



CORA

AMETIST
advanced methods for timed systems

Optimal & Real Time Scheduling

— *using* —
Model Checking Technology



BRICS

Basic Research
in Computer Science



CENTER FOR INDLEJREDE SOFTWARE SYSTEMER

April 2002 – June 2005 IST-2001-35304

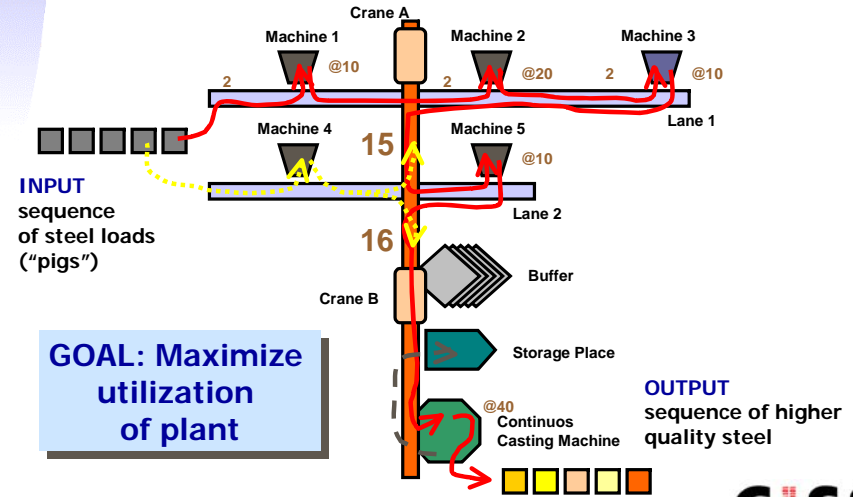
■ Academic partners: ■ Industrial Partners

- Nijmegen
 - Aalborg
 - Dortmund
 - Grenoble
 - Marseille
 - Twente
 - Weizmann
- Axxom
 - Bosch
 - Cybernetix
 - Terma

OBJECTIVES

- powerful, unifying mathematical modelling
- efficient computerized problem-solving tools
- distributed real-time systems
- time-dependent behaviour and dynamic resource allocation
- TIMED AUTOMATA**

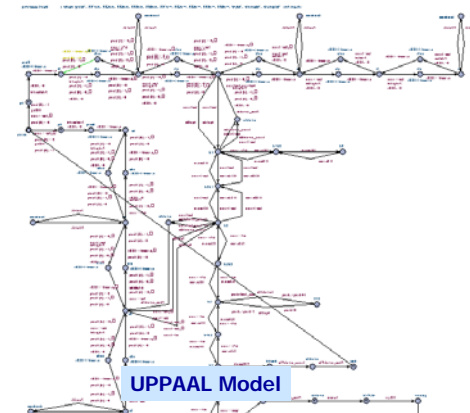
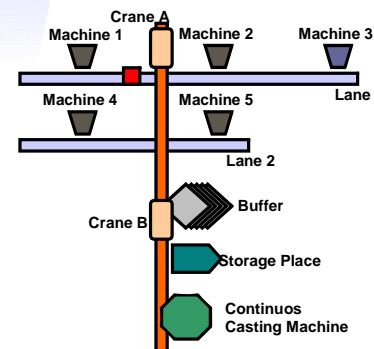
SIDMAR Overview



LTR Project VHS (Verification of Hybrid systems)

SIDMAR Modelling

A Single Load



Case S

- Cybern
 - Smart Pers

- Terma:
 - Mem

- Bosch:
 - Car F
 - Sens

- AXXOM
 - Lacq

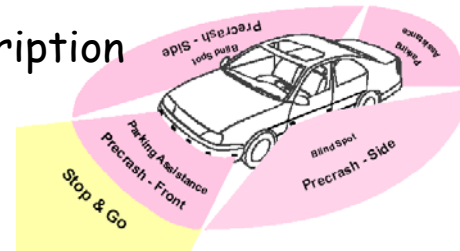
- Benchmarks



CPS: Informal description

● CPS obtains and makes available for other systems information about environment of a car. This information may be used for:

- Parking assistance
- Pre-crash detection
- Blind spot supervision
- Lane change assistance
- Stop & go
- Etc

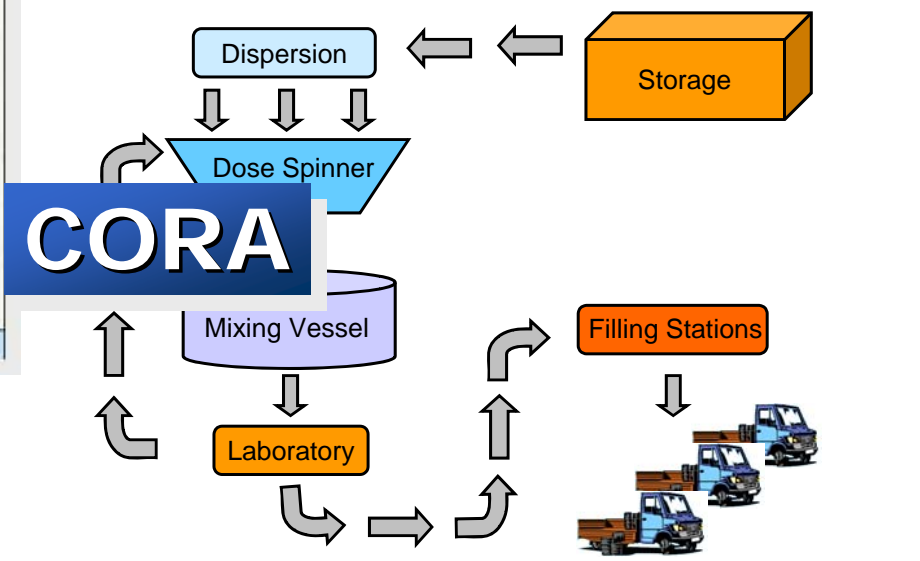


● The CPS considered in this case study

- One sensor group only (currently 2 sensors)
- Only the front sensors and corresponding controllers
- Application: pre-crash detection, parking assistance, stop & go

CLASSIC

Product flow of a Product axxom



CORA

UPPAAL CORA - Mozilla

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RELATED SITES: UPPAAL | AMETIST

UPPAAL CORA

UPPAAL for Cost Optimal Reachability Analysis

Main Page Introduction Language Guide Option Guide Publications Case Studies Download Contact us

Welcome!

[UPPAAL](#) is an integrated tool environment for modeling, validation and verification of real-time systems modeled as networks of timed automata, extended with data types (bounded integers, arrays, etc.).

UPPAAL CORA is a branch of UPPAAL for Cost Optimal Reachability Analysis developed by the UPPAAL team as part of the [VHS](#) and [AMETIST](#) projects. Whereas UPPAAL supports model checking of timed automata, UPPAAL CORA uses an extension of timed automata called [LPTA](#). LPTA allows you to annotate the model with the notion of cost. This can be the cost of delay in certain situations or the cost of particular actions. UPPAAL CORA then finds optimal paths matching goal conditions.

UPPAAL CORA has been used in a number of case studies. Some of these are described on the case study page of this site. If you come up with interesting uses, please contact us. We are interested in hearing what you do!

Due to different internal data structures, UPPAAL CORA currently consists of two different versions:

- A version for the simplified case of time optimal reachability analysis.
- A version for the full language of LPTA.

Latest News

UPPAAL Got New Home Page

25 Jan 2005

The main [UPPAAL site](#) adopted the same layout as the UPPAAL CORA site. At the same time, the UPPAAL CORA site has adopted a new color scheme.

Updated case studies

30 Nov 2004

The models of the case studies can now be downloaded from the case study page.

[More News >](#)

Done

Overview

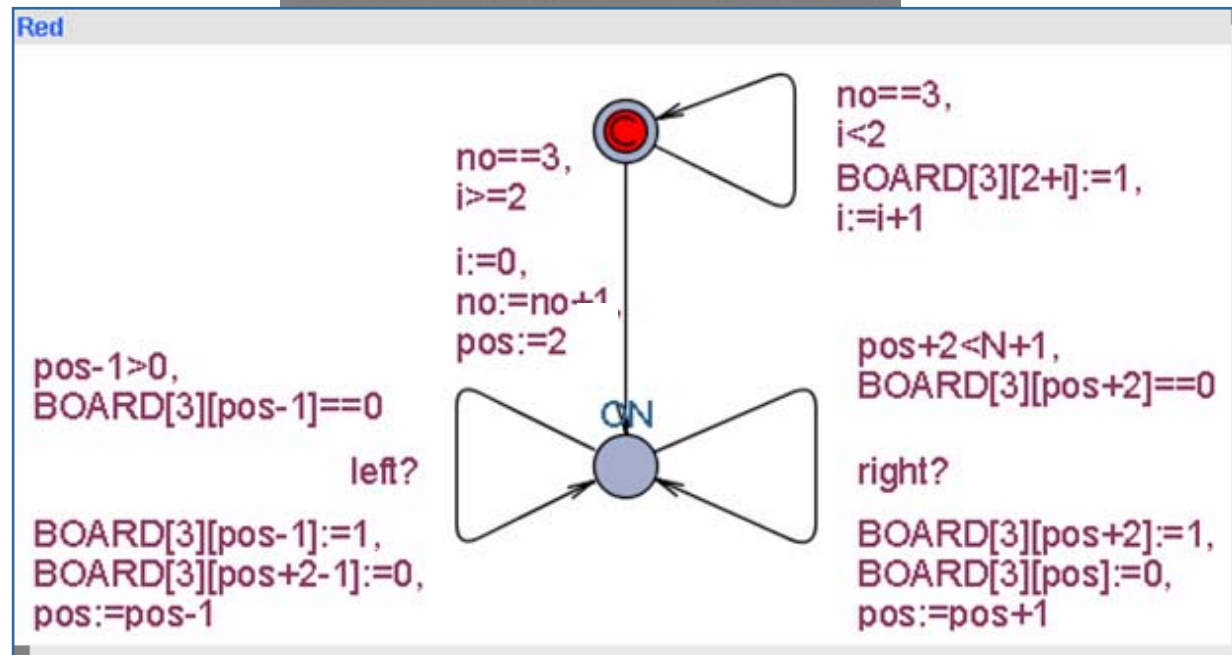
- Timed Automata & Scheduling
- Priced Timed Automata and Optimal Scheduling
- Optimal Infinite Scheduling
- **Optimal Scheduling Using Priced Timed Automata.**
 G. Behrmann, K. G. Larsen, J. I. Rasmussen,
 ACM SIGMETRICS Performance Evaluation Review
- On-Line Scheduling and Off-Line Test Generation

Rush Hour

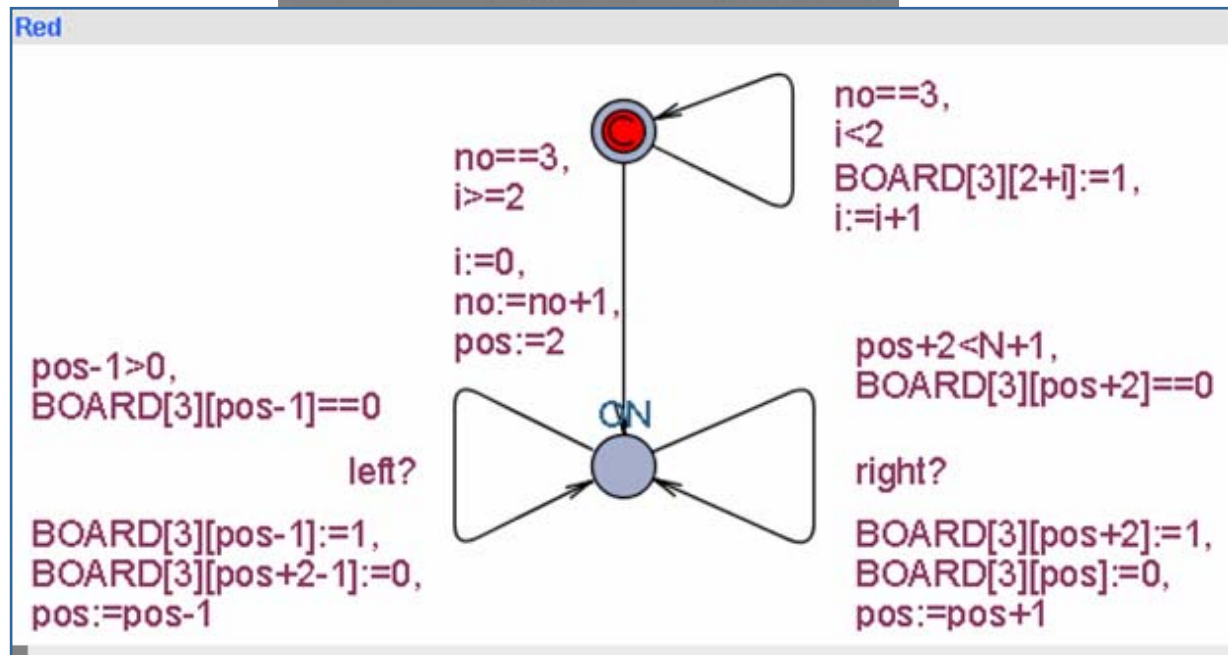
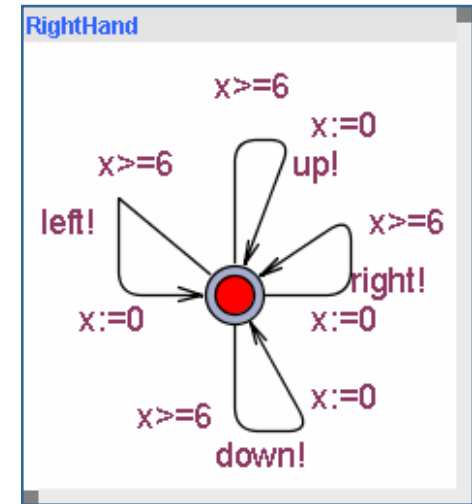
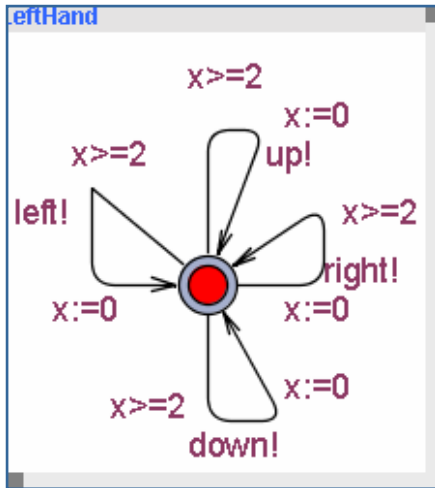


OBJECTIVE:
Get your
CAR out

Rush Hour

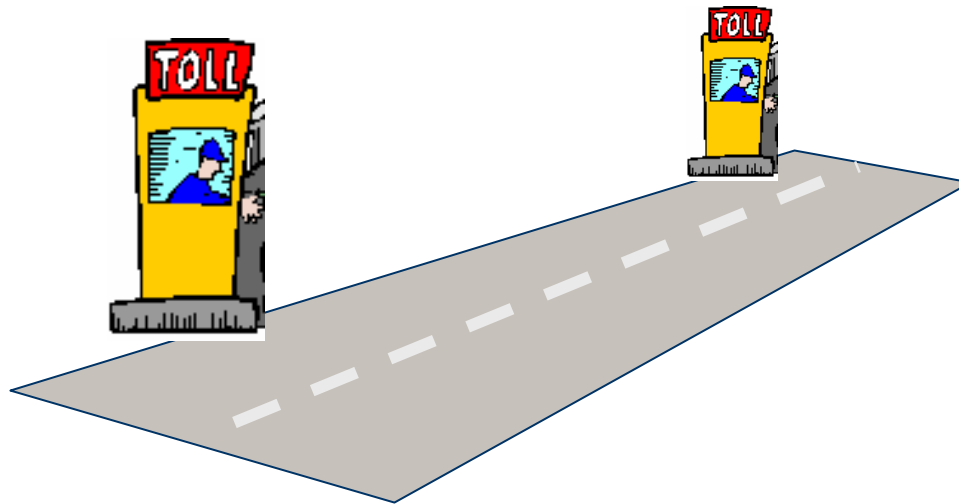


Rush Hour



Real Time Scheduling

- Only 1 "Pass"
- Cheat is possible
 (drive close to car with "Pass")



UNSAFE

Crossing Times



5



10



Pass



20



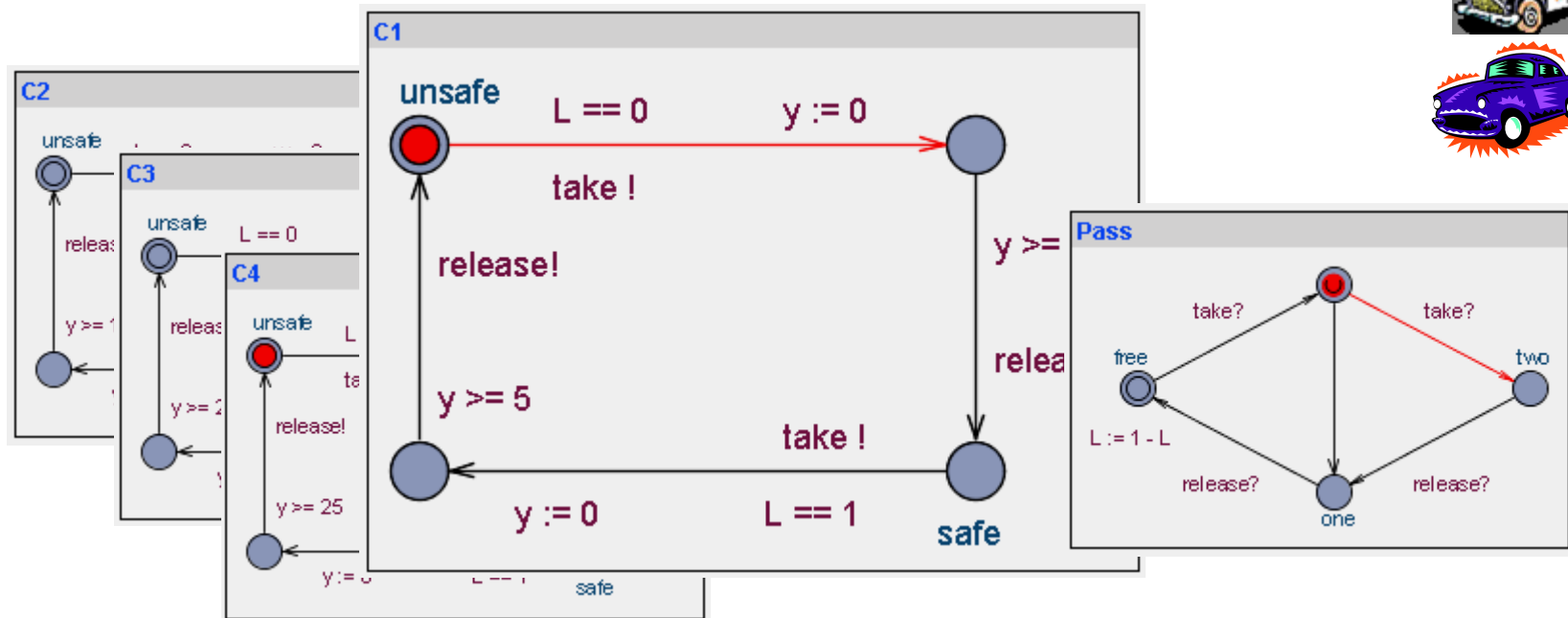
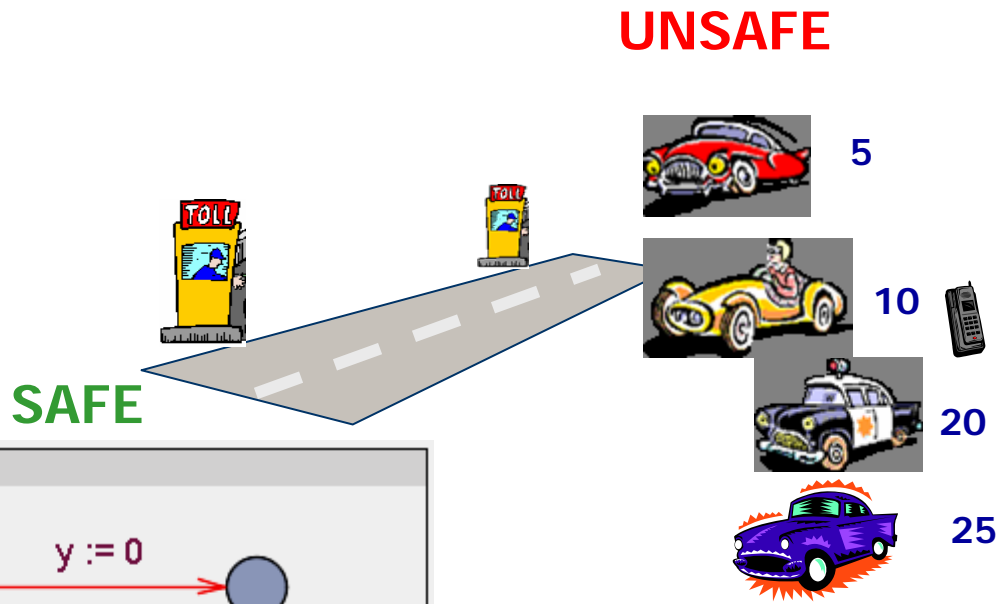
25

SAFE

CAN THEY MAKE IT TO SAFE WITHIN 70 MINUTES ???

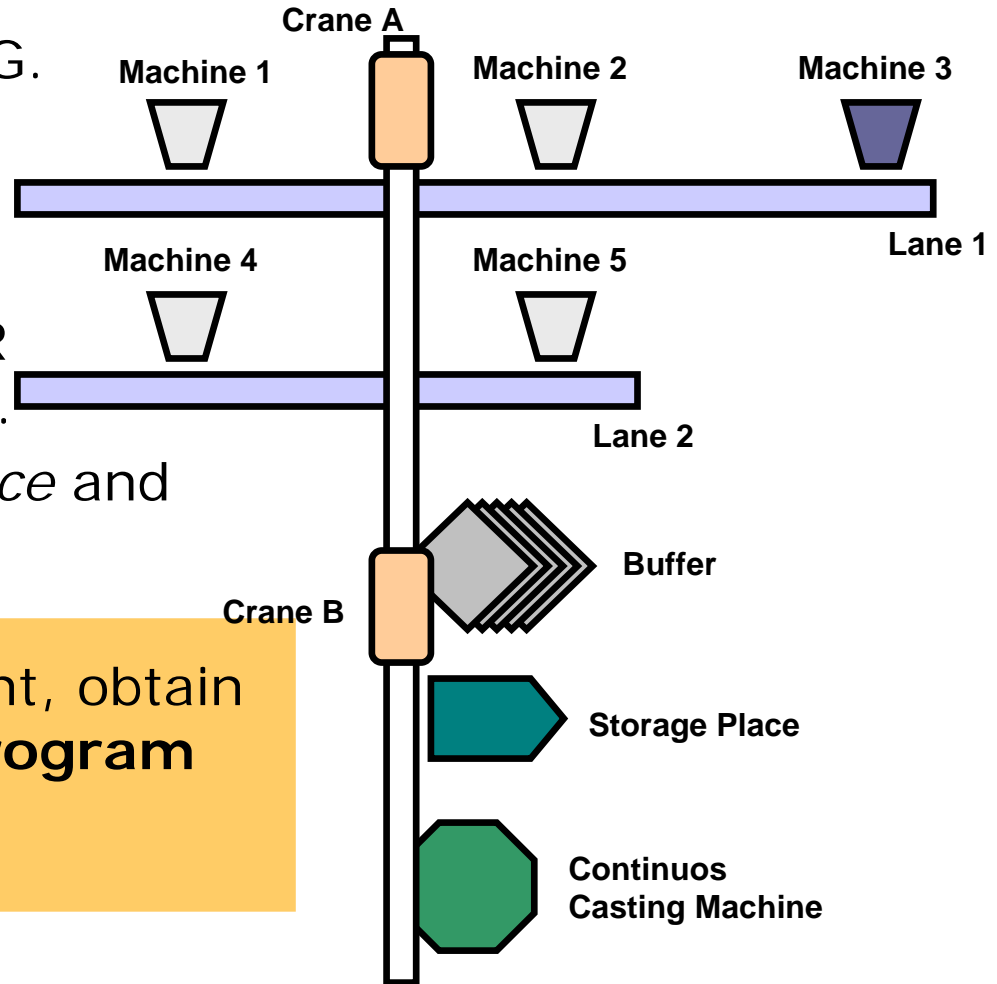
Real Time Scheduling

Solve Scheduling Problem using **UPPAAL**



Steel Production Plant

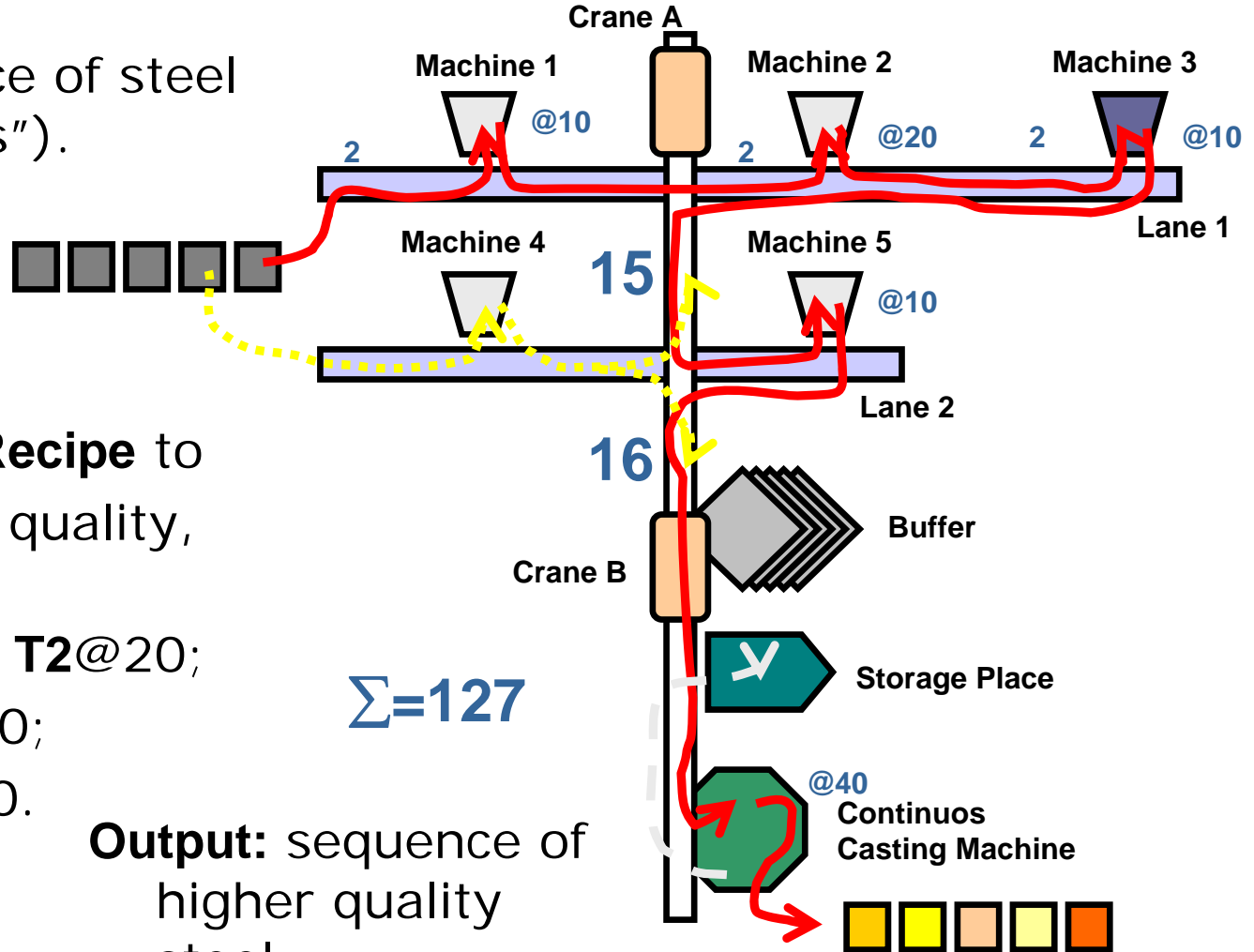
- A. Fehnker, T. Hune, K. G. Larsen, P. Pettersson
- Case study of Esprit-LTR project 26270 VHS
- Physical plant of SIDMAR located in Gent, Belgium.
- Part between *blast furnace* and *hot rolling mill*.



Objective: model the plant, obtain **schedule** and control **program** for plant.

Steel Production Plant

Input: sequence of steel loads ("pigs").



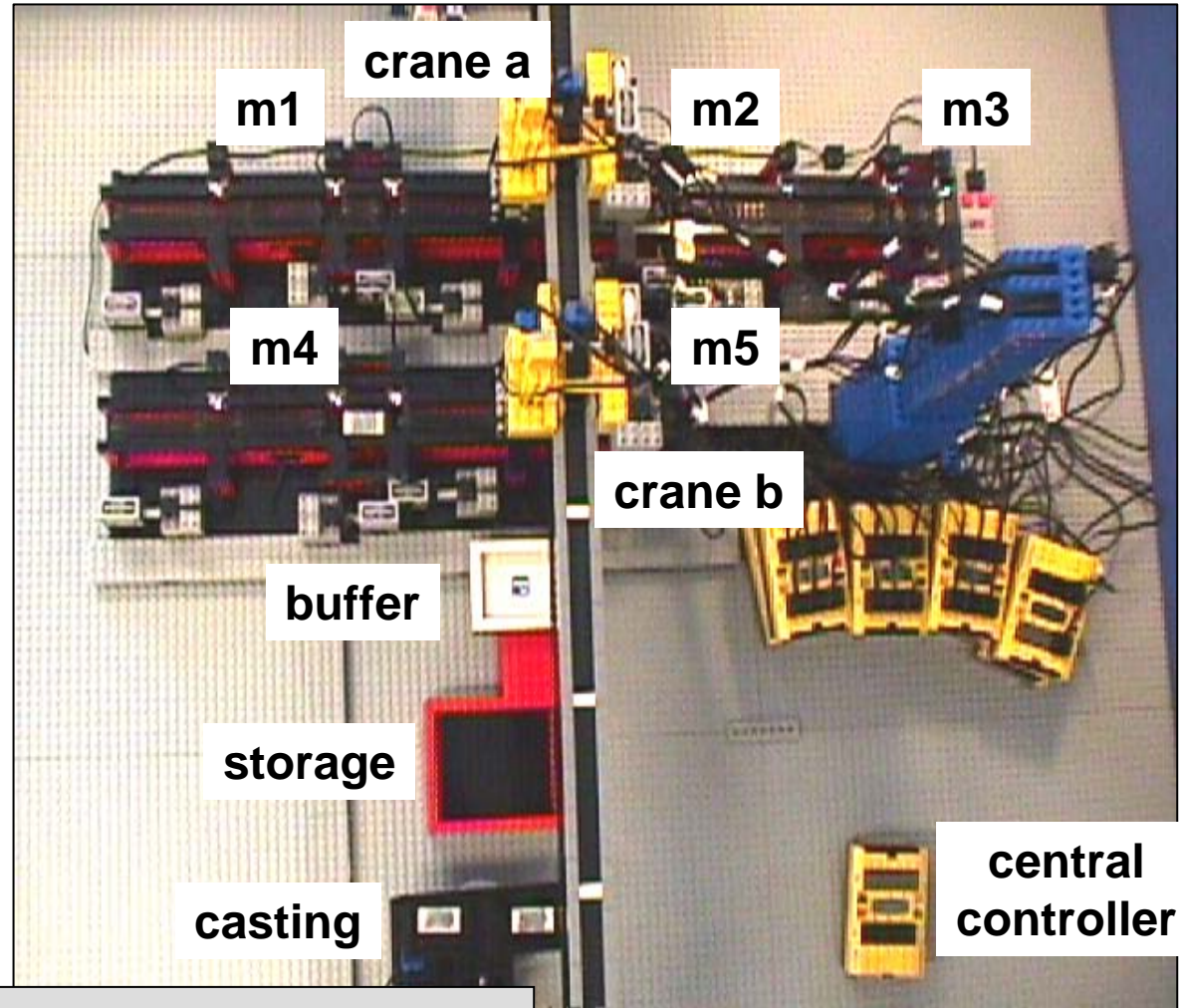
Load follows **Recipe** to obtain certain quality, e.g:

start; **T1@10**; **T2@20**;
T3@10; **T2@10**;
 end within 120.

Output: sequence of higher quality steel.

Controller Synthesis for LEGO Model

- LEGO RCX Mindstorms.
- Local controllers with control programs.
- IR protocol for remote invocation of programs.
- Central controller.



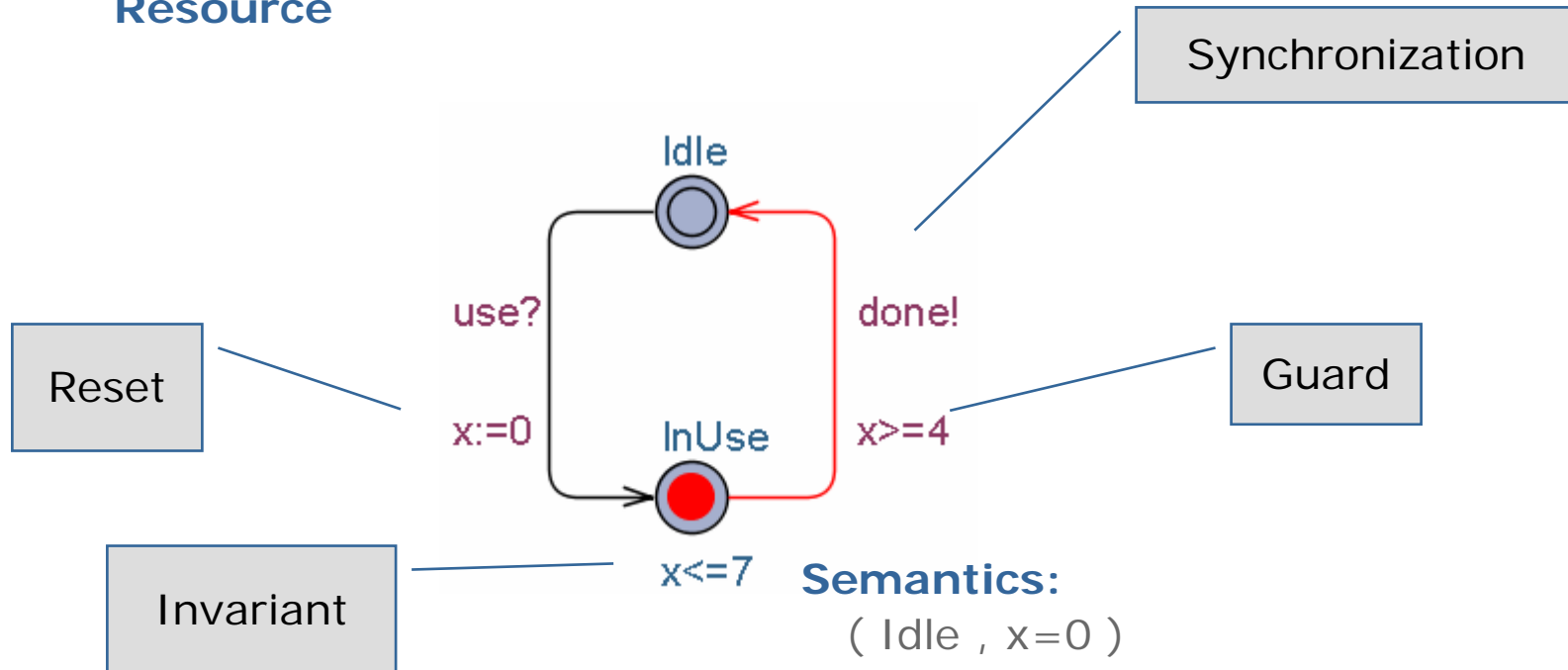
1971 lines of RCX code (n=5),
 24860 - “ - (n=60).

[Synthesis](#)

Timed Automata

[Alur & Dill'89]

Resource

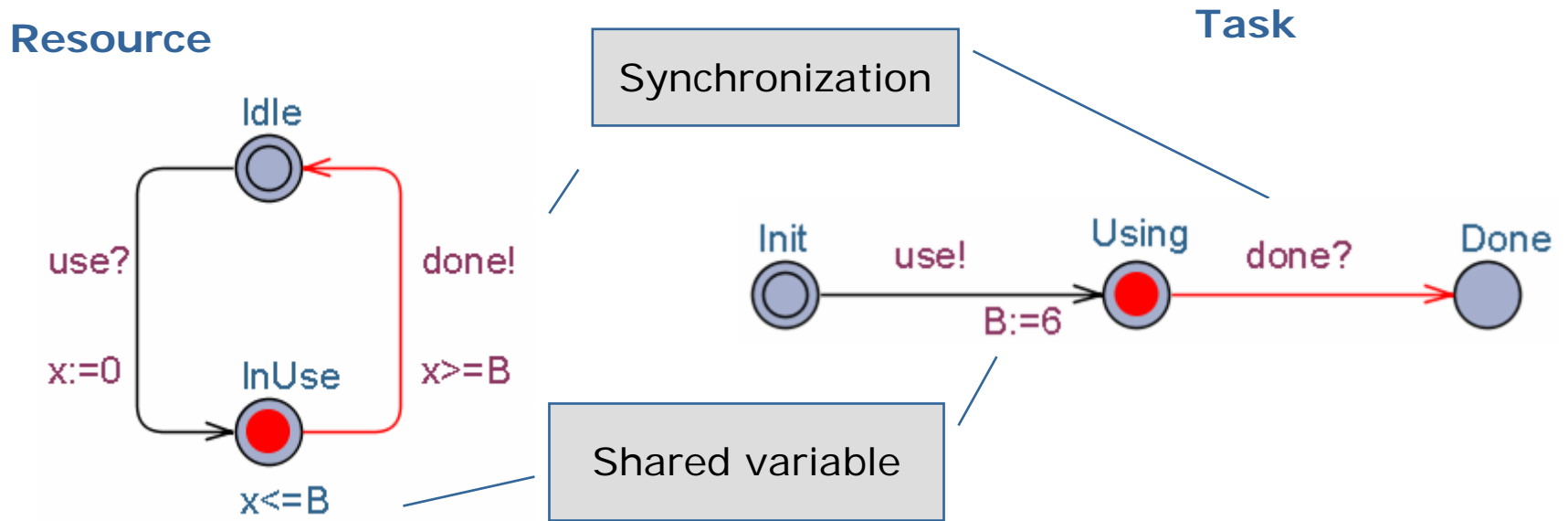


Semantics:

(Idle , x=0)

- (Idle , x=2.5) d(2.5)
- (InUse , x=0) use?
- (InUse , x=5) d(5)
- (Idle , x=5) **done!**
- (Idle , x=8) d(3)
- (InUse , x=0) use?

Composition



Semantics:

- (Idle , Init , B=0 , x=0)
- (Idle , Init , B=0 , x=3.1415) d(3)
- (InUse , Using , B=6 , x=0) use
- (InUse , Using , B=6 , x=6) d(6)
- (Idle , Done , B=6 , x=6) done

Jobshop Scheduling

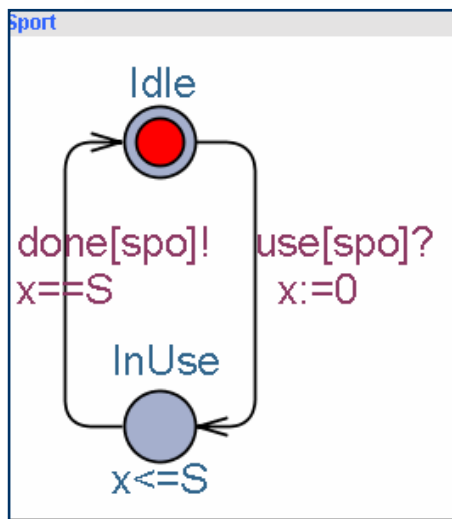
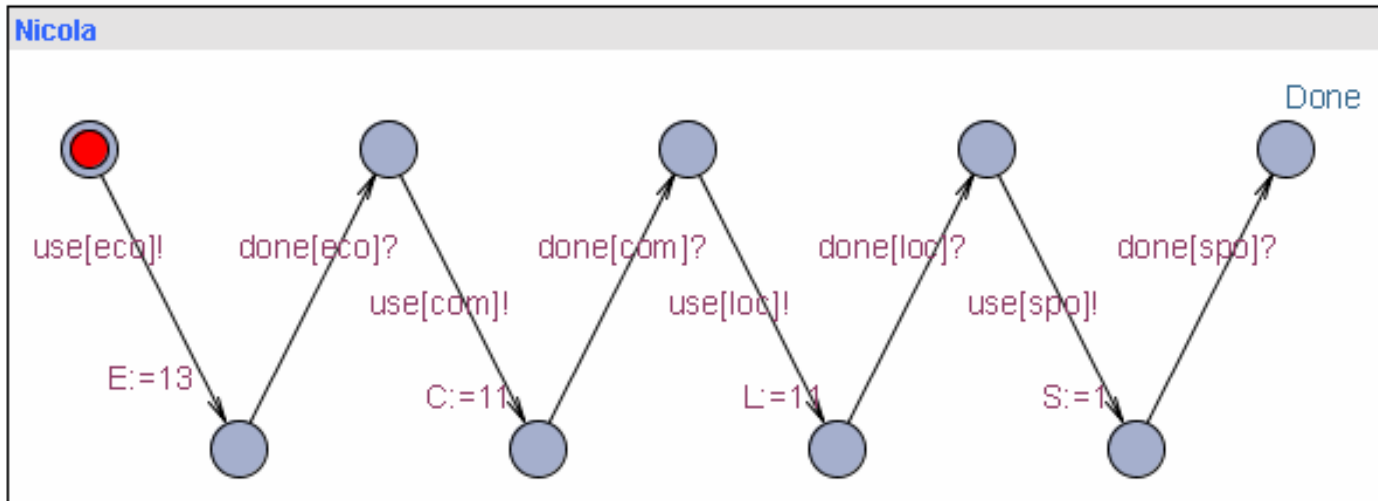
RESOURCES

JOB S

	Sport	Economy	Local News	Comic Strip
Kim	2. 5 min	4. 1 min	3. 3 min	1. 10 min
Maria	1. 10 min	2. 20 min	3. 1 min	4. 1 min
Nicola	4. 1 min	1. 13 min	3. 11 min	2. 11 min

Problem: compute the minimal **MAKESPAN**

Jobshop Scheduling in UPPAAL



	Sport	Economy	Local News	Comic Strip
Kim	2. 5 min	4. 1 min	3. 3 min	1. 10 min
Maria	1. 10 min	2. 20 min	3. 1 min	4. 1 min
Nicola	4. 1 min	1. 13 min	3. 11 min	2. 11 min

Experiments

[TACAS'2001]

B-&-B algorithm running for 60 sec.

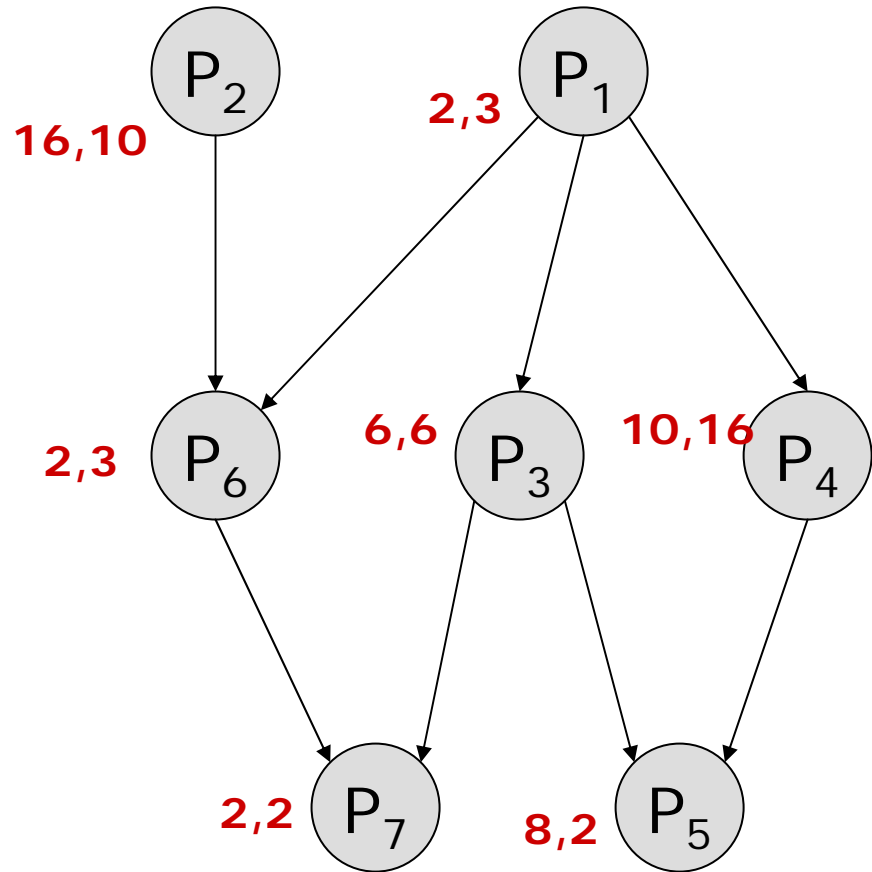
problem instance	BF		MC		MC+		DF		RDF		comb. heur.		minimal makespan
	cost	states	cost	states	cost	states	cost	states	cost	states	cost	states	
la01	-	-	-	-	-	-	2466	-	842	-	666	292	666
la02	-	-	-	-	-	-	2360	-	806	-	672	-	655
la03	-	-	-	-	-	-	2094	-	769	-	626	-	597
la04	-	-	-	-	-	-	2212	-	783	-	639	-	590
la05	-	-	-	-	593	9791	1955	-	696	-	593	284	593
la06	-	-	-	-	-	-	3656	-	1076	-	926	480	926
la07	-	-	-	-	-	-	3410	-	1113	-	890	-	890
la08	-	-	-	-	-	-	3520	-	1009	-	863	400	863
la09	-	-	-	-	-	-	3984	-	1154	-	951	425	951
la10	-	-	-	-	-	-	3681	-	1063	-	958	454	958
la11	-	-	-	-	-	-	4974	-	1303	-	1222	642	1222
la12	-	-	-	-	-	-	4557	-	1271	-	1039	633	1039
la13	-	-	-	-	-	-	4846	-	1227	-	1150	662	1150
la14	-	-	-	-	1292	10653	5145	-	1377	-	1292	688	1292
la15	-	-	-	-	-	-	5264	-	1459	-	1289	-	1207
la16	-	-	-	-	-	-	4849	-	1298	-	1022	-	945
la17	-	-	-	-	-	-	4299	-	938	-	786	-	784
la18	-	-	-	-	-	-	4763	-	1034	-	922	-	848
la19	-	-	-	-	-	-	4566	-	1140	-	904	-	842
la20	-	-	-	-	-	-	5056	-	1378	-	964	-	902
la21	-	-	-	-	-	-	7608	-	1326	-	1149	-	(1040,1053)
la22	-	-	-	-	-	-	6920	-	1413	-	1047	-	927
la23	Lawrence Job Shop Problems						7676	-	1357	-	1075	-	1032
la24							7237	-	1346	-	1061	-	935
la25	-	-	-	-	-	-	7141	-	1290	-	1070	-	977

m=5 { j=10 }
 m=10 { j=15 }
 m=10 { j=20 }
 m=10 { j=10 }
 m=10 { j=15 }

Task Graph Scheduling

Optimal Static Task Scheduling

- Task $\mathbf{P} = \{P_1, \dots, P_m\}$
- Machines $\mathbf{M} = \{M_1, \dots, M_n\}$
- Duration $\Delta : (\mathbf{P} \times \mathbf{M}) \rightarrow \mathbf{N}_\infty$
- $<$: p.o. on \mathbf{P} (pred.)



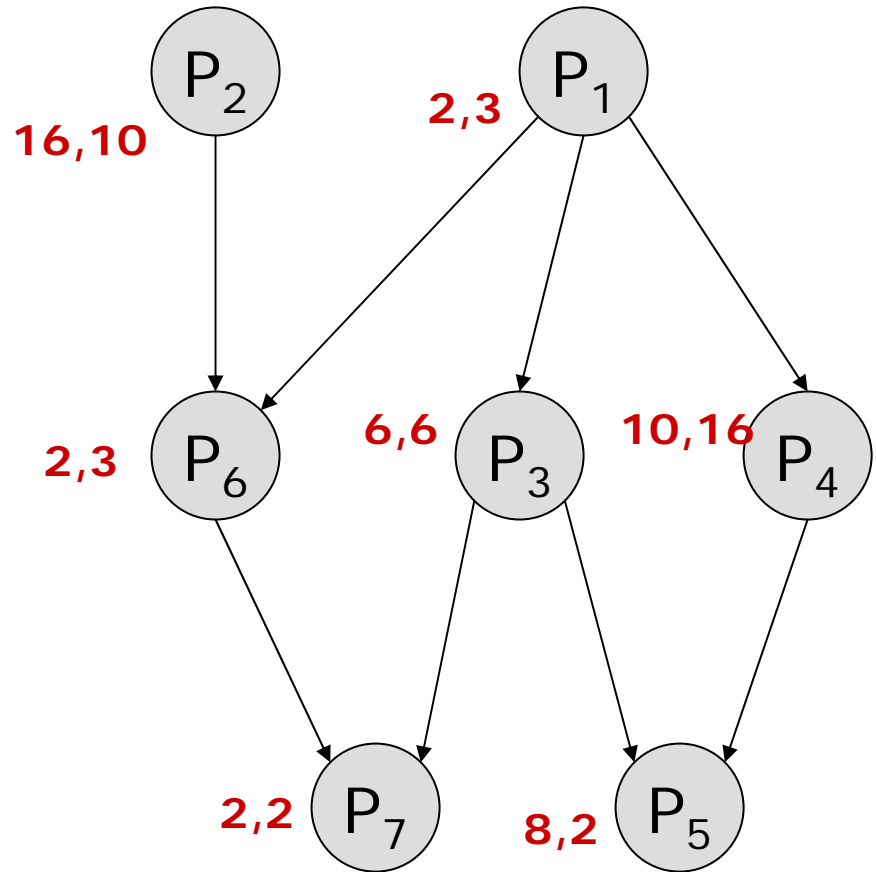
$\mathbf{M} = \{M_1, M_2\}$



Task Graph Scheduling

Optimal Static Task Scheduling

- Task $\mathbf{P} = \{P_1, \dots, P_m\}$
 - Machines $\mathbf{M} = \{M_1, \dots, M_n\}$
 - Duration $\Delta : (\mathbf{P} \times \mathbf{M}) \rightarrow \mathbf{N}_\infty$
 - $< : \text{p.o. on } \mathbf{P} \text{ (pred.)}$
-
- A task can be executed only if all predecessors have completed
 - Each machine can process at most one task at a time
 - Task cannot be preempted.



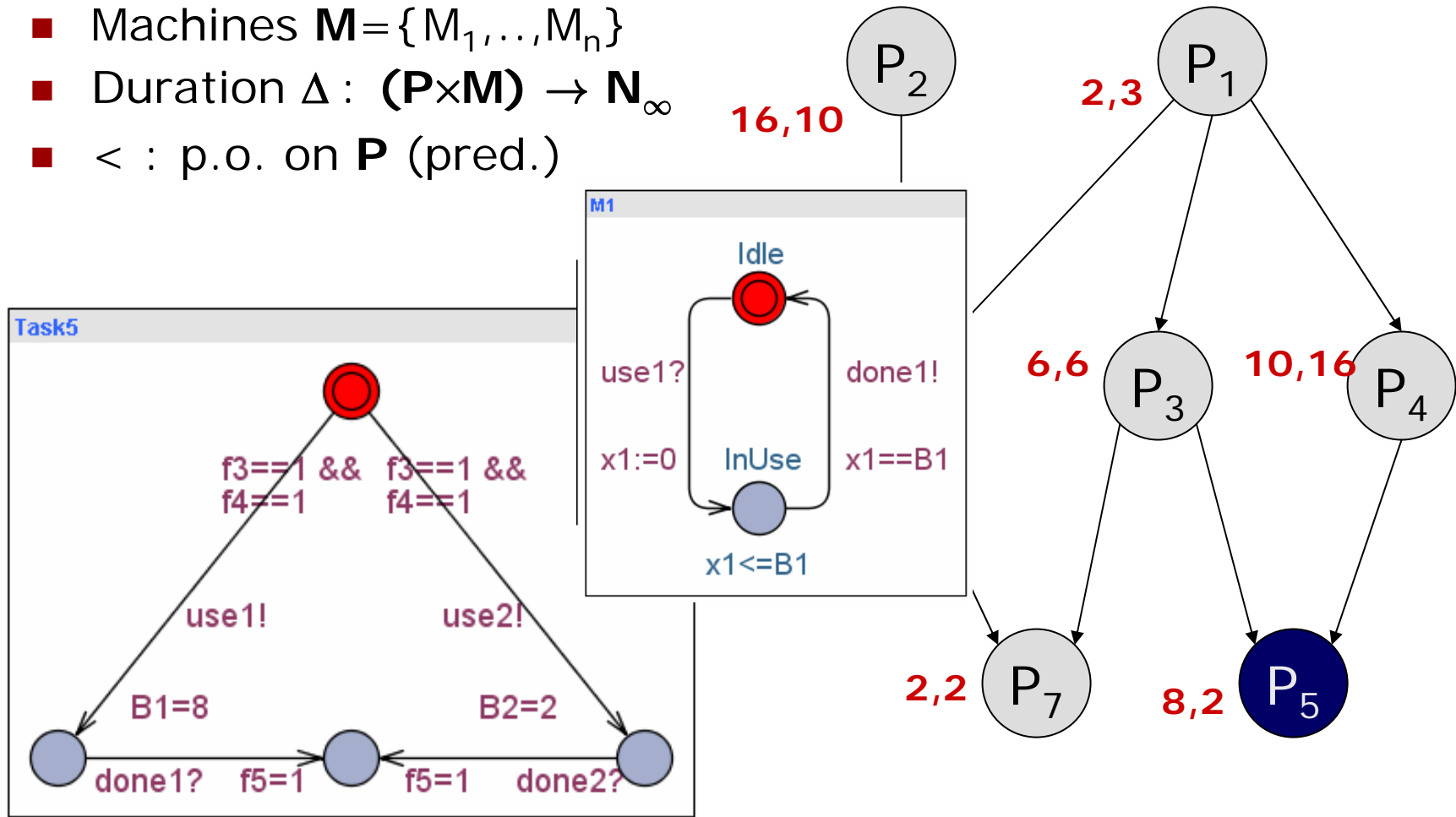
$$\mathbf{M} = \{M_1, M_2\}$$



Task Graph Scheduling

Optimal Static Task Scheduling

- Task $\mathbf{P} = \{P_1, \dots, P_m\}$
- Machines $\mathbf{M} = \{M_1, \dots, M_n\}$
- Duration $\Delta : (\mathbf{P} \times \mathbf{M}) \rightarrow \mathbf{N}_\infty$
- $<$: p.o. on \mathbf{P} (pred.)



$$\mathbf{M} = \{M_1, M_2\}$$

Experimental Results

name	#tasks	#chains	# machines	optimal	TA
001	437	125	4	1178	1182
000	452	43	20	537	537
018	730	175	10	700	704
074	1007	66	12	891	894
021	1145	88	20	605	612
228	1187	293	8	1570	1574
071	1193	124	20	629	634
271	1348	127	12	1163	1164
237	1566	152	12	1340	1342
231	1664	101	16	t.o.	1137
235	1782	218	16	t.o.	1150
233	1980	207	19	1118	1121
294	2014	141	17	1257	1261
295	2168	965	18	1318	1322
292	2333	318	3	8009	8009
298	2399	303	10	2471	2473

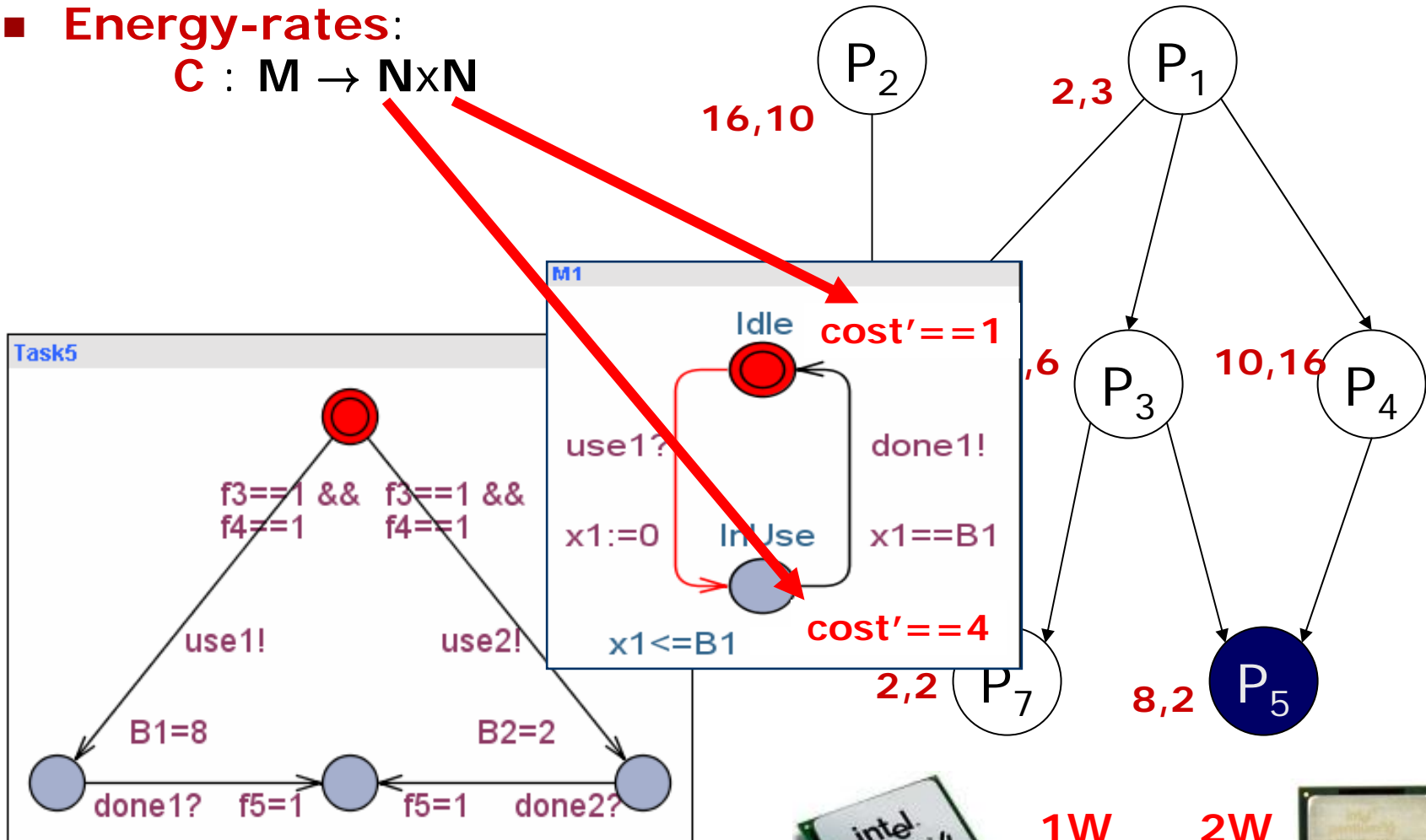
Abdeddaim, Kerbaa, Maler

Optimal Task Graph Scheduling

Power-Optimality

■ Energy-rates:

$$C : M \rightarrow N \times N$$



1W
4W

2W
3W



Priced Timed Automata

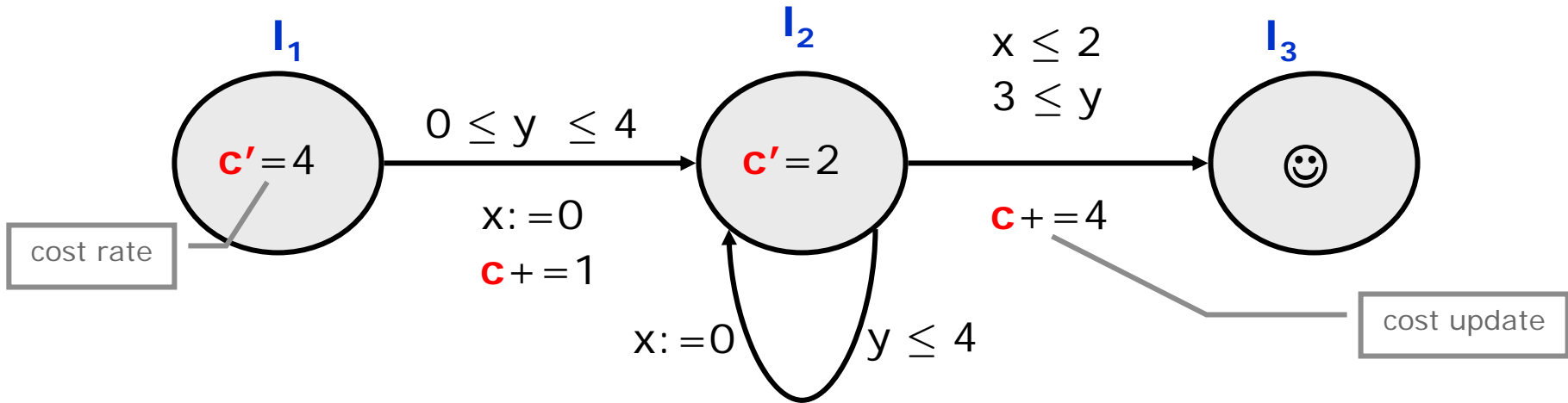
Optimal Scheduling

Priced Timed Automata

Timed Automata + **COST** variable

Behrmann, Fehnker, et al (HSCC'01)

Alur, Torre, Pappas (HSCC'01)

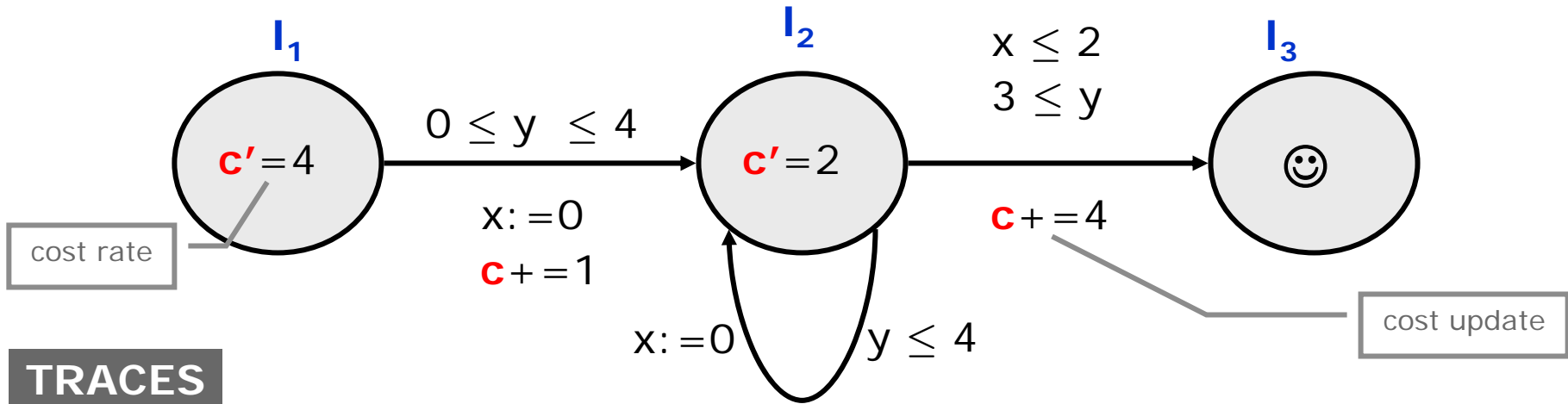


Priced Timed Automata

Timed Automata + **COST** variable

Behrmann, Fehnker, et al (HSCC'01)

Alur, Torre, Pappas (HSCC'01)



TRACES

$$(l_1, x=y=0) \xrightarrow[12]{\varepsilon(3)} (l_1, x=y=3) \xrightarrow[1]{} (l_2, x=0, y=3) \xrightarrow[4]{} (l_3, \dots)$$

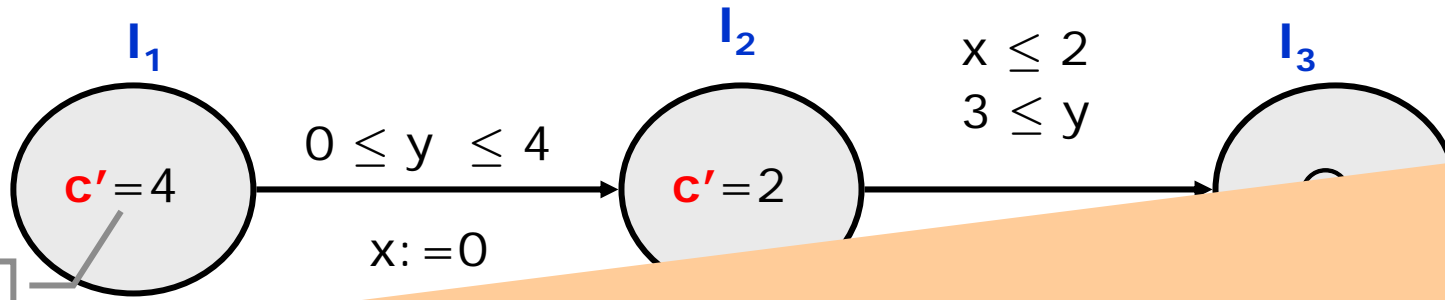
$\Sigma c = 17$

Priced Timed Automata

Timed Automata + **COST** variable

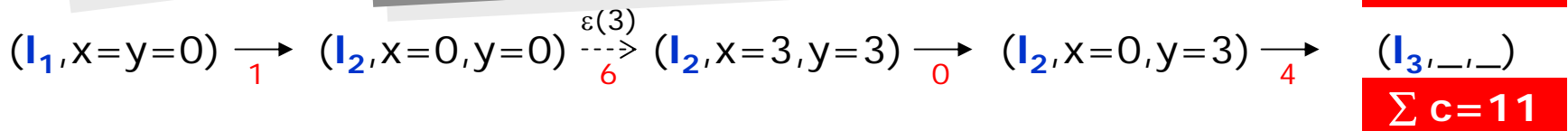
Behrmann, Fehnker, et al (HSCC'01)

Alur, Torre, Pappas (HSCC'01)



Problem :
Find the **minimum** cost of reaching location l_3

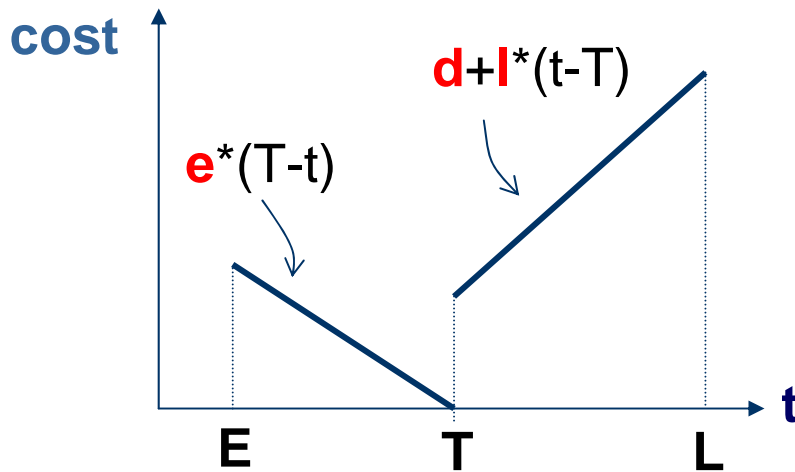
Efficient Implementation:
CAV'01 and TACAS'04



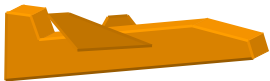
$(l_3, _, _)$
 $\Sigma c = 16$

$(l_3, _, _)$
 $\Sigma c = 11$

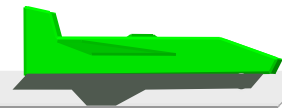
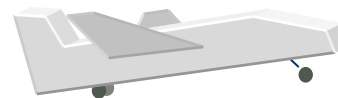
Aircraft Landing Problem



- E** earliest landing time
- T** target time
- L** latest time
- e** cost rate for being early
- l** cost rate for being late
- d** fixed cost for being late

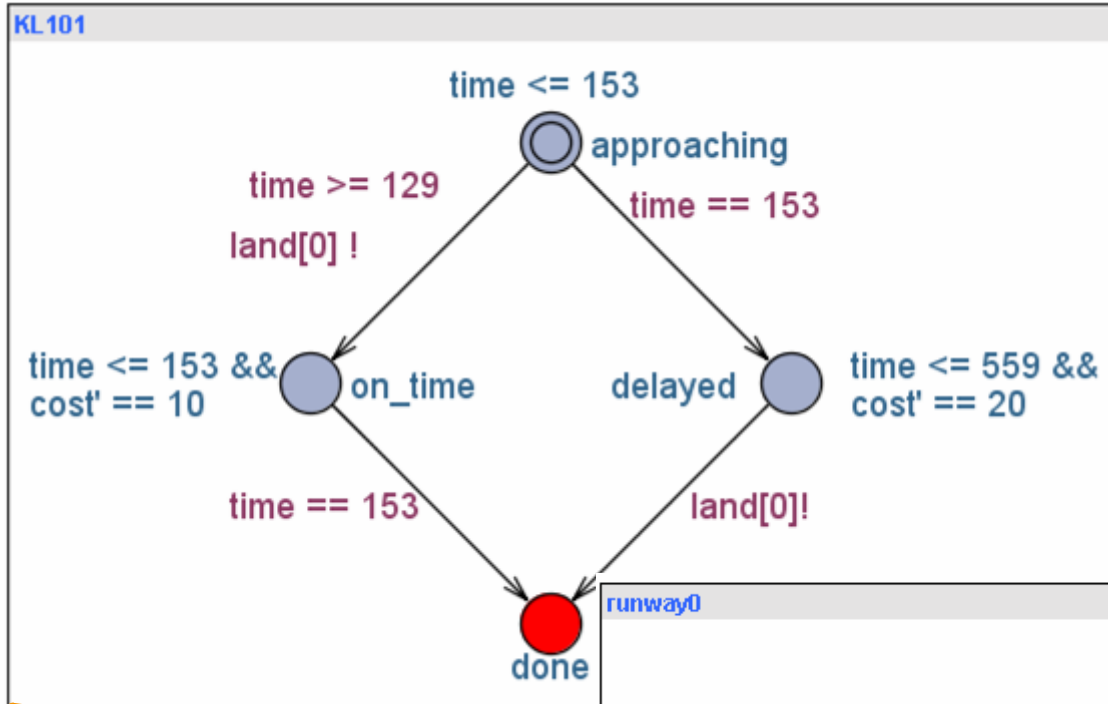


Planes have to keep separation distance to avoid turbulences caused by preceding planes



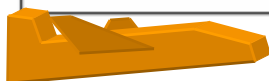
Runway

Modeling ALP with PTA

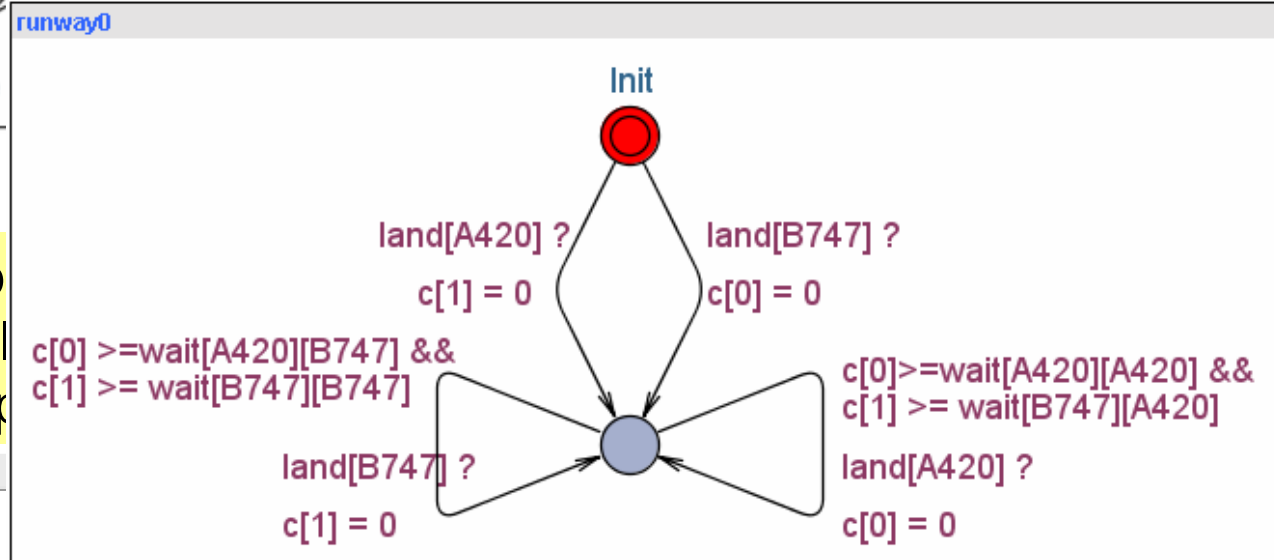


- 129**: Earliest landing time
- 153**: Target landing time
- 559**: Latest landing time
- 10**: Cost rate for early
- 20**: Cost rate for late

Runway handles 2 types of planes



Planes have to keep sep distance to avoid turbul caused by preceding p



Symbolic "A*"

Zones

Definition

A *zone* Z over a set of clocks C is a finite conjunction of simple constraints of the forms:

$$x \geq l \quad x \leq u \quad x - y \geq l' \quad x - y \leq u'$$

where $x, y \in C$, $l, u \in \mathbb{N}$ and $l', u' \in \mathbb{Z}$.

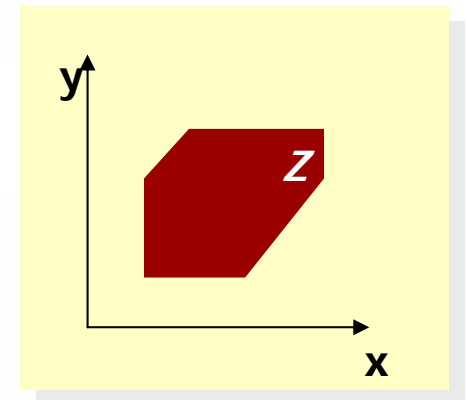
For $u \in \mathbb{R}^C$ and Z a zone we write $u \models Z$ if u satisfies all constraints of Z .

Operations

Reset: $\{x\}Z = \{u[0/x] \mid u \models Z\}$

Delay: $Z^\dagger = \{u + d \mid u \models Z\}$

Offset: $\Delta_Z \models Z$ such that $\forall u \models Z. \forall x \in C. \Delta_Z(x) \leq u(x)$.



Priced Zone

Definition

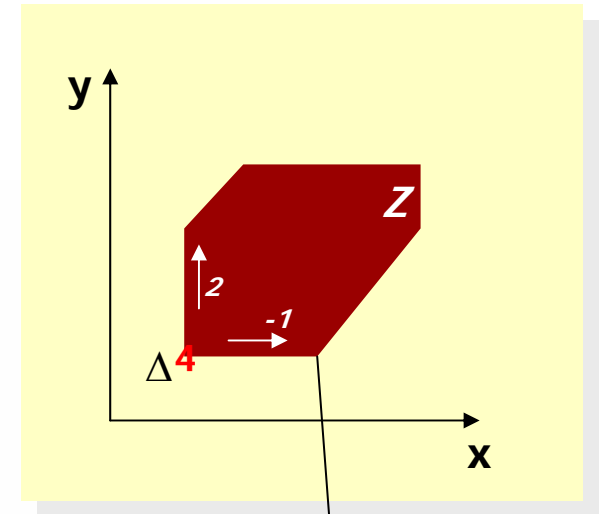
A priced zone P is a tuple (Z, c, r) , where:

- Z is a zone
- $c \in \mathbb{N}$ describes the cost of Δ_Z
- $r : C \rightarrow \mathbb{Z}$ gives a *rate* for any clock $x \in C$.

We write $u \models P$ whenever $u \models Z$. For $u \models P$ we define $\text{Cost}(u, P)$ as follows:

$$\text{Cost}(u, P) = c + \sum_{x \in C} r(x) \cdot (u(x) - \Delta_Z(x))$$

$$\text{Cost}(x, y) = 2y - x + 2$$



Branch & Bound Algorithm

Cost := ∞

Passed := \emptyset

Waiting := $\{(l_0, Z_0)\}$

while Waiting $\neq \emptyset$ **do**

select (l, Z) from Waiting

if $l = l_g$ and $\text{minCost}(Z) < \text{Cost}$ **then**

 Cost := $\text{minCost}(Z)$

if $\text{minCost}(Z) + \text{Rem}_{(l,Z)} \geq \text{Cost}$ **then** break

if for all (l, Z') in Passed: $Z' \not\subseteq Z$ **then**

add (l, Z) to Passed

add all (l', Z') with $(l, Z) \rightarrow (l', Z')$ to Waiting

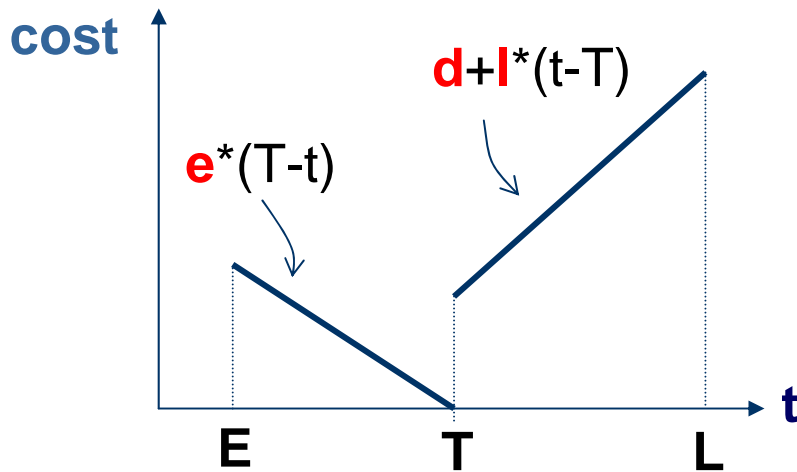
return Cost

Experimental Results

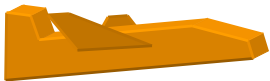
Experiments *MC Order*

COST-rates				SCHEDULE	COST	TIME	#Expl	#Pop'd
G ₅	C ₁	B ₂	D ₂					
0	0	0	5					
Min Time				CG> G< BD> C< CG>	/	60	1762 1538	2638
1	1	1	1	CG> G< BG> G< GD>	55	65	252	378
9	2	3	10	GD> G< CG> G< BG>	195	65	149	233
1	2	3	4	CG> G< BD> C< CG>	140	60	232	350
1	2	3	10	CD> C< CB> C< CG>	170	65	263	408
1	20	30	40	BD> B< CB> C< CG>	975 1085	85 time<85	-	-
0	0	0	0	-	0	-	406	447

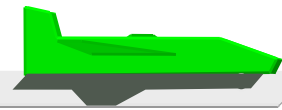
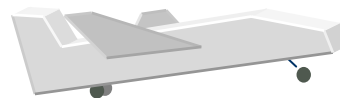
Example: Aircraft Landing



- E** earliest landing time
- T** target time
- L** latest time
- e** cost rate for being early
- l** cost rate for being late
- d** fixed cost for being late

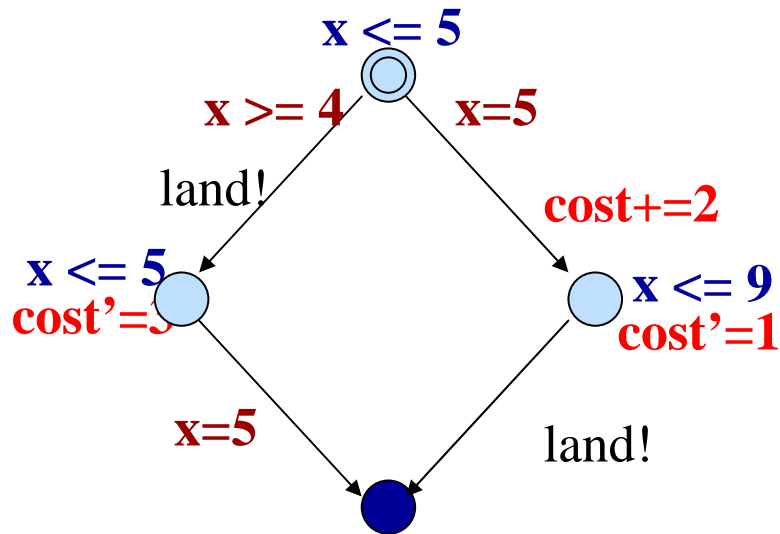


Planes have to keep separation distance to avoid turbulences caused by preceding planes

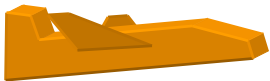


Runway

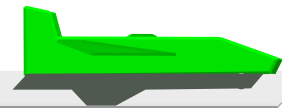
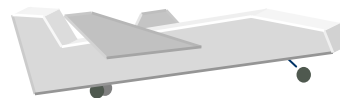
Example: Aircraft Landing



- 4** earliest landing time
- 5** target time
- 9** latest time
- 3** cost rate for being early
- 1** cost rate for being late
- 2** fixed cost for being late



Planes have to keep separation distance to avoid turbulences caused by preceding planes



Runway

Aircraft Landing

CAV'01

Source of examples:
Baesley et al'2000

	problem instance	1	2	3	4	5	6	7
	number of planes	10	15	20	20	20	30	44
	number of types	2	2	2	2	2	4	2
1	optimal value	700	1480	820	2520	3100	24442	1550
	explored states	481	2149	920	5693	15069	122	662
	cputime (secs)	4.19	25.30	11.05	87.67	220.22	0.60	4.27
2	optimal value	90	210	60	640	650	554	0
	explored states	1218	1797	669	28821	47993	9035	92
	cputime (secs)	17.87	39.92	11.02	755.84	1085.08	123.72	1.06
3	optimal value	0	0	0	130	170	0	
	explored states	24	46	84	207715	189602	62	N/A
	cputime (secs)	0.36	0.70	1.71	14786.19	12461.47	0.68	
4	optimal value				0	0		
	explored states	N/A	N/A	N/A	65	64	N/A	N/A
	cputime (secs)				1.97	1.53		

Branch & Bound Algorithm

Zone based Linear Programming Problems

Cost := ∞

Passed := \emptyset

Waiting := $\{(l_0, Z_0)\}$

while Waiting $\neq \emptyset$ **do**

select (l, Z) from Waiting

if $l = l_g$ and $\text{minCost}(Z) < \text{Cost}$ **then**

 Cost := $\text{minCost}(Z)$

if $\text{minCost}(Z) + \text{Rem}_{(l, Z)} \geq \text{Cost}$ **then** break

if for all (l', Z') in Passed: $Z' \not\subseteq Z$ **then**

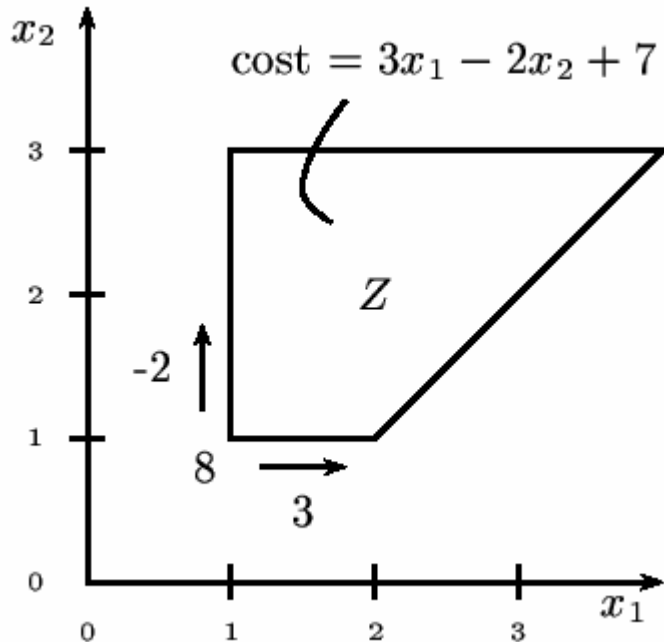
add (l, Z) to Passed

add all (l', Z') with $(l, Z) \rightarrow (l', Z')$ to Waiting

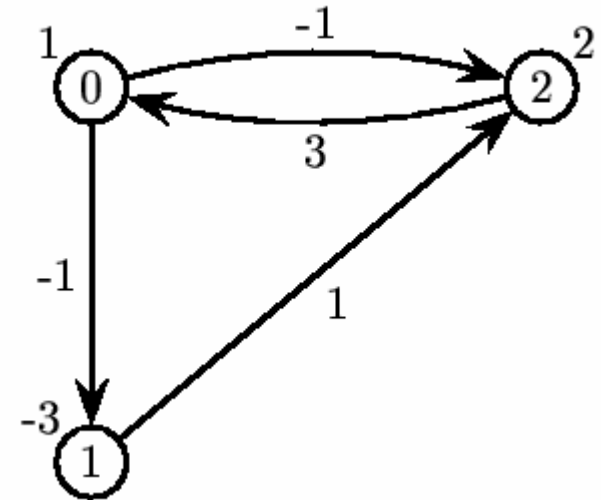
return Cost

Zone LP \rightarrow Min Cost Flow

Exploiting duality



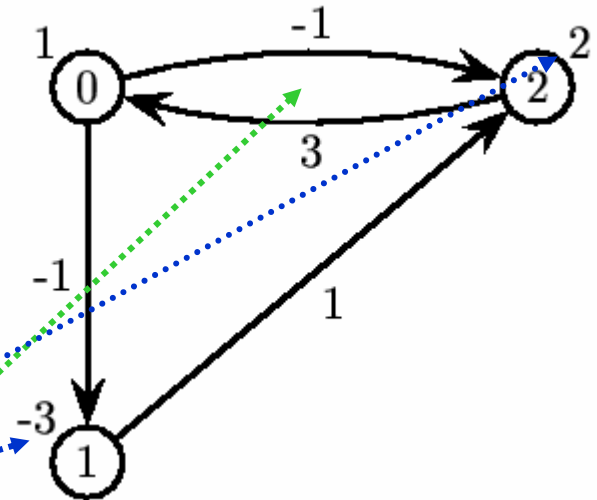
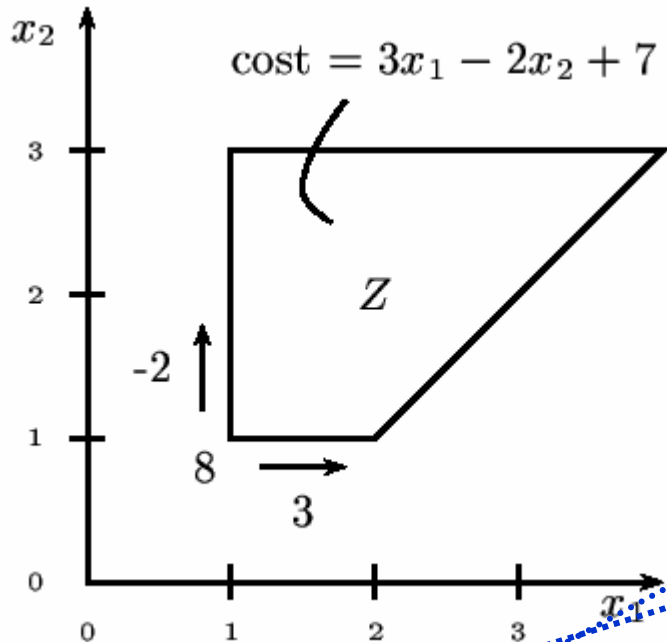
minimize $3x_1 - 2x_2 + 7$
 when $x_1 - x_2 \leq 1$
 $1 \leq x_2 \leq 3$
 $x_2 \geq 1$



minimize $3y_{2,0} - y_{0,2} + y_{1,2} - y_{0,1}$
 when $y_{2,0} - y_{0,1} - y_{0,2} = 1$
 $y_{0,2} + y_{1,2} = 2$
 $y_{0,1} - y_{1,2} = -3$

Zone LP \rightarrow Min Cost Flow

Exploiting duality



minimize $3x_1 - 2x_2 + 7$
 when $x_1 - x_2 \leq 1$
 $1 \leq x_2 \leq 3$
 $x_2 \geq 1$

minimize $3y_{2,0} - y_{0,2} + y_{1,2} - y_{0,1}$
 when $y_{2,0} - y_{0,1} - y_{0,2} = 1$
 $y_{0,2} + y_{1,2} = 2$
 $y_{0,1} - y_{1,2} = -3$

Aircraft Landing

Using MCF/Netsimplex

Rasmussen, Larsen, Subramani
TACAS04

RW	Planes	10	15	20	20	20	30	44
	Types	2	2	2	2	2	4	2
1	simplex	0.844s	5.210s	2.135s	17.888s	44.878s	0.451s	0.670s
	netsimplex	0.156s	0.657s	0.369s	2.363s	5.503s	0.127s	0.322s
factor		5.41	7.93	5.79	7.57	8.16	3.55	2.08
2	simplex	2.577s	7.436s	2.175s	94.357s	120.004s	2.322s	0.264s
	netsimplex	0.332s	1.036s	0.436s	13.376s	18.033s	0.600s	0.179s
factor		8.00	7.18	4.99	7.054	6.65	3.87	1.474
3	simplex	0.120s	0.181s	0.357s	740.100s	516.678s	0.166s	N/A
	netsimplex	0.064s	0.104s	0.129s	170.176s	124.805s	0.079s	N/A
factor		1.87	1.74	2.77	4.34	4.14	2.10	
4	simplex	N/A	N/A	N/A	1.603s	0.318s	N/A	N/A
	netsimplex	N/A	N/A	N/A	0.378s	0.093s	N/A	N/A
factor					4.24	3.42		

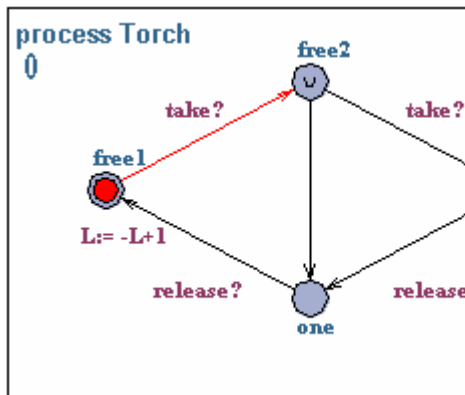
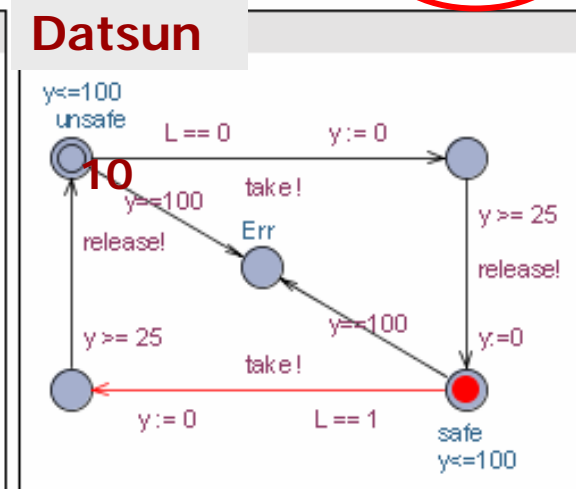
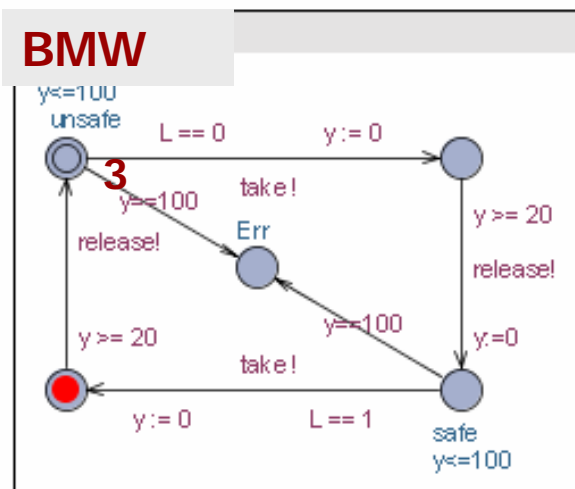
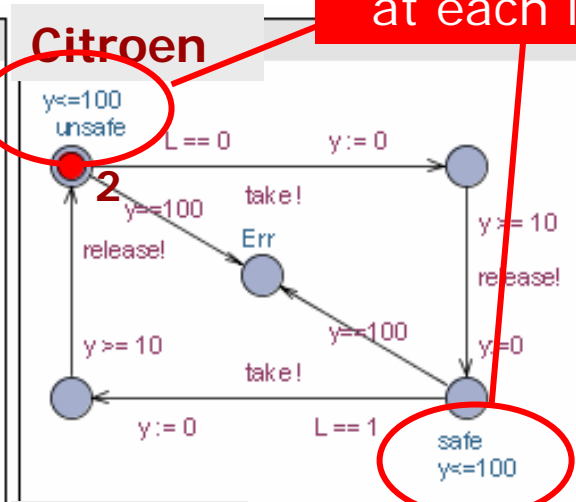
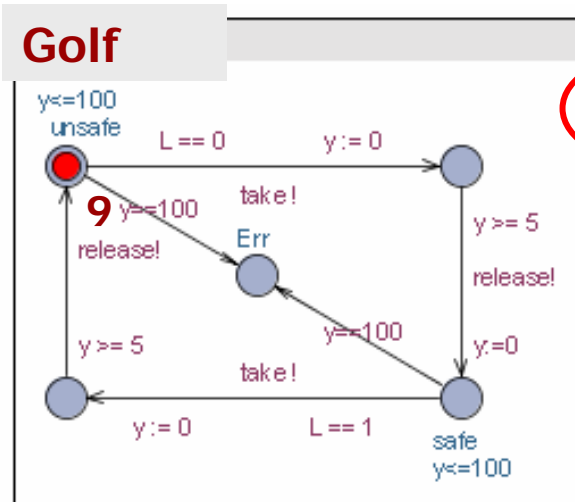
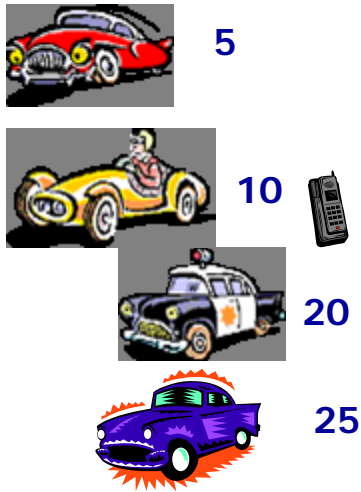
A. Loebel (2000). MCF Version 1.2 - A network simplex implementation. (<http://www.zib.de>)

Optimal Infinite Scheduling

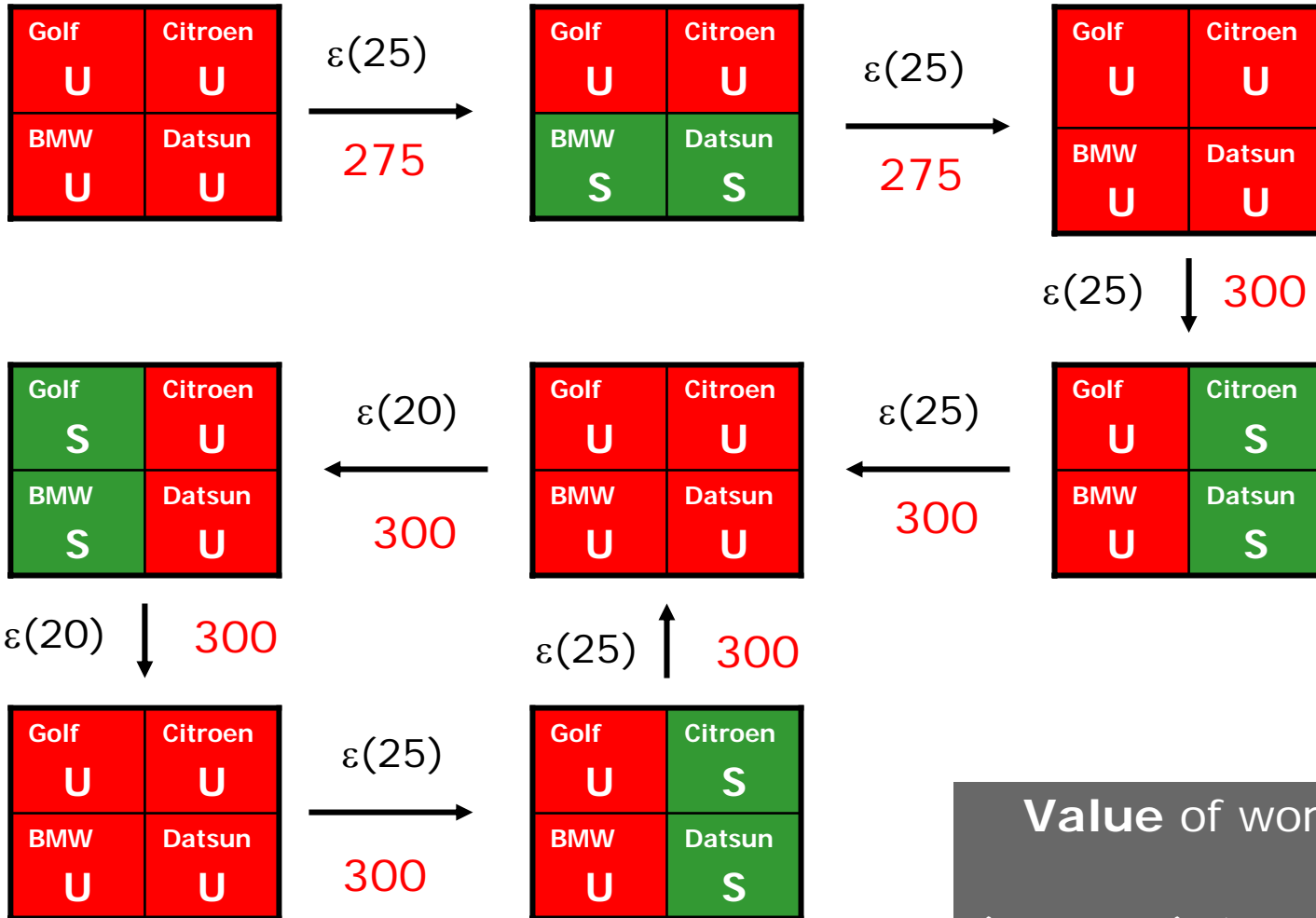
**with Ed Brinksma
Patricia Bouyer
Arne Skou**

EXAMPLE: Optimal WORK plan for cars with different subscription rates for city driving !

maximal 100 min. at each location



Workplan I



Value of workplan:
 $(4 \times 300) / 90 = 13.33$

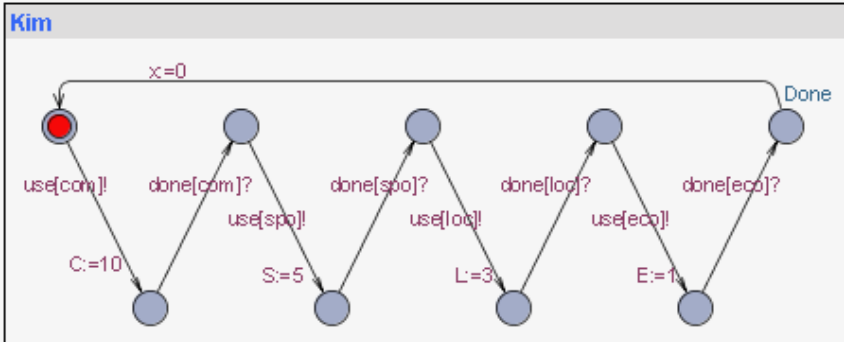
Workplan II



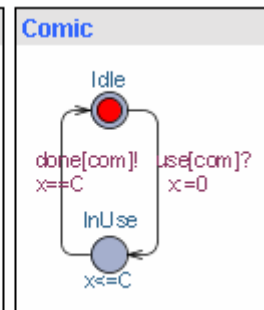
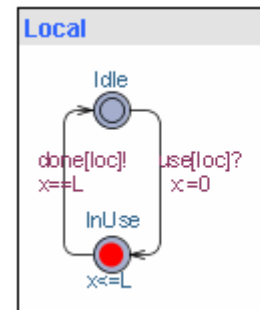
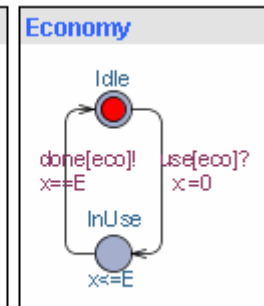
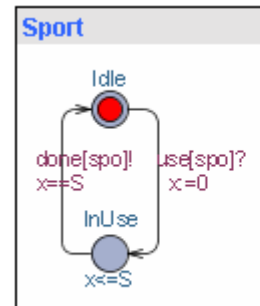
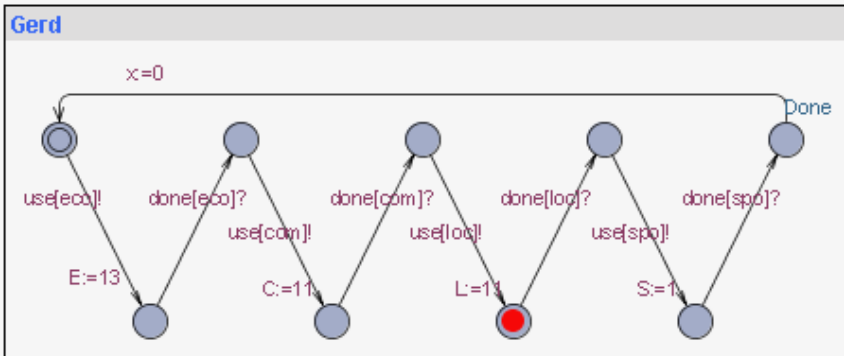
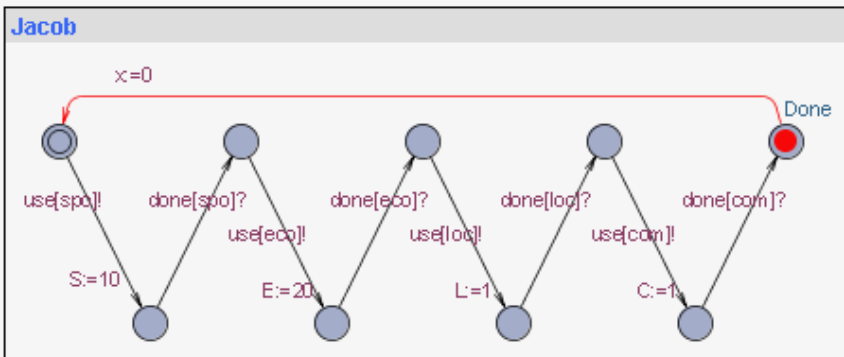
Value of workplan:
 $560 / 100 = 5.6$



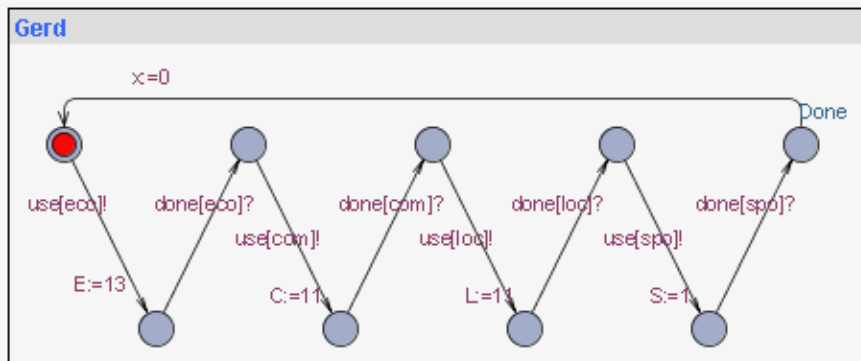
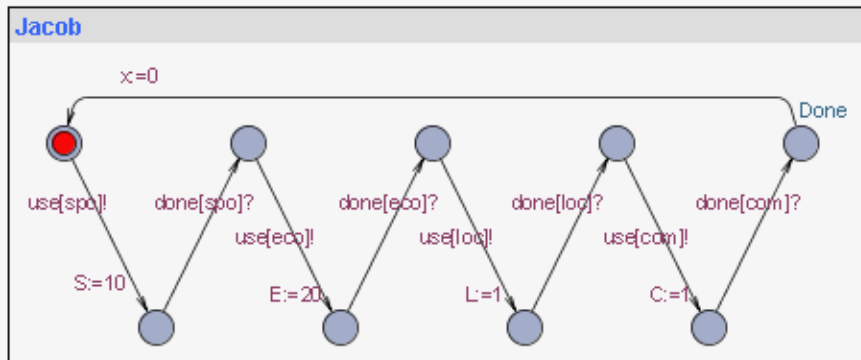
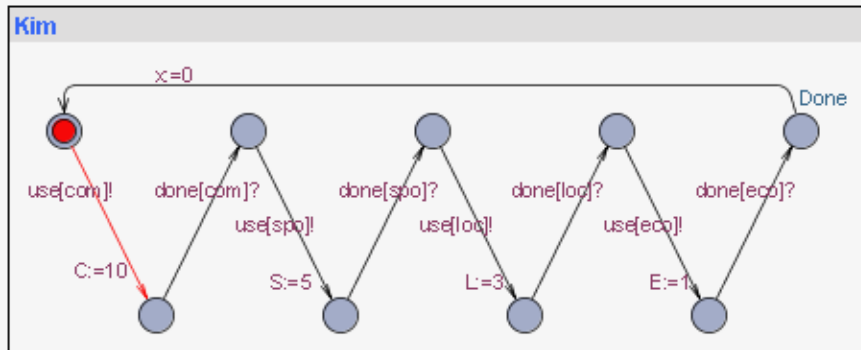
Infinite Scheduling



$E[]$ (Kim. $x \leq 100$ and
 Jacob. $x \leq 90$ and
 Gerd. $x \leq 100$)

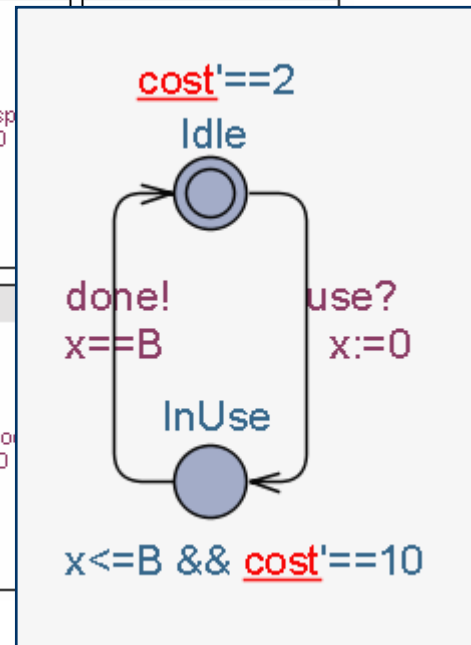
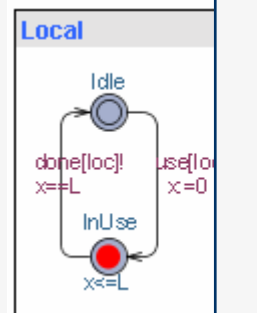
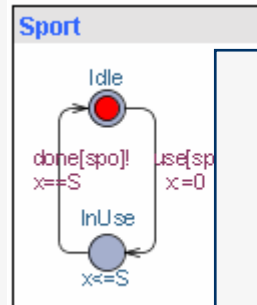


Infinite Optimal Scheduling

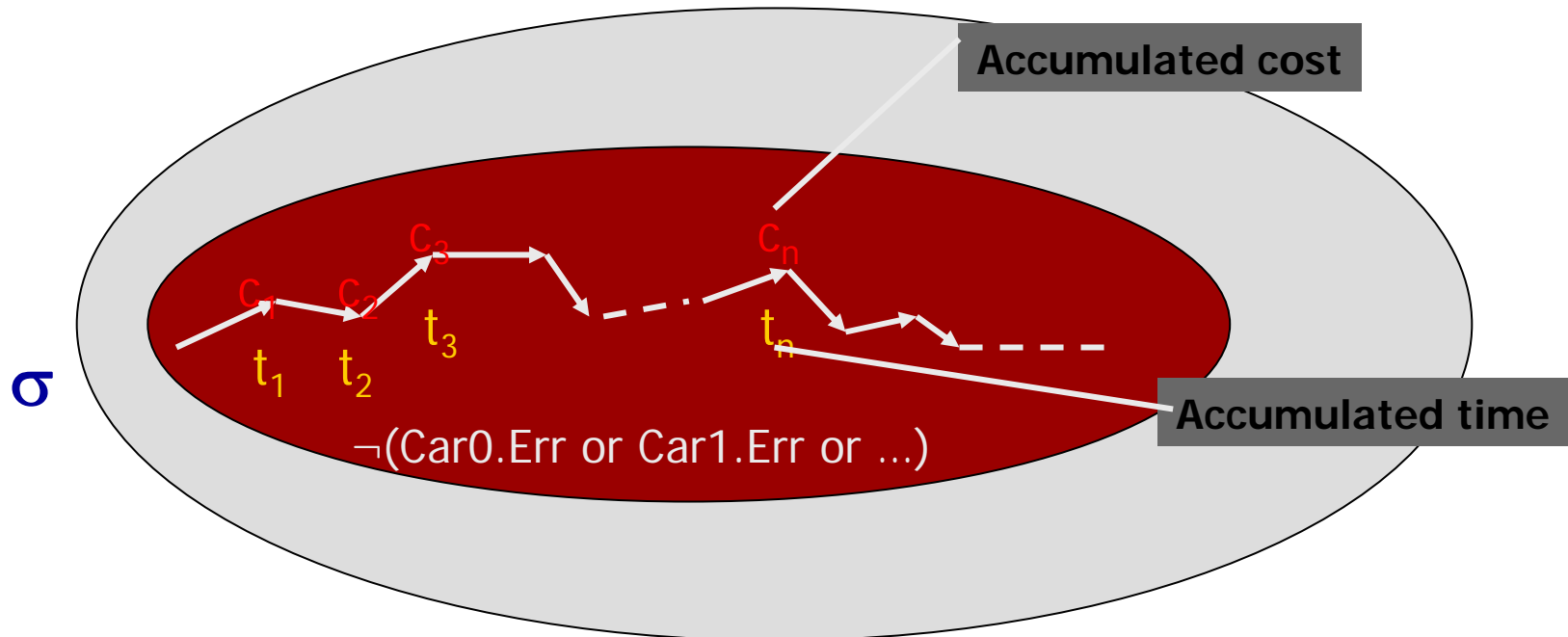


$E[]$ (Kim. $x \leq 100$ and
Jacob. $x \leq 90$ and
Gerd. $x \leq 100$)

**+ most minimal limit
of **cost/time****



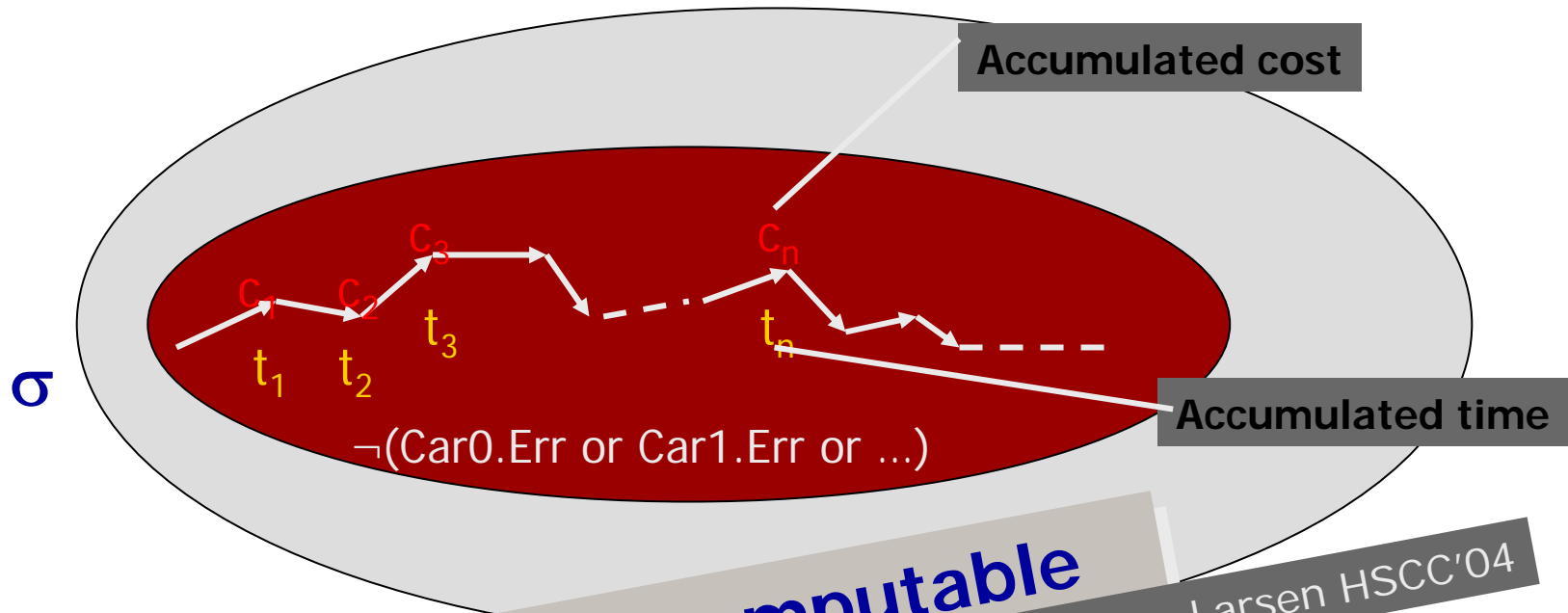
Cost Optimal Scheduling = *Optimal Infinite Path*



Value of path σ : $\text{val}(\sigma) = \lim_{n \rightarrow \infty} c_n / t_n$

Optimal Schedule σ^* : $\text{val}(\sigma^*) = \inf_{\sigma} \text{val}(\sigma)$

Cost Optimal Scheduling = *Optimal Infinite Path*



THEOREM: σ^* is computable

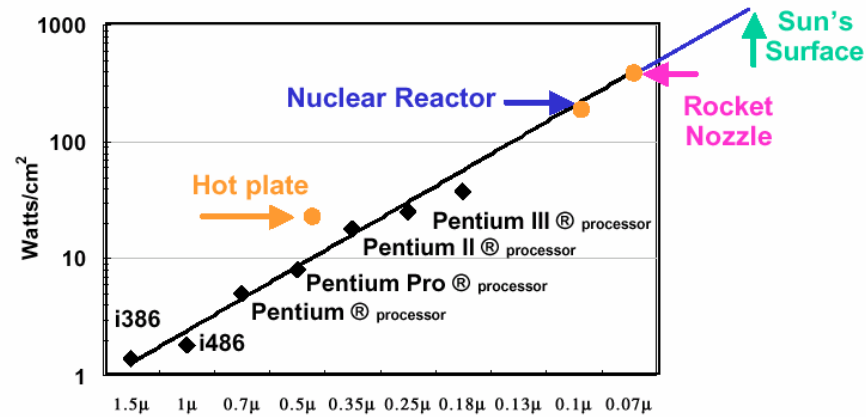
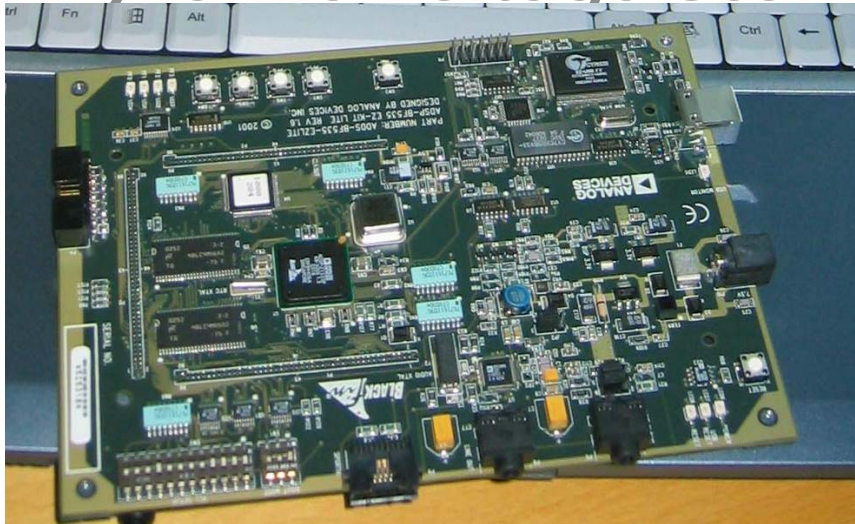
Bouyer, Brinksma, Larsen HSCC'04

For path σ : $\text{val}(\sigma) = \lim_{n \rightarrow \infty} C_n / t_n$

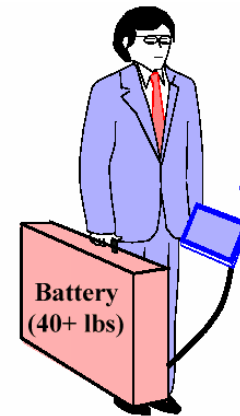
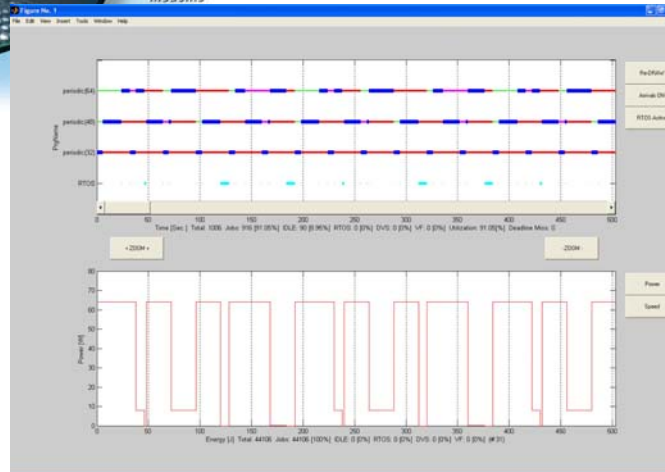
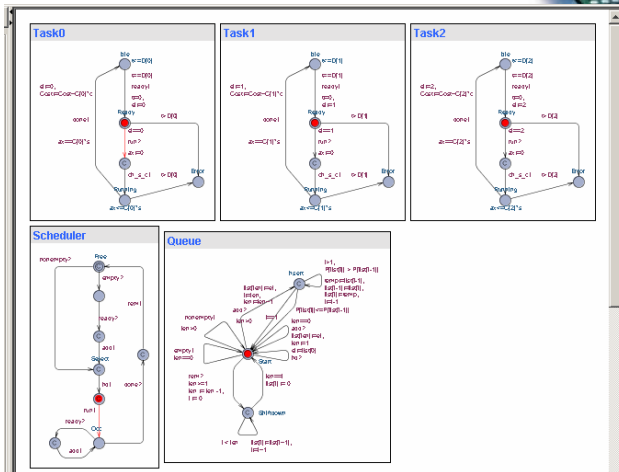
Optimal Schedule σ^* : $\text{val}(\sigma^*) = \inf_{\sigma} \text{val}(\sigma)$

Application

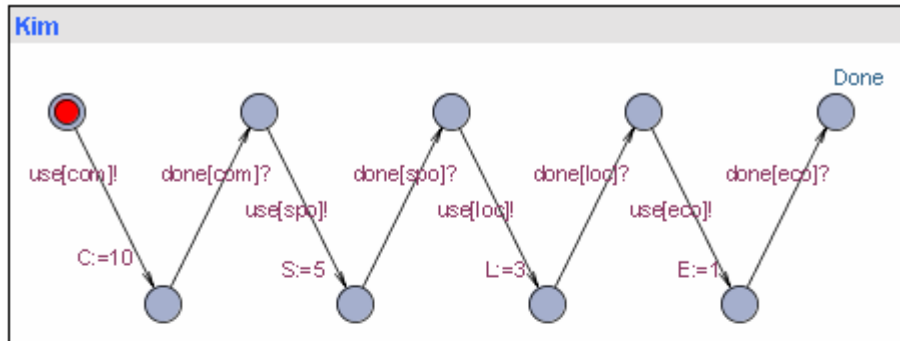
Dynamic Voltage Scaling



Automotive Electronics
 Biometrics
 Security and Surveillance
BLACK ANALOG DEVICES
 Information Appliances
 Embedded Modems



Dynamic Scheduling



$E \leftrightarrow$ (Kim.Done and
Jacob.Done and
Gerd.Done)

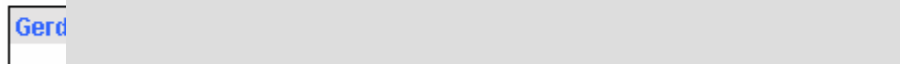
+ winning / optimal
strategy



Cost-Optimal Reachability Strategies for Priced Timed
Game Automata

[Alur et al, ICALP'04]

[Bouyer, Cassez, Fleury, Larsen, FSTTCS'04]



Time-Optimal Reachability Strategies for Timed *Games*

[Cassez, David, Fleury, Larsen, Lime, CONCUR'05]



Uncontrollable
timing uncertainty

$x \leq B+3$