Optional exam (Sangiorgi’s lectures)

Consider the following variant of bisimulation. I write $P \xrightarrow{\mu} P'$ if there is some $P'$ s.t. $P \xrightarrow{\mu} P'$, and similarly for $P \xrightarrow{\mu}$ and $P \xrightarrow{\mu}$. I recall that $\sim$ is strong bisimilarity.

**Definition** [quasi-bisimulation] A relation $\mathcal{R}$ on the states of an LTS is a quasi-bisimulation if whenever $P \mathcal{R} Q$:

1. if $P \xrightarrow{\mu} P'$, then there is $Q'$ such that $Q \xrightarrow{\mu} Q'$ and $P' \mathcal{R} Q'$.
2. if $Q \xrightarrow{\mu}$ then $P \xrightarrow{\mu}$

$P$ and $Q$ are quasi-bisimilar, written $P \preceq Q$ (or $Q \succeq P$), if $P \mathcal{R} Q$, for some quasi-bisimulation $\mathcal{R}$.

**Exercise 1** Show that $\preceq$ is reflexive and transitive, but not symmetric

**Exercise 2** Show processes $P, Q$ (each written as an LTS, or as a CCS process, as you prefer) s.t. both $P \preceq Q$ and $P \succeq Q$ hold, but not $P \sim Q$.

**Exercise 3** Take a finite CCS language (ie, without recursion).

- Is $\preceq$ preserved by the operators of this language?
- Same question for the relation $\preceq \cup \succeq$ (which relates two processes $P, Q$ if both $P \preceq Q$ and $P \succeq Q$)

**Exercise 4** Consider the following version of weak quasi-bisimilarity:

**Definition** A relation $\mathcal{R}$ on the states of an LTS is a weak quasi-bisimulation if $P \mathcal{R} Q$ implies:

1. if $P \xrightarrow{\mu} P'$, then there is $Q'$ s.t. $Q \xrightarrow{\mu} Q'$ and $P' \mathcal{R} Q'$;
2. if $Q \xrightarrow{\mu}$ then $P \xrightarrow{\mu}$

$P$ and $Q$ are weakly quasi-bisimilar, written $P \preceq\approx Q$ (or $Q \succeq\approx P$), if $P \mathcal{R} Q$, for some weak quasi-bisimulation $\mathcal{R}$.

Is $\preceq\approx$ preserved by the parallel composition operators of CCS? If yes, prove it, if not, give a counterexample.

**Exercise 5** We have seen in the lectures an inductive characterisation of bisimilarity on image-finite LTSs ($\sim\equiv\sim_\omega$, where $\sim_\omega \triangleq \bigcap_n \sim_n$). Define and prove a similar inductive characterisation for $\preceq$.

**Exercise 6** We consider here processes described by LTSs that are image-finite and where the alphabet of actions $\text{Act}$ is finite. On these processes, bisimilarity can be characterised using the following modal logic:

$$F ::= \langle a \rangle.F \ | \ [a].F \ | \ F_1 \land F_2 \ | \ F_1 \lor F_2 \ | \ \text{true} \ | \ \text{false}$$
where $a \in \text{Act}$. If we define $P \prec_L Q$ thus:

$$
\text{for all } F, P \models F \implies Q \models F
$$

then we have

$$
P \sim Q \iff (P \prec_L Q \text{ and } Q \prec_L P)
$$

(we also have $P \sim Q \iff P \prec_L Q$, in fact, since each formula has a dual.)

For details: see Milner’s CCS book, or Caires’s lectures at Globan. (note: i am not asking to prove these results)

Define and prove and similar modal logic characterisation for $\preceq$; that is a logic with the property that

$$
P \preceq Q \iff P \prec_L Q
$$

(Hints: read a proof of the characterisation of bisimilarity using the logic; then the logic for $\preceq$ is obtained by the grammar above by appropriately pruning it)