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A hybrid optimal method to the vehicle routing problem with time windows

The shortest path decomposition of the VRPTW by Lagrangian relaxation require the finding of the optimal Lagrangian multipliers. This problem is a convex non-differentiable optimization problem. The optimal multipliers are found using the non-differentiable method denoted the cutting-plane method with trust-region.

The cutting-plane method has been coupled with a Dantzig-Wolfe algorithm in a branch-and-bound scheme. The root node of the branch-and-bound tree is solved by the cutting-plane-method and, if an integer solution is not obtained, shifting to a Dantzig-Wolfe algorithm in the tree nodes occurs. The hybrid method combining the cutting-plane and Dantzig-Wolfe algorithm has been tested on the well-known Solomon VRPTW benchmark problems and a range of extended Solomon problems.

We have succeeded in solving several previously unsolved problems and an extended Solomon problem with 1000 customers, which is the largest problem ever solved to optimality. The computational times were reduced significantly by the cutting-plane method in the root node compared to the Dantzig-Wolfe method due to the easier subproblems. It seems therefore very efficient to combine the strengths of an Lagrangian relaxation approach with the strengths of an Dantzig-Wolfe decomposition approach for the VRPTW.