

Optional exam (Sangiorgi's lectures)

Consider the following variant of bisimulation. I write $P \xrightarrow{\mu}$ if there is some P' s.t. $P \xrightarrow{\mu} P'$, and similarly for $P \xRightarrow{\mu}$ and $P \xRightarrow{\hat{\mu}}$. I recall that \sim is strong bisimilarity.

Definition [quasi-bisimulation] A relation \mathcal{R} on the states of an LTS is a quasi-bisimulation if whenever $P \mathcal{R} Q$:

1. if $P \xrightarrow{\mu} P'$, then there is Q' such that $Q \xrightarrow{\mu} Q'$ and $P' \mathcal{R} Q'$.
2. if $Q \xrightarrow{\mu}$ then $P \xrightarrow{\mu}$

P and Q are quasi-bisimilar, written $P \lesssim Q$ (or $Q \gtrsim P$), if $P \mathcal{R} Q$, for some quasi-bisimulation \mathcal{R} .

Exercise 1 Show that \lesssim is reflexive and transitive, but not symmetric

Exercise 2 Show processes P, Q (each written as an LTS, or as a CCS process, as you prefer) s.t. both $P \lesssim Q$ and $P \gtrsim Q$ hold, but not $P \sim Q$.

Exercise 3 Take a finite CCS language (ie, without recursion).

- Is \lesssim preserved by the operators of this language?
- Same question for the relation $\lesssim \cup \gtrsim$ (which relates two processes P, Q if both $P \lesssim Q$ and $P \gtrsim Q$)

Exercise 4 Consider the following version of weak quasi-bisimilarity:

Definition A relation \mathcal{R} on the states of an LTS is a weak quasi-bisimulation if PRQ implies:

1. if $P \xrightarrow{\mu} P'$, then there is Q' s.t. $Q \xRightarrow{\hat{\mu}} Q'$ and $P' \mathcal{R} Q'$;
2. if $Q \xRightarrow{\mu}$ then $P \xRightarrow{\hat{\mu}}$

P and Q are weakly quasi-bisimilar, written $P \lesssim Q$ (or $Q \gtrsim P$), if $P \mathcal{R} Q$, for some weak quasi-bisimulation \mathcal{R} .

Is \lesssim preserved by the parallel composition operators of CCS? If yes, prove it, if not, give a counterexample.

Exercise 5 We have seen in the lectures an inductive characterisation of bisimilarity on image-finite LTSs ($\sim = \sim_\omega$, where $\sim_\omega \triangleq \bigcap_n \sim_n$). Define and prove a similar inductive characterisation for \lesssim .

Exercise 6 We consider here processes described by LTSs that are image-finite and where the alphabet of actions Act is finite. On these processes, bisimilarity can be characterised using the following modal logic:

$$F ::= \langle a \rangle . F \mid [a] . F \mid F_1 \wedge F_2 \mid F_1 \vee F_2 \mid \text{true} \mid \text{false}$$

where $a \in \mathbf{Act}$. If we define $P <_L Q$ thus:

$$\text{for all } F, P \models F \text{ implies } Q \models F$$

then we have

$$P \sim Q \text{ iff } (P <_L Q \text{ and } Q <_L P)$$

(we also have $P \sim Q$ iff $P <_L Q$, in fact, since each formula has a dual.)
For details: see Milner's CCS book, or Caires's lectures at Globan. (note: i am not asking to prove these results)

Define and prove and similar modal logic characterisation for \lesssim ; that is a logic with the property that

$$P \lesssim Q \text{ iff } P <_L Q$$

(Hints: read a proof of the characterisation of bisimilarity using the logic; then the logic for \lesssim is obtained by the grammar above by appropriately pruning it)