Classical and modern heuristics for the vehicle routing problem

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The *Vehicle Routing Problem* (VRP) holds a central place in distribution management. It is also an important combinatorial optimization problem that has stimulated several powerful exact and approximate solution methodologies. The most studied version is the capacitated VRP on an undirected graph which can be defined as follows. Let $G = (V, E)$ be a graph where $V = \{v_0, v_1, ..., v_n\}$ is the vertex set and $E = \{(v_i, v_j) : v_i, v_j \in V, i < j\}$ is the edge set. Vertex v_0 represents a depot at which are based m identical vehicles of capacity Q. In some versions of the problems, m is a known value, in others it is a decision variable. The remaining vertices represent cities or customers. With every vertex v_i is associated a non-negative demand q_i. Also, a non-negative cost or distance c_{ij} is associated with every edge (v_i, v_j). The VRP consists of determining a set of m vehicle routes i) starting and ending at the depot; ii) such that every customer is visited exactly once by one vehicle; iii) such that the total demand of each vehicle route does not exceed Q; iv) having a minimum total cost.

The VRP is an NP-hard problem that is exceedingly difficult to solve to optimality. To this day, no exact algorithm can consistently solve VRP instances in excess of 50 cities, although several larger instances, some involving more than 100 cities, have been solved optimally. Thus, the only practical approach is the use of heuristics. These belong to two broad classes: classical heuristics and modern heuristics (or metaheuristics).

Classical VRP heuristics can be broadly classified into three categories. *Constructive heuristics* gradually build a feasible solution while keeping an eye on solution cost, but do not contain an improvement phase *per se*. In *two-phase heuristics*, the problem is decomposed into its two natural components: clustering of vertices into feasible routes and actual route construction, with possible feedback loops between the two stages. Two-phase heuristics can be divided into two classes: *cluster-first, route-second* methods and *route-first, cluster-second* methods. In the first case, vertices are first organized into feasible clusters, and a vehicle route is constructed for each of them. In the second case, a tour is first built on all vertices and is then segmented into feasible vehicle routes. Finally, *improvement methods* attempt to upgrade any feasible solution by performing a sequence of edge or vertex exchanges within or between vehicle routes. The distinction between constructive and improvements methods is, however, often blurred since most constructive algorithms incorporate improvements steps (typically 3–opt) at various stages.

As far as we are aware, six main types of metaheuristics have been applied to the VRP: 1) Simulated Annealing (SA), 2) Deterministic Annealing (DA), 3) Tabu Search (TS), 4) Genetic Algorithms (GA), 5) Ant Systems (AS), and 6) Neural Networks (NN). The first three algorithms, SA, DA and TS, start from an initial solution x_1, and move at each iteration t from x_t to a solution x_{t+1} in the neighbourhood $N(x_t)$ of x_t, until a stopping condition is satisfied. If $f(x)$ denotes the cost of x, then $f(x_{t+1})$ is not necessarily less than $f(x_t)$. As a result, care must be taken to avoid cycling. GA examines at each step a population of solutions. Each population is derived from the preceding one by combining its best elements and discarding the worst. AS is a constructive approach in which several new solutions are created at each iteration using some of the information gathered at previous iterations. As was pointed out by Taillard *et al.*, TS, GA and AS are methods that record, as the search proceeds, information on solutions encountered and use it to obtain improved solution. NN is a learning mechanism that gradually adjusts a of weights until an acceptable solution is reached. The rules governing the search differ in each case and these must also be tailored to the shape of the problem at hand. Also, a fair amount of creativity and experimentation is required.

The purpose of this talk is to provide an over view of the most important classical and modern heuristics for the VRP. Several comparative computational results will be presented. Some future research directions will be outlined.