Optional exam (Sangiorgi's lectures)

Consider the following variant of bisimulation. I write $P \xrightarrow{\mu}$ if there is some P' s.t. $P \xrightarrow{\mu} P'$, and similarly for $P \overset{\mu}{\Longrightarrow}$ and $P \overset{\widehat{\mu}}{\Longrightarrow}$. I recall that $\sim$ is strong bisimilarity.

Definition *[quasi-bisimulation] A relation $\mathcal{R}$ on the states of an LTS is a* quasi-bisimulation *if whenever $P \mathcal{R} Q$:*

1. *if $P \xrightarrow{\mu} P'$, then there is Q' such that $Q \xrightarrow{\mu} Q'$ and $P' \mathcal{R} Q'$.*
2. *if $Q \xrightarrow{\mu}$ then $P \xrightarrow{\mu}$*

P and Q are quasi-bisimilar*, written $P \lesssim Q$ (or $Q \gtrsim P$), if $P \mathcal{R} Q$, for some quasi-bisimulation $\mathcal{R}$.*

Exercise 1 Show that $\lesssim$ is reflexive and transitive, but not symmetric

Exercise 2 Show processes P, Q (each written as an LTS, or as a CCS process, as you prefer) s.t. both $P \lesssim Q$ and $P \gtrsim Q$ hold, but not $P \sim Q$.

Exercise 3 Take a finite CCS language (ie, without recursion).

- Is $\lesssim$ preserved by the operators of this language?
- Same question for the relation $\lesssim \cup \gtrsim$ (which relates two processes P, Q if both $P \lesssim Q$ and $P \gtrsim Q$)

Exercise 4 Consider the following version of weak quasi-bisimilarity:

Definition *A relation $\mathcal{R}$ on the states of an LTS is a a* weak quasi-bisimulation *if $P\mathcal{R}Q$ implies:*

1. *if $P \xrightarrow{\mu} P'$, then there is Q' s.t. $Q \overset{\widehat{\mu}}{\Longrightarrow} Q'$ and $P' \mathcal{R} Q'$;*
2. *if $Q \overset{\mu}{\Longrightarrow}$ then $P \overset{\widehat{\mu}}{\Longrightarrow}$*

P and Q are weakly quasi-bisimilar*, written $P \lessapprox Q$ (or $Q \gtrapprox P$), if $P \mathcal{R} Q$, for some weak quasi-bisimulation $\mathcal{R}$.*

Is $\lessapprox$ preserved by the parallel composition operators of CCS ? If yes, prove it, if not, give a counterexample.

Exercise 5 We have seen in the lectures an inductive characterisation of bisimilarity on image-finite LTSs ($\sim = \sim_\omega$, where $\sim_\omega \triangleq \bigcap_n \sim_n$). Define and prove a similar inductive characterisation for $\lesssim$.

Exercise 6 We consider here processes described by LTSs that are image-finite and where the alphabet of actions `Act` is finite. On these processes, bisimilarity can be characterised using the following modal logic:

$$F ::= \langle a \rangle .\, F \;\big|\; [a].\, F \;\big|\; F_1 \wedge F_2 \;\big|\; F_1 \vee F_2 \;\big|\; \texttt{true} \;\big|\; \texttt{false}$$

where $a \in \texttt{Act}$. If we define $P \ <_{\mathrm{L}} \ Q$ thus:

$$\text{for all } F,\ P \models F \text{ implies } Q \models F$$

then we have

$$P \sim Q \text{ iff } (P \ <_{\mathrm{L}} \ Q \text{ and } Q \ <_{\mathrm{L}} \ P)$$

(we also have $P \sim Q$ iff $P \ <_{\mathrm{L}} \ Q$, in fact, since each formula has a dual.) For details: see Milner's CCS book, or Caires's lectures at Globan. (note: i am not asking to prove these results)

Define and prove and similar modal logic characterisation for $\lesssim$; that is a logic with the property that

$$P \lesssim Q \text{ iff } P \ <_{\mathrm{L}} \ Q$$

(Hints: read a proof of the characterisation of bisimilarity using the logic; then the logic for $\lesssim$ is obtained by the grammar above by appropriately pruning it)