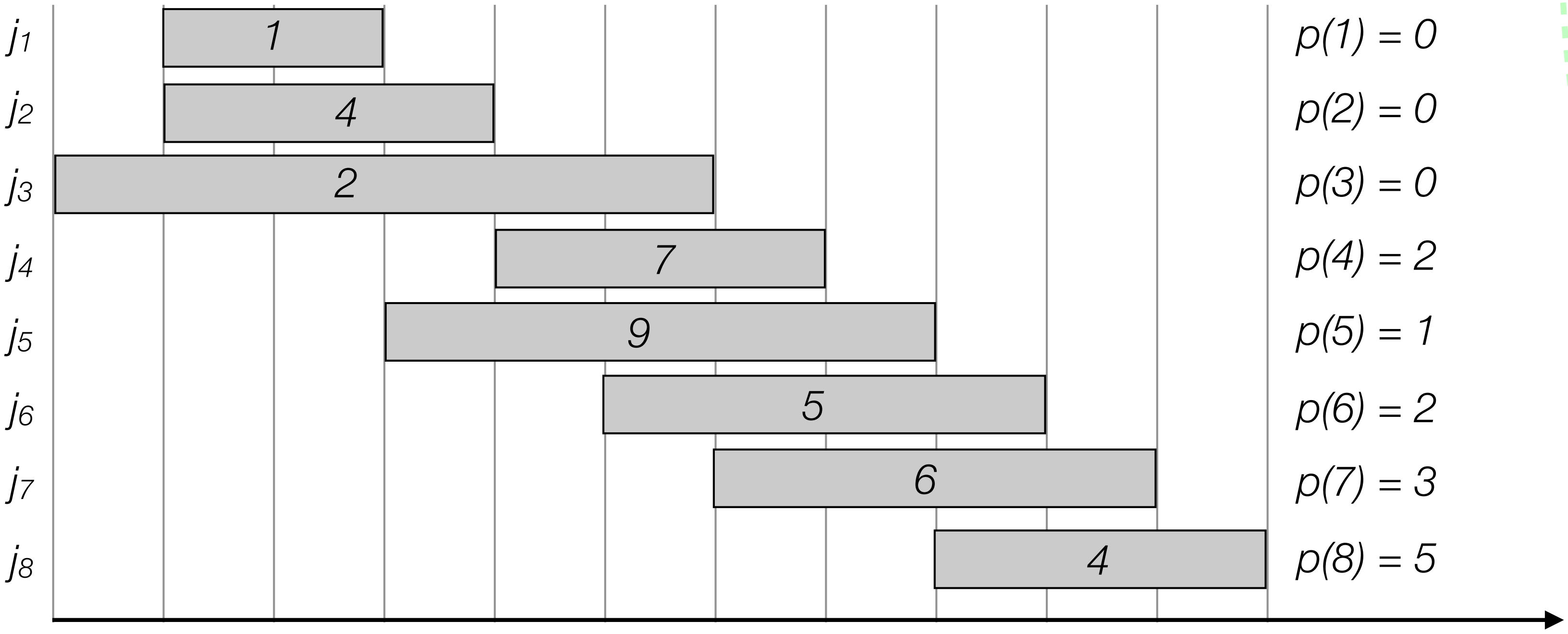


i	M[i]
0	0
1	1
2	4
3	4
4	11
5	11
6	11
7	11
8	15



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$p(5) = 1$

$p(6) = 2$

$p(7) = 3$

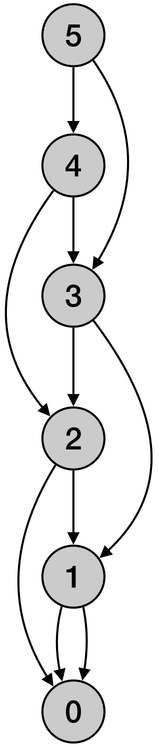
$p(8) = 5$

Weighted interval scheduling: bottom-up

```
Compute-Bottom-Up—Opt(n, s[1..n], f[1..n], v[1..n])

Sort jobs by finish time so that f[1] ≤ f[2]≤ … ≤ f[n]
Compute p[1], p[2], …, p[n]

M[0] = 0.
for j=1 to n
  M[j] = max(v[j] + M(p[j]), M(j-1))
return M[n]
```



Weighted interval scheduling: bottom-up

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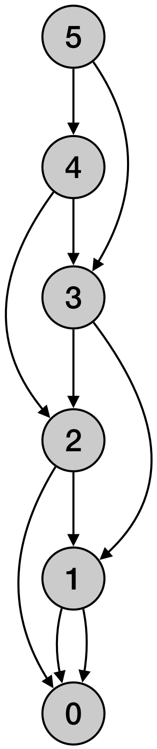

- Running time O(n log n):

Weighted interval scheduling: bottom-up

```
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```

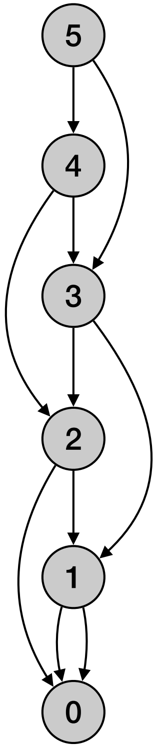

- Running time O(n log n):
  - Sorting takes O(n log n) time.

Weighted interval scheduling: bottom-up

```
Compute-Bottom-Up—Opt(n, s[1..n], f[1..n], v[1..n])

Sort jobs by finish time so that f[1] ≤ f[2]≤ … ≤ f[n]
Compute p[1], p[2], …, p[n]

M[0] = 0.
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  M[j] = max(v[j] + M(p[j]), M(j-1))
return M[n]
```
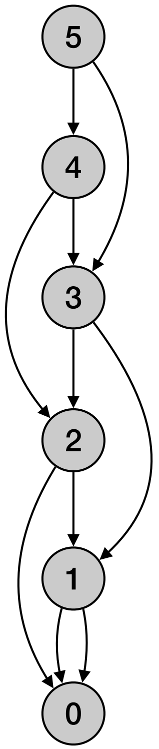
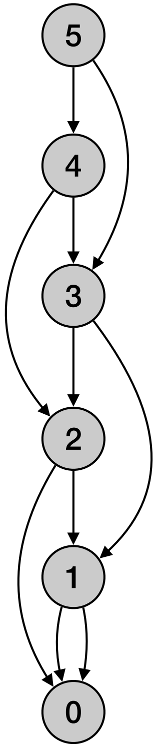



- Running time O(n log n):
  - Sorting takes O(n log n) time.
  - Computing p(n): O(n log n)

Weighted interval scheduling: bottom-up

```
Compute-Bottom-Up—Opt(n, s[1..n], f[1..n], v[1..n])

Sort jobs by finish time so that f[1] ≤ f[2]≤ … ≤ f[n]
Compute p[1], p[2], …, p[n]

M[0] = 0.
for j=1 to n
  M[j] = max(v[j] + M(p[j]), M(j-1))
return M[n]
```



- Running time O(n log n):
  - Sorting takes O(n log n) time.
  - Computing p(n): O(n log n)
  - For loop: O(n) time

Weighted interval scheduling: bottom-up

```
Compute-Bottom-Up—Opt(n, s[1..n], f[1..n], v[1..n])

Sort jobs by finish time so that f[1] ≤ f[2]≤ … ≤ f[n]
Compute p[1], p[2], …, p[n]

M[0] = 0.
for j=1 to n
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return M[n]
```

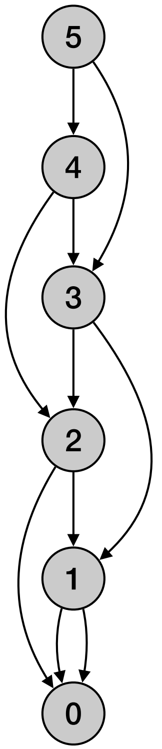

- Running time O(n log n):
    - Sorting takes O(n log n) time.
    - Computing p(n): O(n log n)
    - For loop: O(n) time
        - Each iteration takes constant time.

Weighted interval scheduling: bottom-up

```
Compute-Bottom-Up–Opt(n, s[1..n], f[1..n], v[1..n])

Sort jobs by finish time so that f[1] ≤ f[2]≤ … ≤ f[n]
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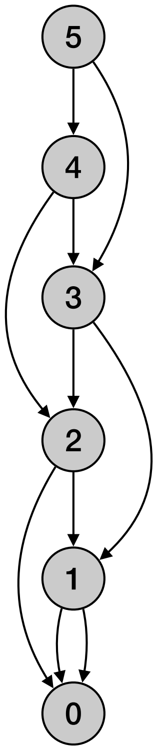
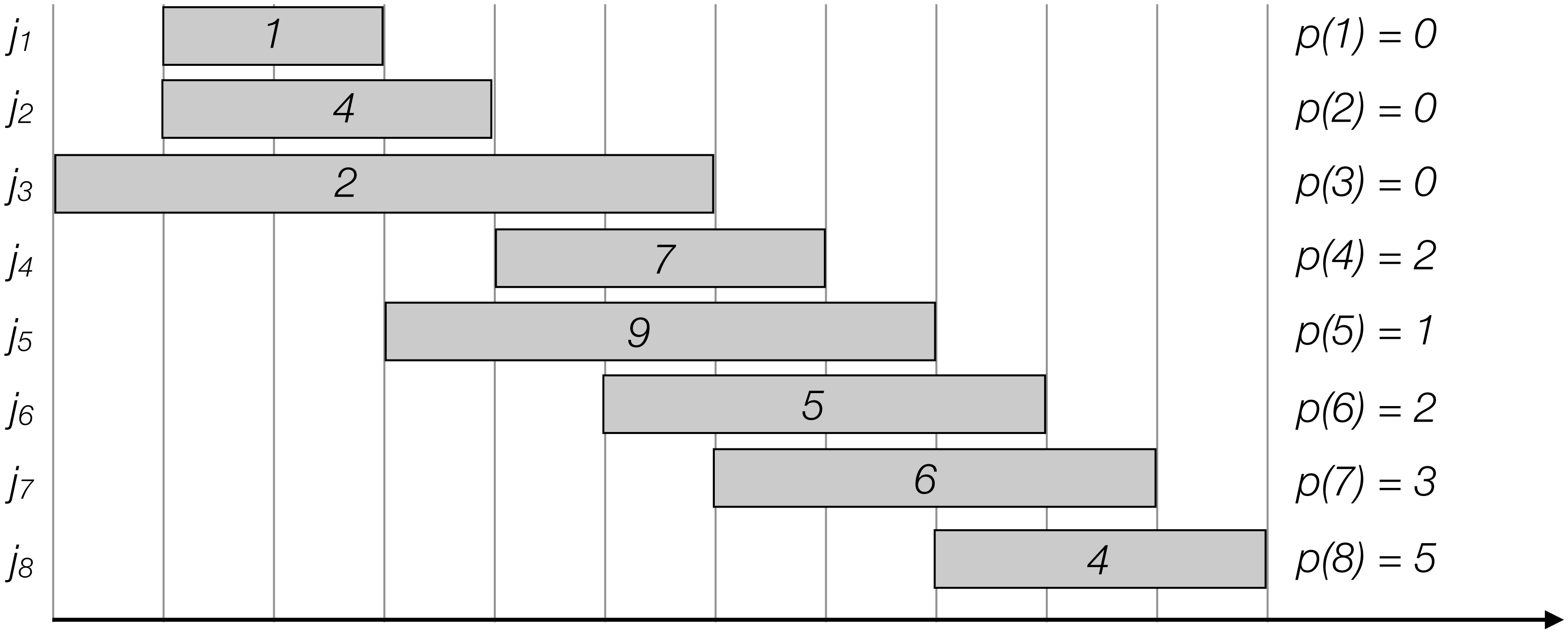


- Running time O(n log n):
  - Sorting takes O(n log n) time.
  - Computing p(n): O(n log n)
  - For loop: O(n) time
    - Each iteration takes constant time.
- Space O(n)

Weighted interval scheduling: find solution



i	M[i]
0	0
1	1
2	4
3	4
4	11
5	11
6	11
7	11
8	15

Weighted interval scheduling: find solution

```
Find-Solution(j)
if j=0
   Return emptyset
else if M[j] > M[j-1]
   return {j} U Find-Solution(p[j])
else
   return Find-Solution(j-1)
```

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1	1
2	4
3	4
4	11
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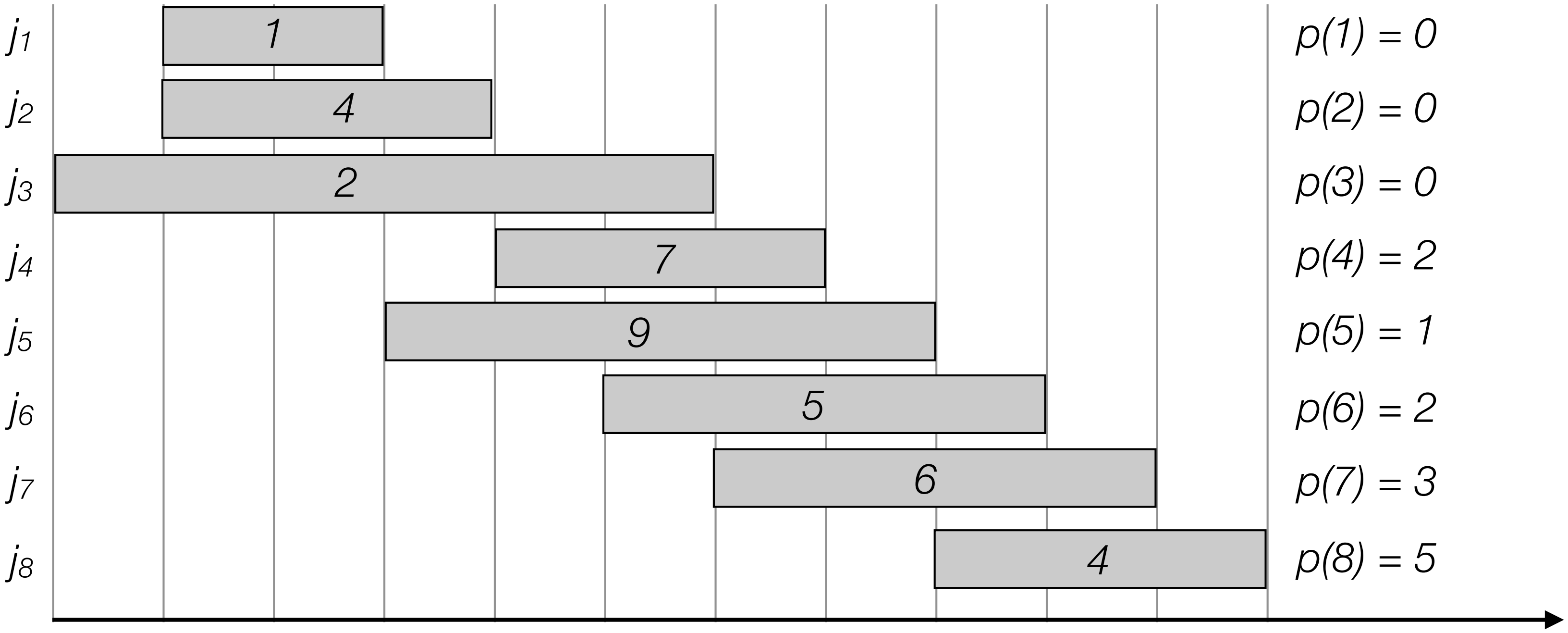
7	11
8	15



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```
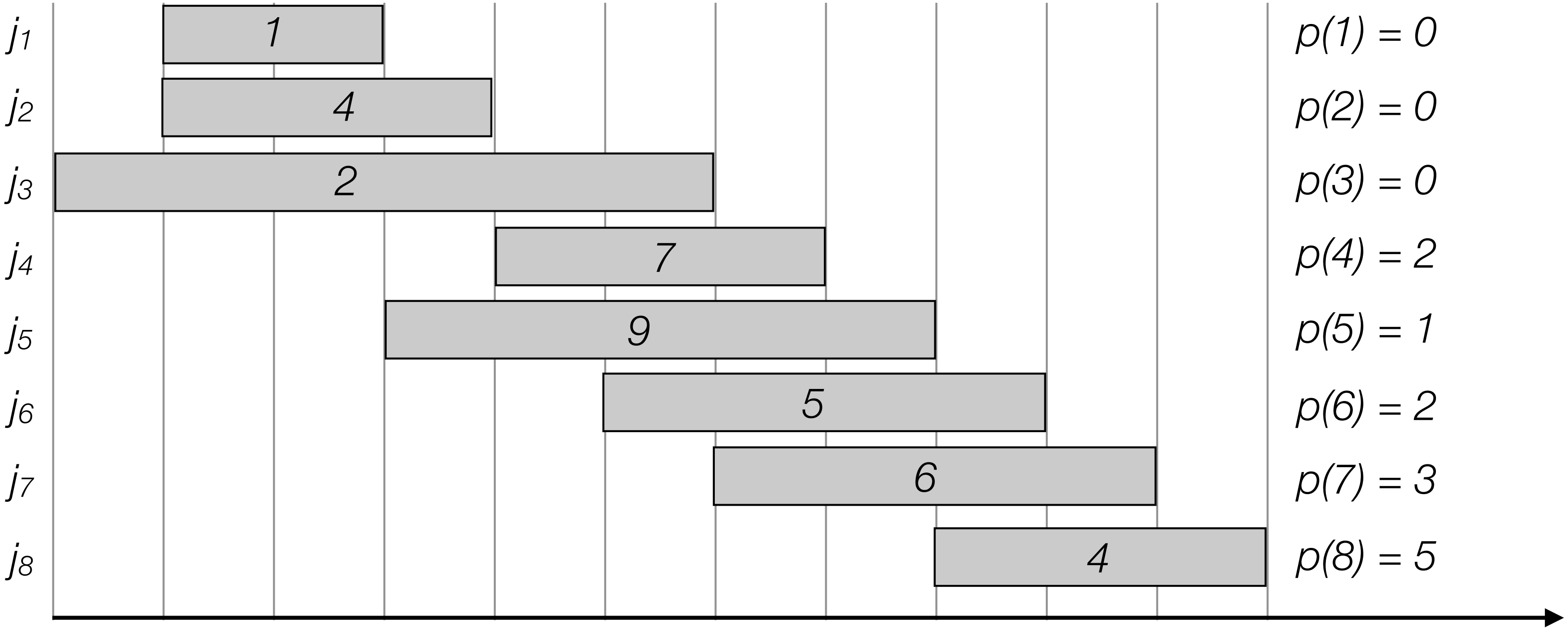

Solution =



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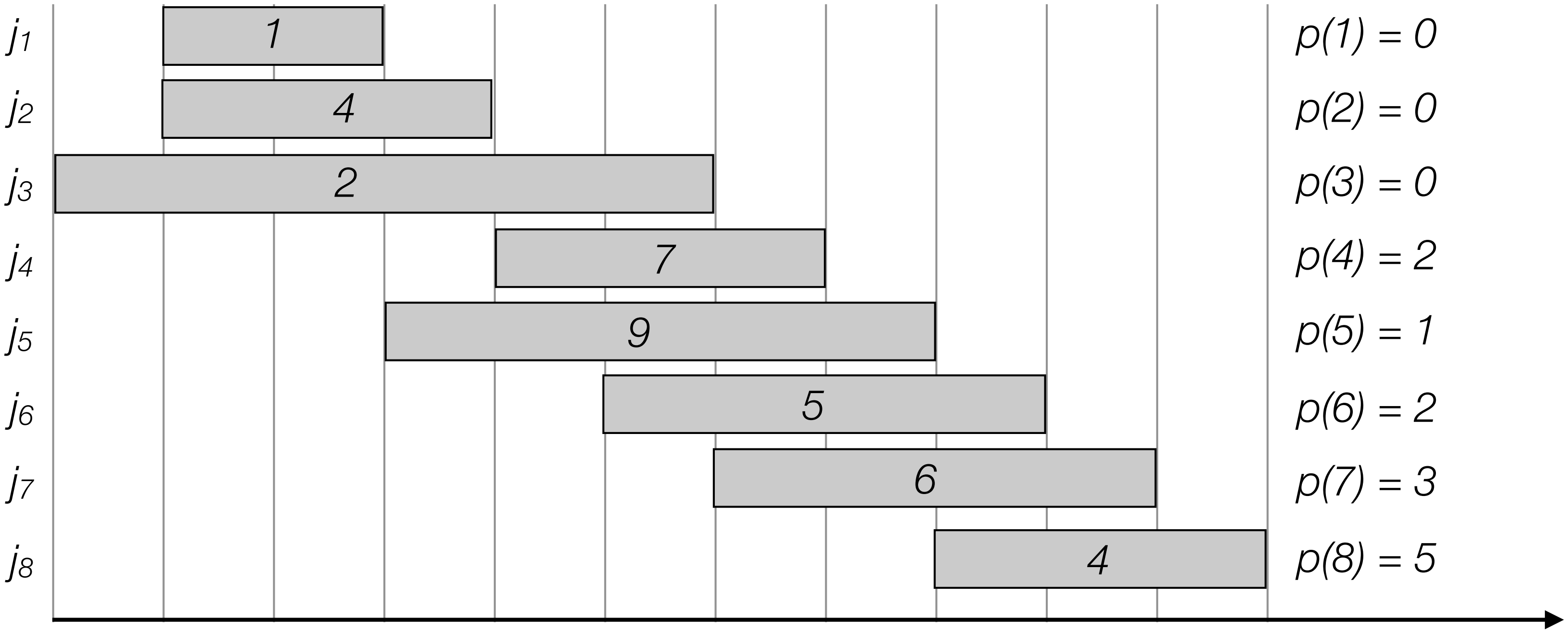

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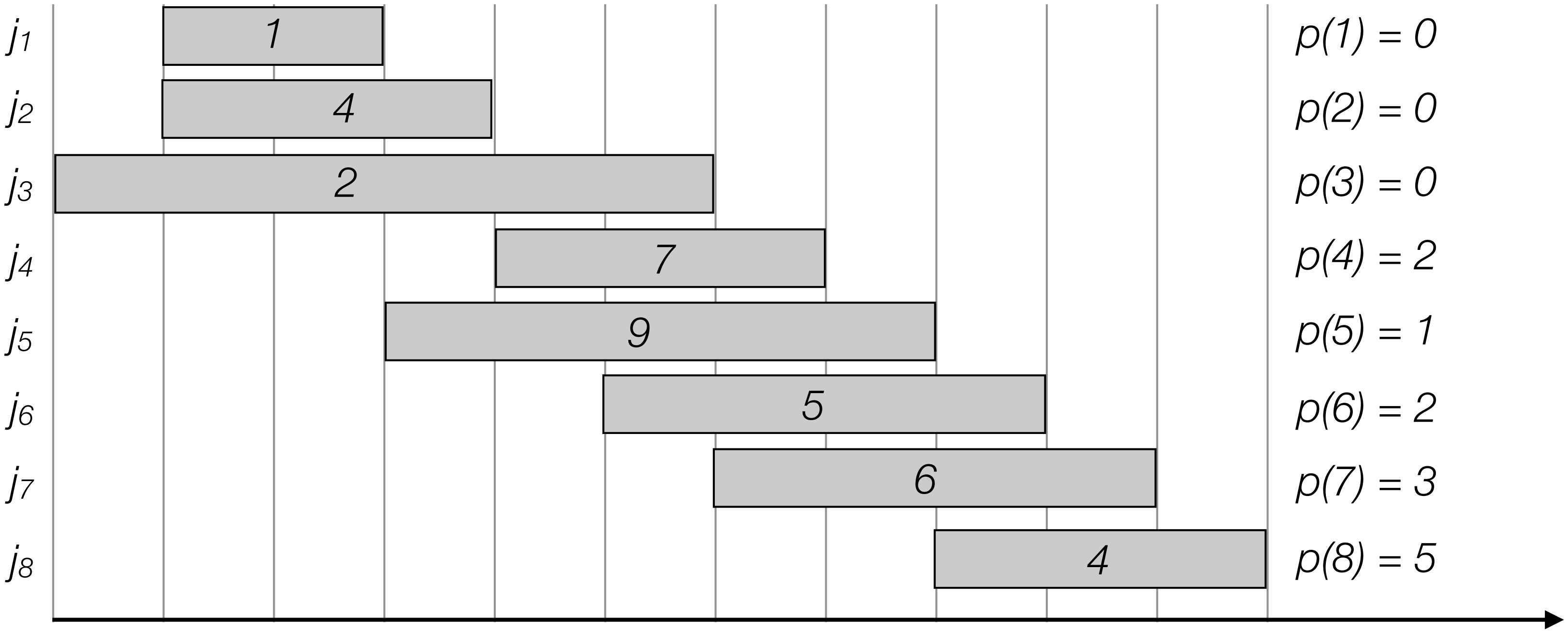

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if j=0
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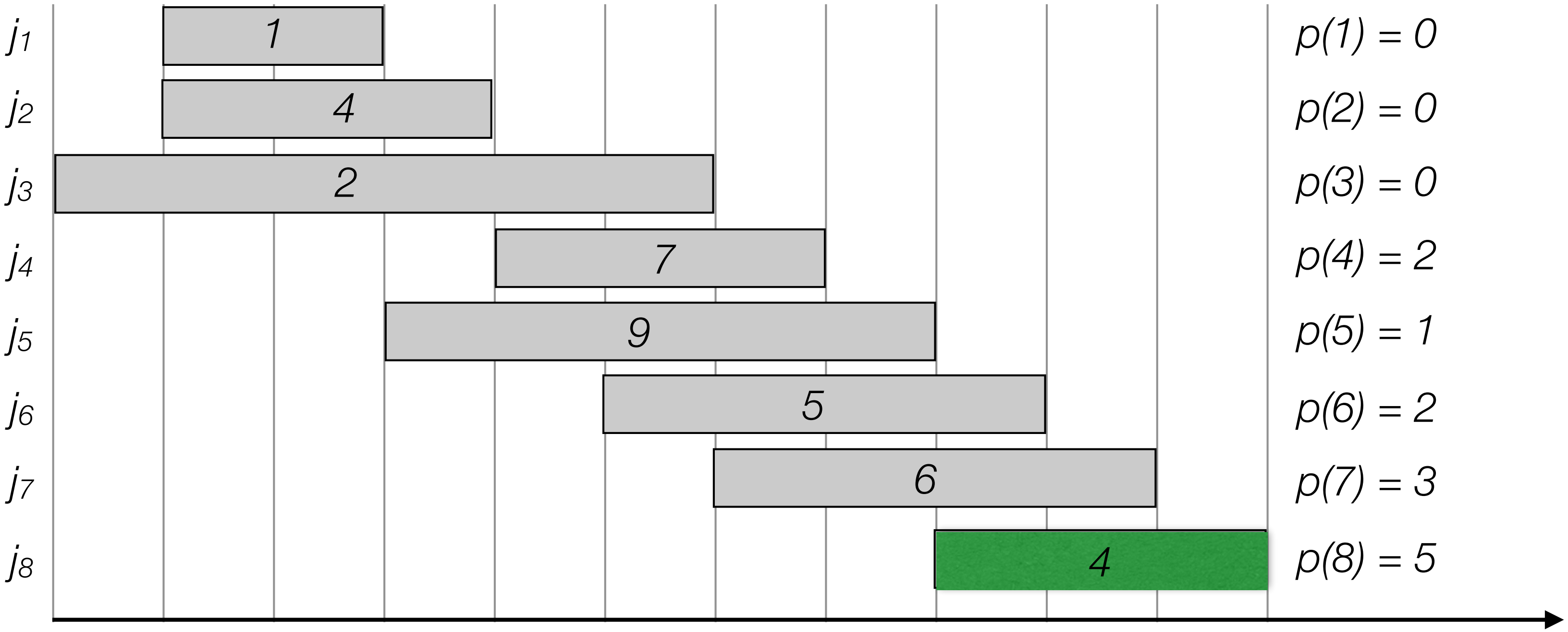
```

Solution = 8



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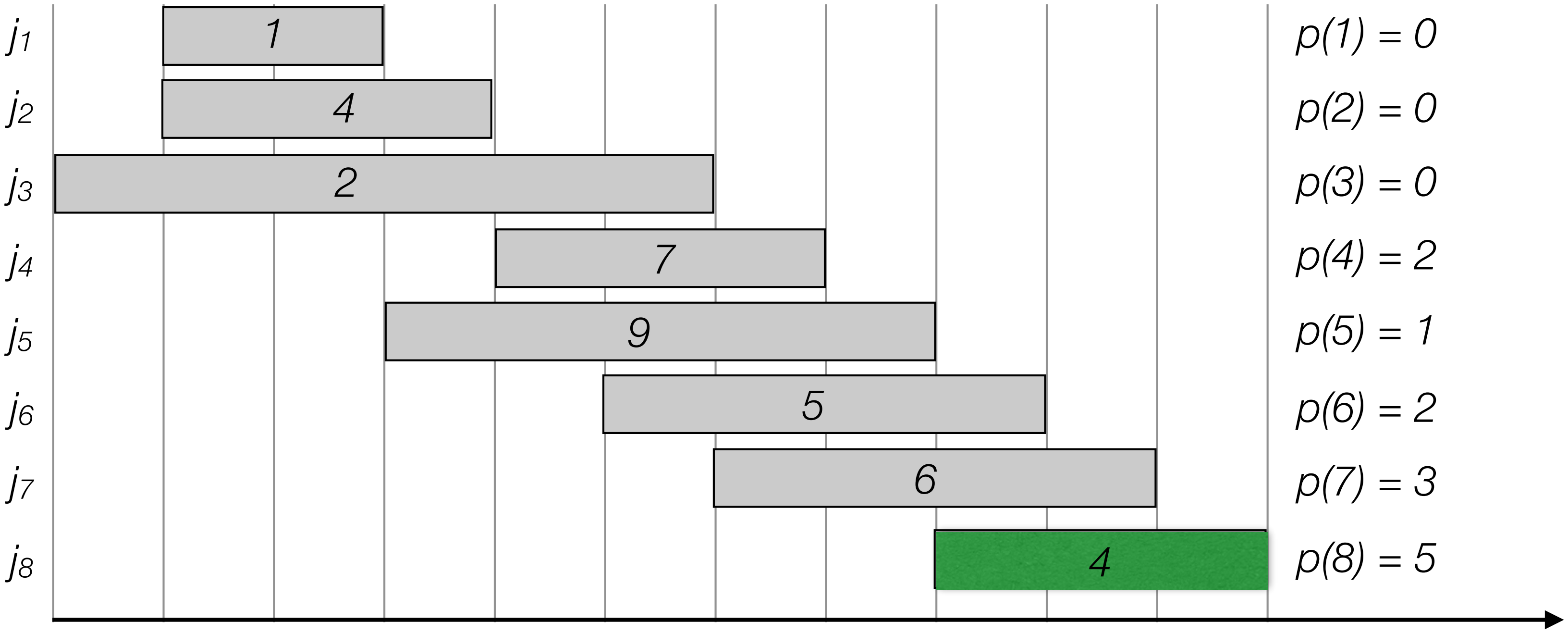

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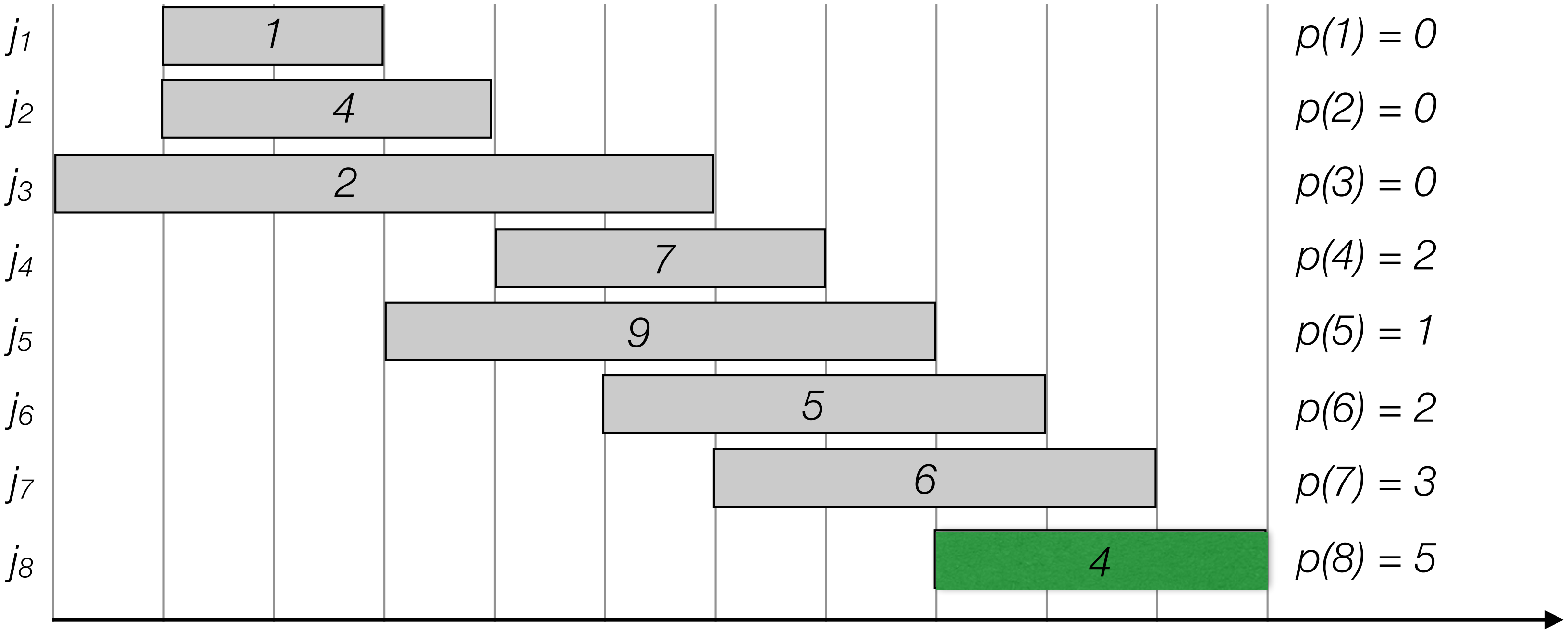

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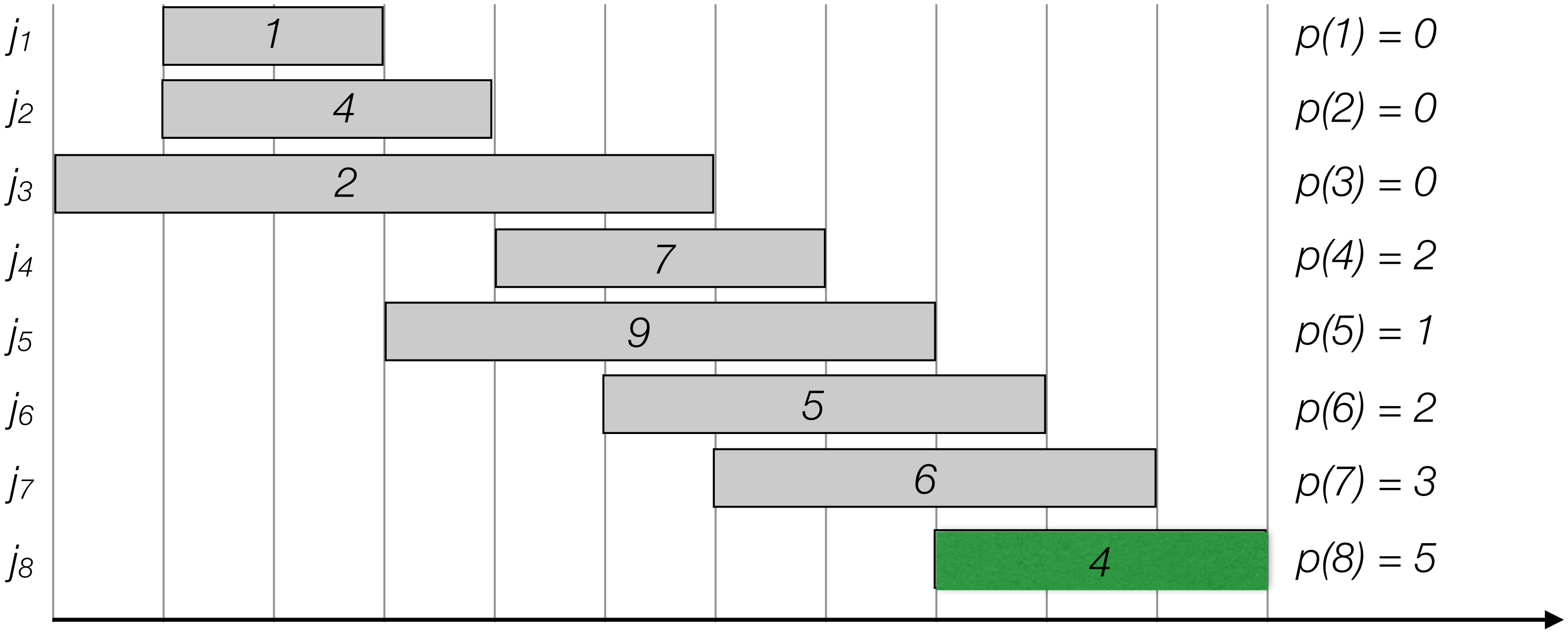

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```
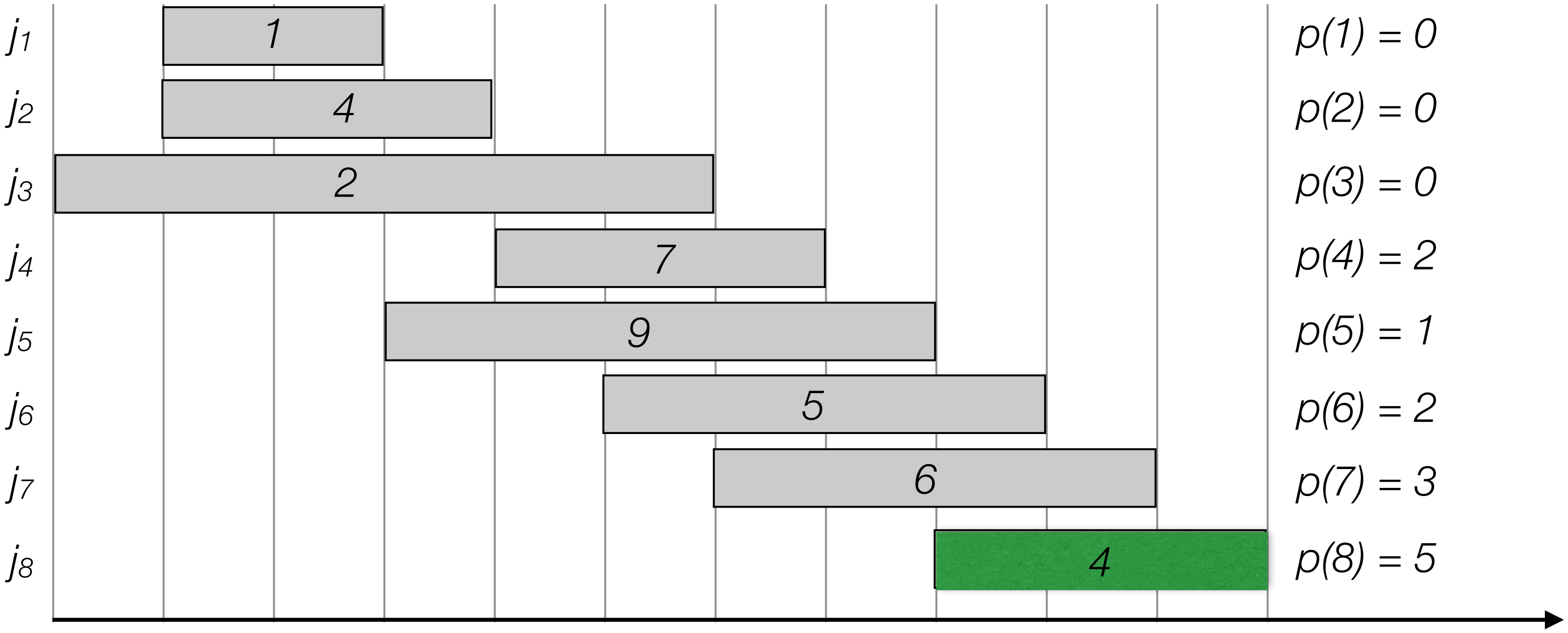

Solution = 8



p(1) = 0

p(2) = 0

p(3) = 0

p(4) = 2

p(5) = 1

p(6) = 2

p(7) = 3

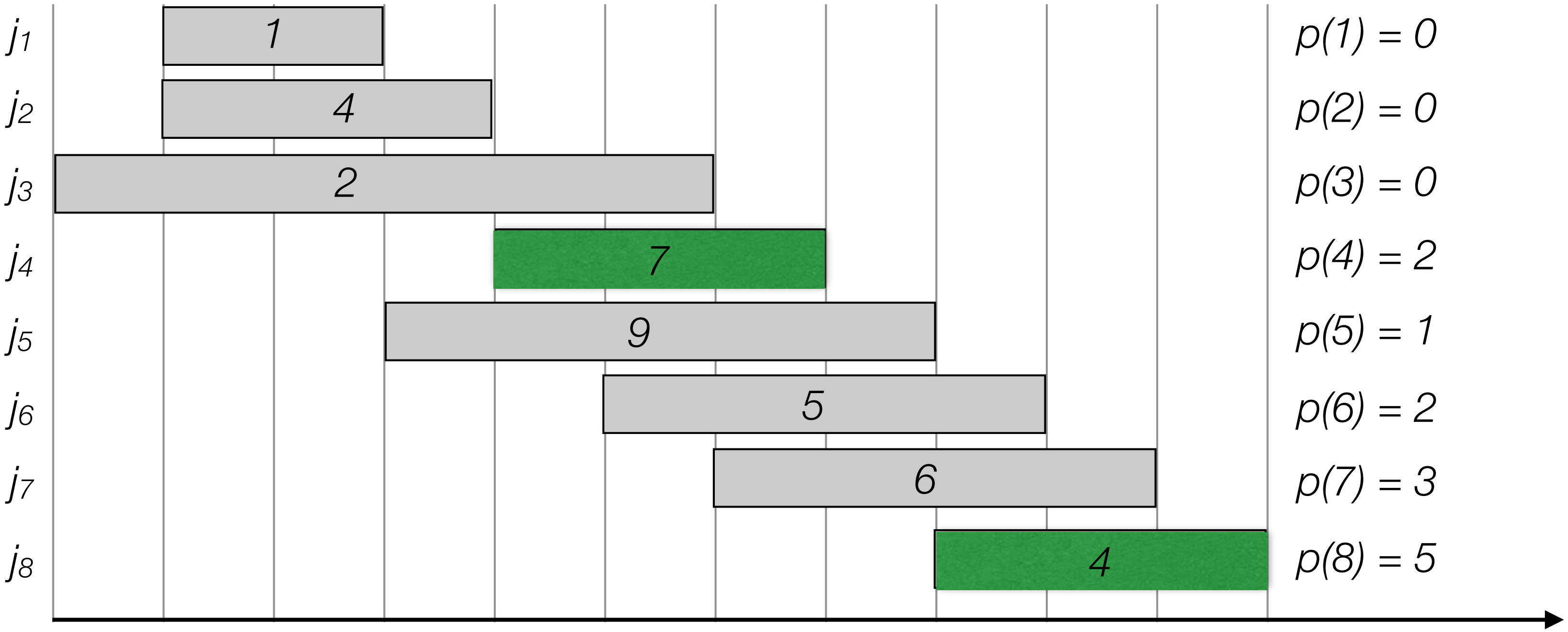
p(8) = 5

i	M[i]
0	0
1	1
2	4
3	4
4	11
5	11
6	11
7	11
8	15

Weighted interval scheduling: find solution

```
Find-Solution(j)
if j=0
  Return emptyset
else if M[j] > M[j-1]
  return {j} U Find-Solution(p[j])
else
  return Find-Solution(j-1)
```

Solution = 8 , 4



$p(1) = 0$

$p(2) = 0$

$p(3) = 0$

$p(4) = 2$

$p(5) = 1$

$p(6) = 2$

$p(7) = 3$

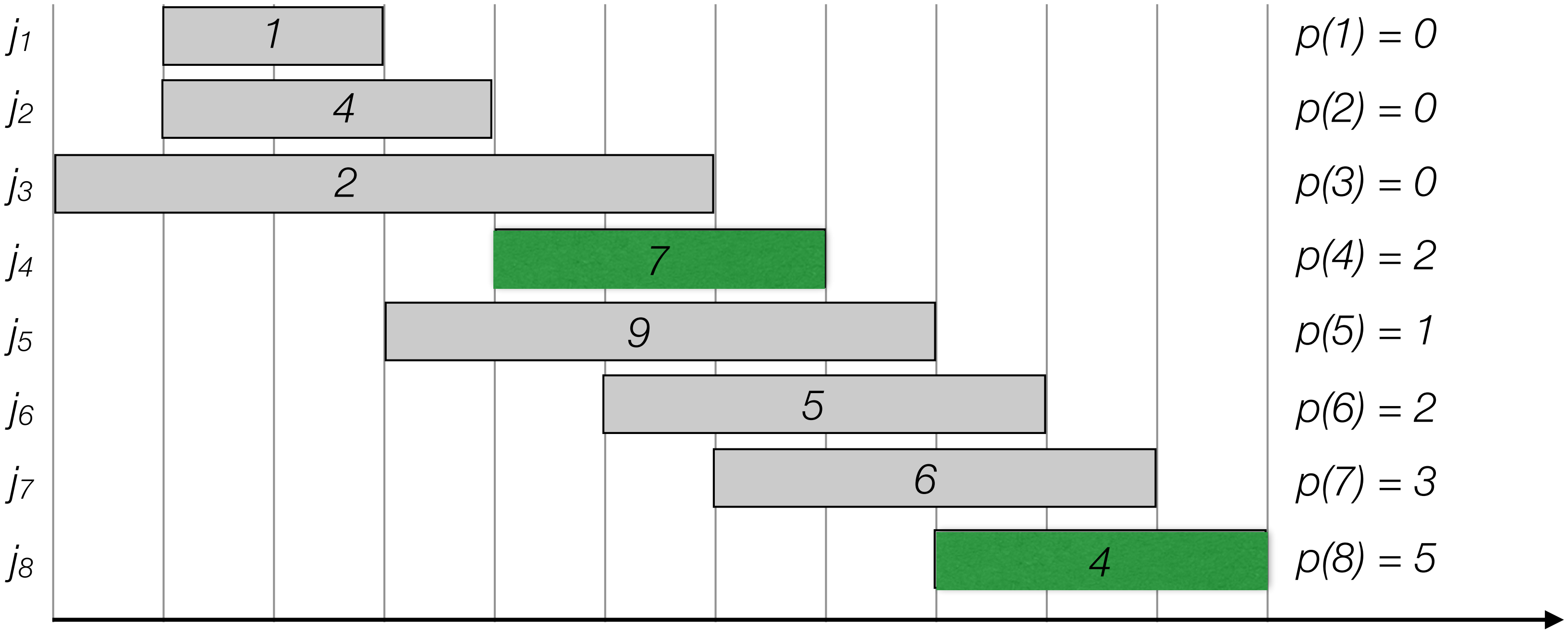
$p(8) = 5$

i	M[i]
0	0
1	1
2	4
3	4
4	11
5	11
6	11
7	11
8	15

Weighted interval scheduling: find solution

```
Find-Solution(j)
if j=0
    Return emptyset
else if M[j] > M[j-1]
    return {j} ∪ Find-Solution(p[j])
else
    return Find-Solution(j-1)
```

Solution = 8 , 4



p(1) = 0

p(2) = 0

p(3) = 0

p(4) = 2

p(5) = 1

p(6) = 2

p(7) = 3

p(8) = 5

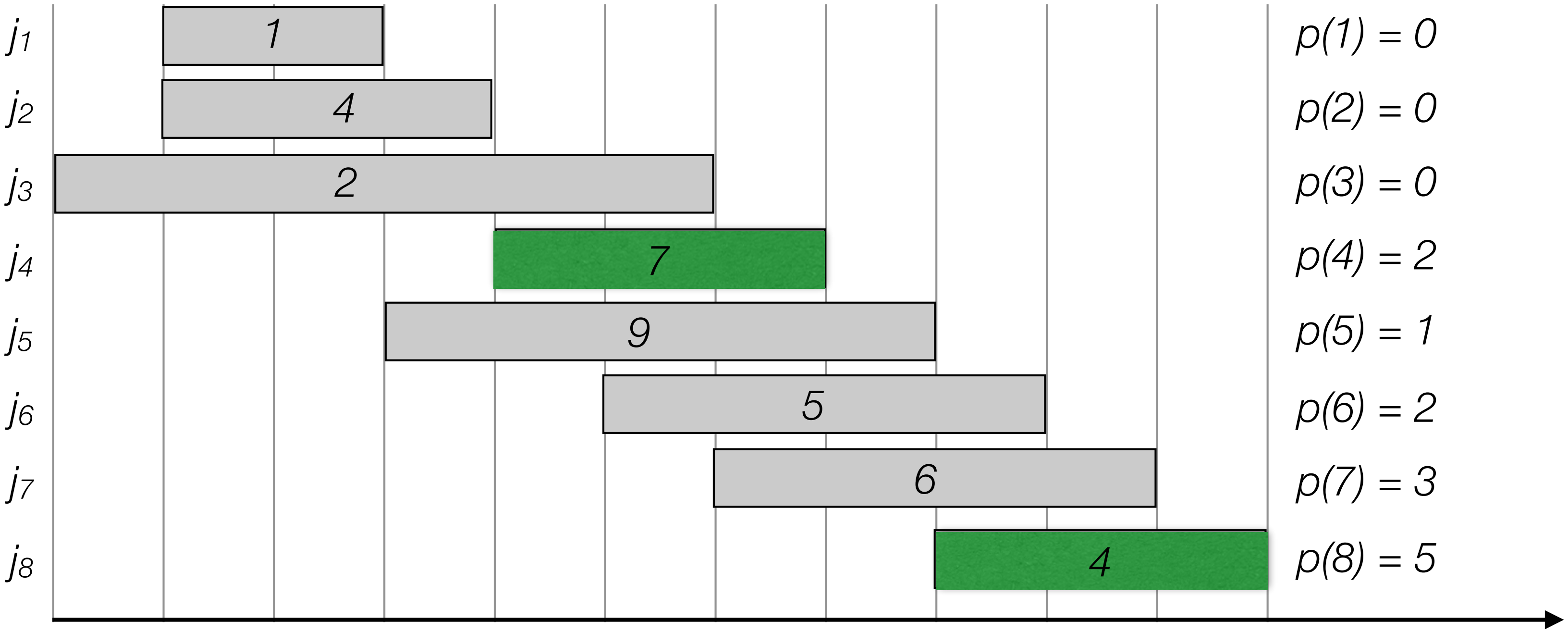
i	M[i]
0	0
1	1
2	4
3	4
4	11
5	11
6	11
7	11
8	15

Weighted interval scheduling: find solution

```
Find-Solution(j)
if j=0
   Return emptyset
else if M[j] > M[j-1]
   return {j} U Find-Solution(p[j])
else
   return Find-Solution(j-1)
```

i	M[i]
0	0
1	1
2	4
3	4
4	11
5	11
6	11
7	11
8	15

Solution = 8 , 4



$p(1) = 0$

$p(2) = 0$

$p(3) = 0$

$p(4) = 2$

$p(5) = 1$

$p(6) = 2$

$p(7) = 3$

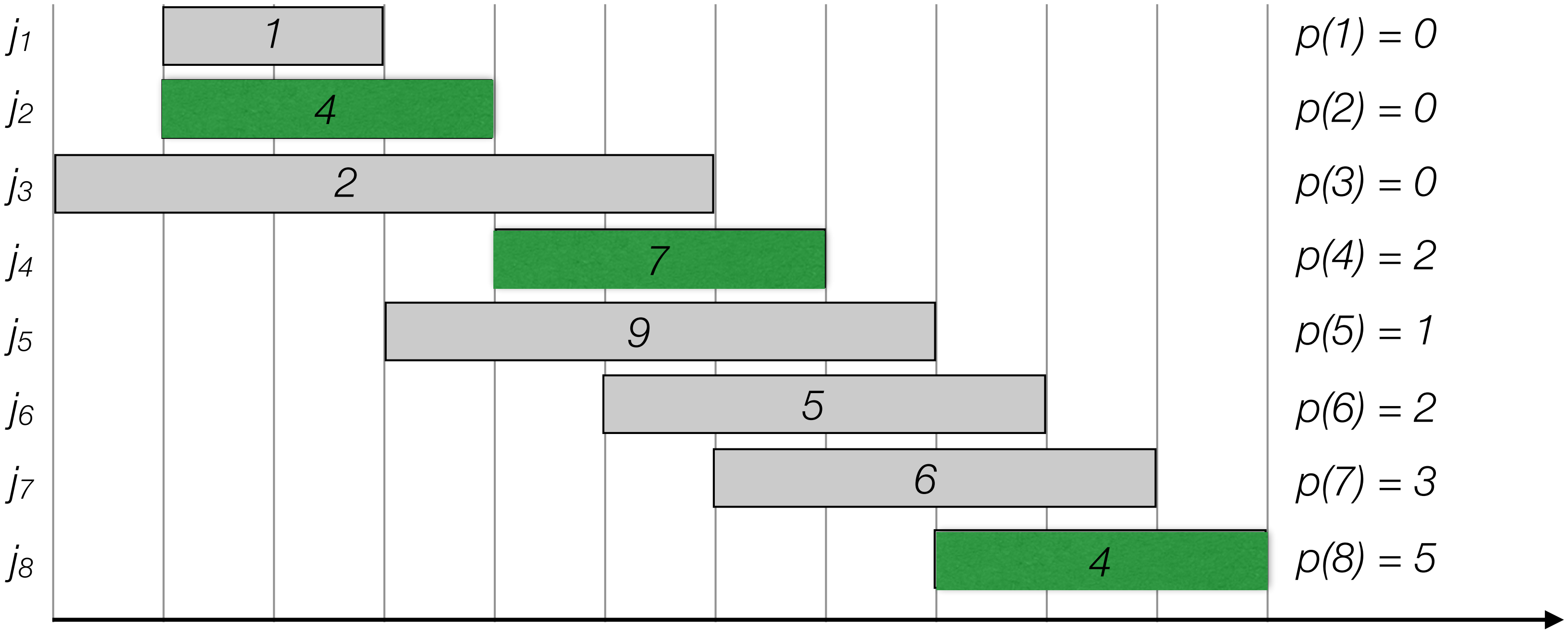
$p(8) = 5$

Weighted interval scheduling: find solution

```
Find-Solution(j)
if j=0
  Return emptyset
else if M[j] > M[j-1]
  return {j} U Find-Solution(p[j])
else
  return Find-Solution(j-1)
```

i	M[i]
0	0
1	1
2	4
3	4
4	11
5	11
6	11
7	11
8	15

Solution = 8 , 4 , 2



$p(1) = 0$

$p(2) = 0$

$p(3) = 0$

$p(4) = 2$

$p(5) = 1$

$p(6) = 2$

$p(7) = 3$

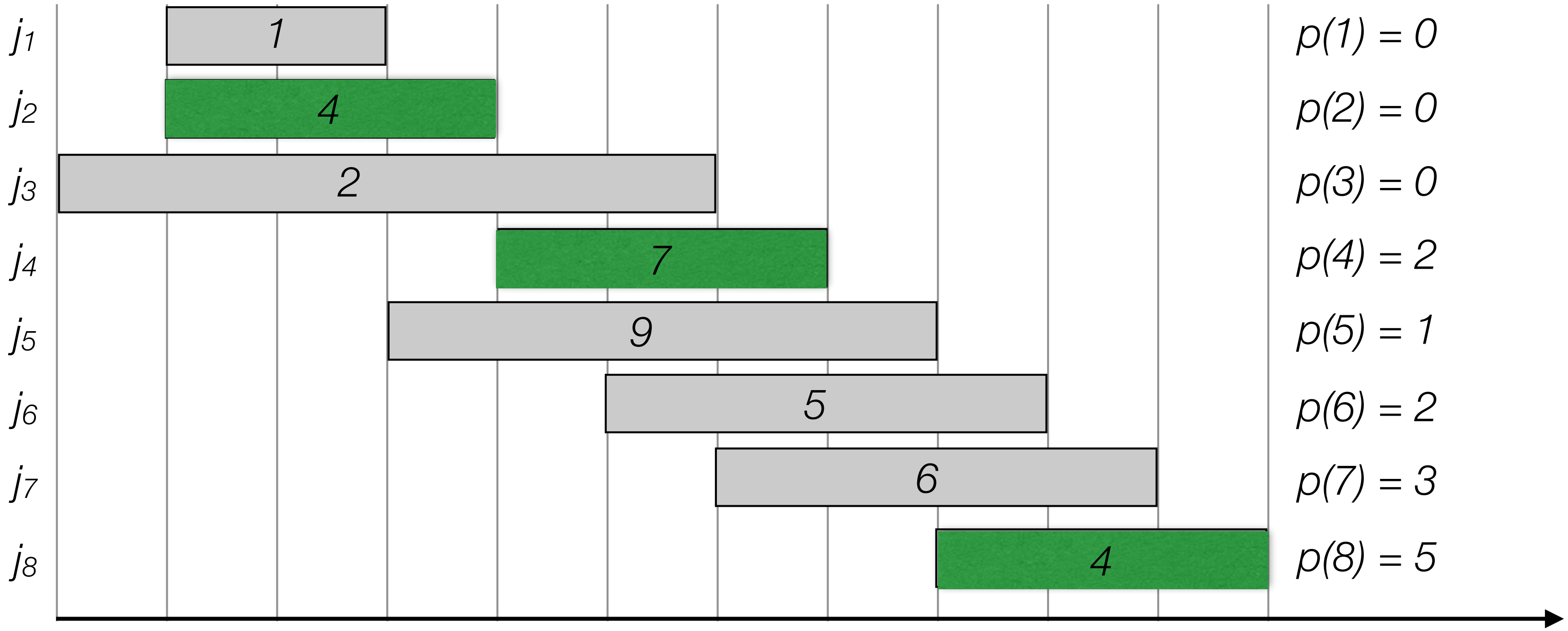
$p(8) = 5$

Weighted interval scheduling: find solution

```
Find-Solution(j)
if j=0
  Return emptyset
else if M[j] > M[j-1]
  return {j} U Find-Solution(p[j])
else
  return Find-Solution(j-1)
```

i	M[i]
0	0
1	1
2	4
3	4
4	11
5	11
6	11
7	11
8	15

Solution = 8 , 4 , 2



j_1
1
j_2
4
j_3
2
j_4
7
j_5
9
j_6
5
j_7
6
j_8
4
$p(1) = 0$
$p(2) = 0$
$p(3) = 0$
$p(4) = 2$
$p(5) = 1$
$p(6) = 2$
$p(7) = 3$
$p(8) = 5$