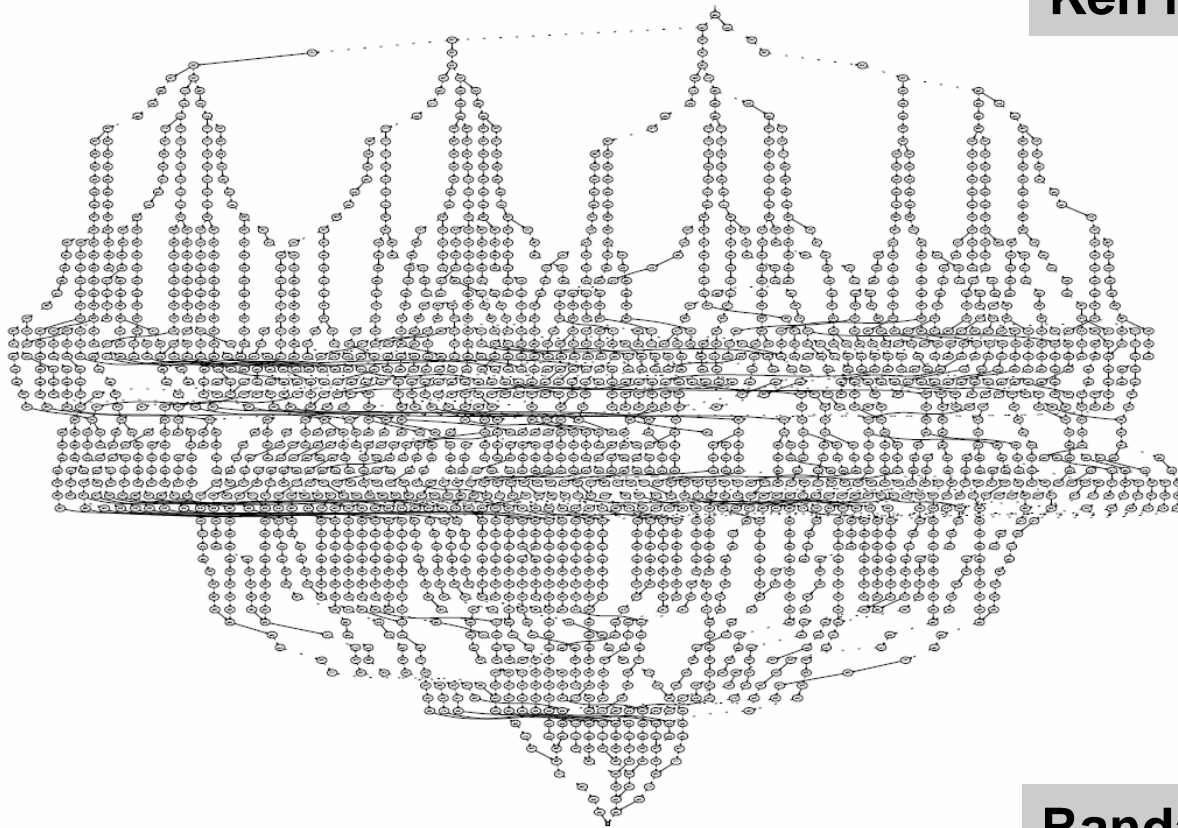


Symbolic Model Checking

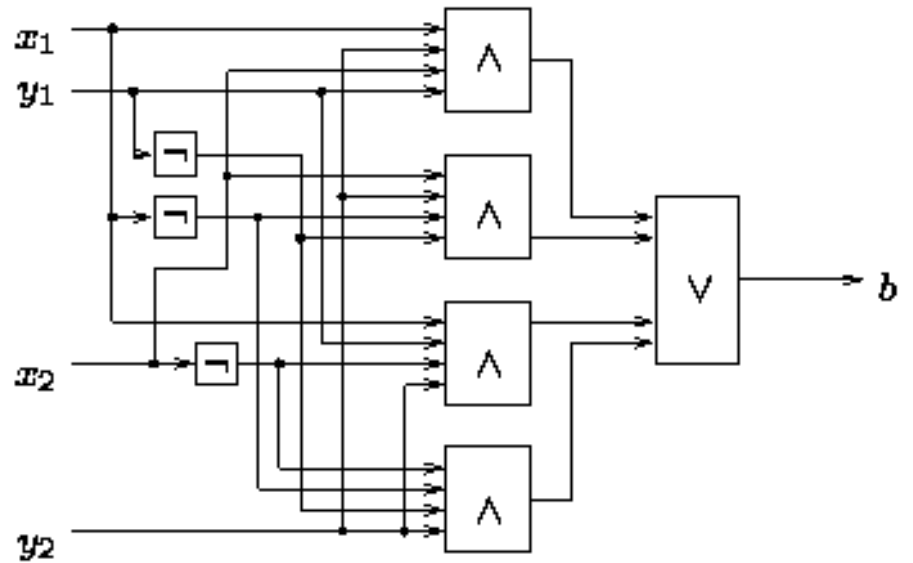
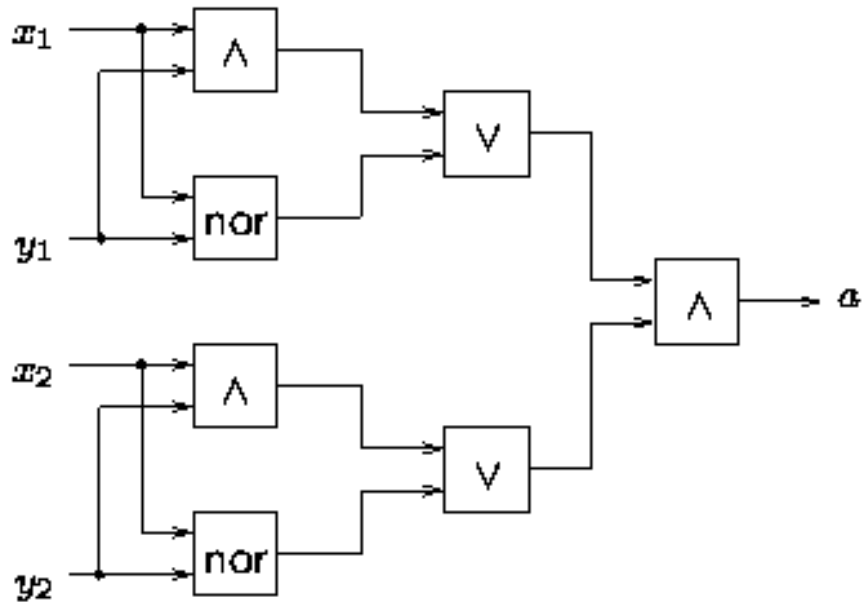
Ken McMillan '90



Randal Bryant '86

Binary Decision Diagrams

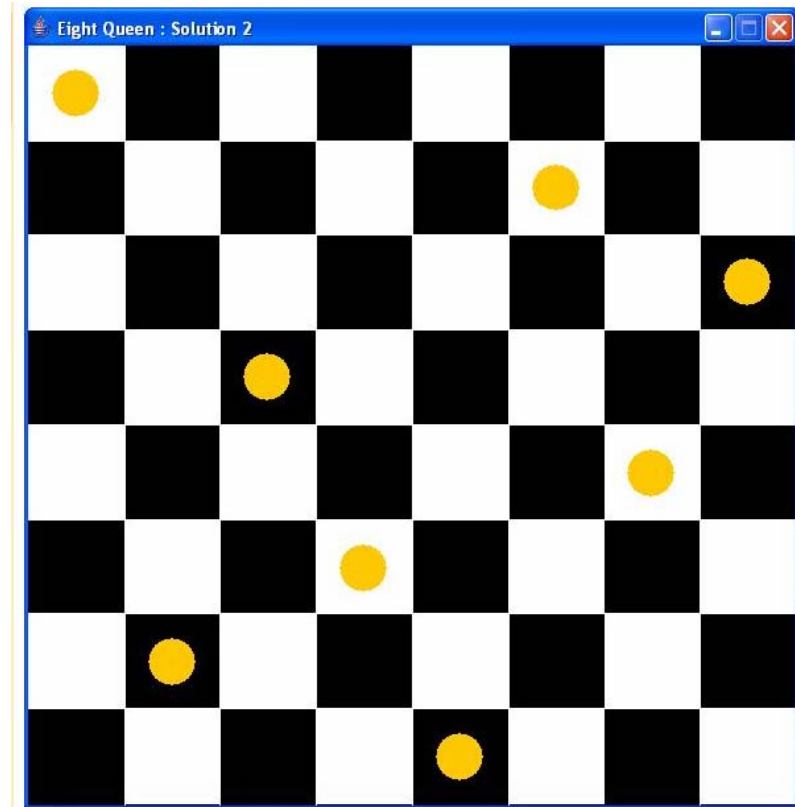
Combinatorial Circuits



Combinatorial Problems

	6		1		4		5	
		8	3		5	6		
2								1
8			4		7			6
		6				3		
7			9		1			4
5								2
		7	2		6	9		
	4		5		8		7	

Sudoku



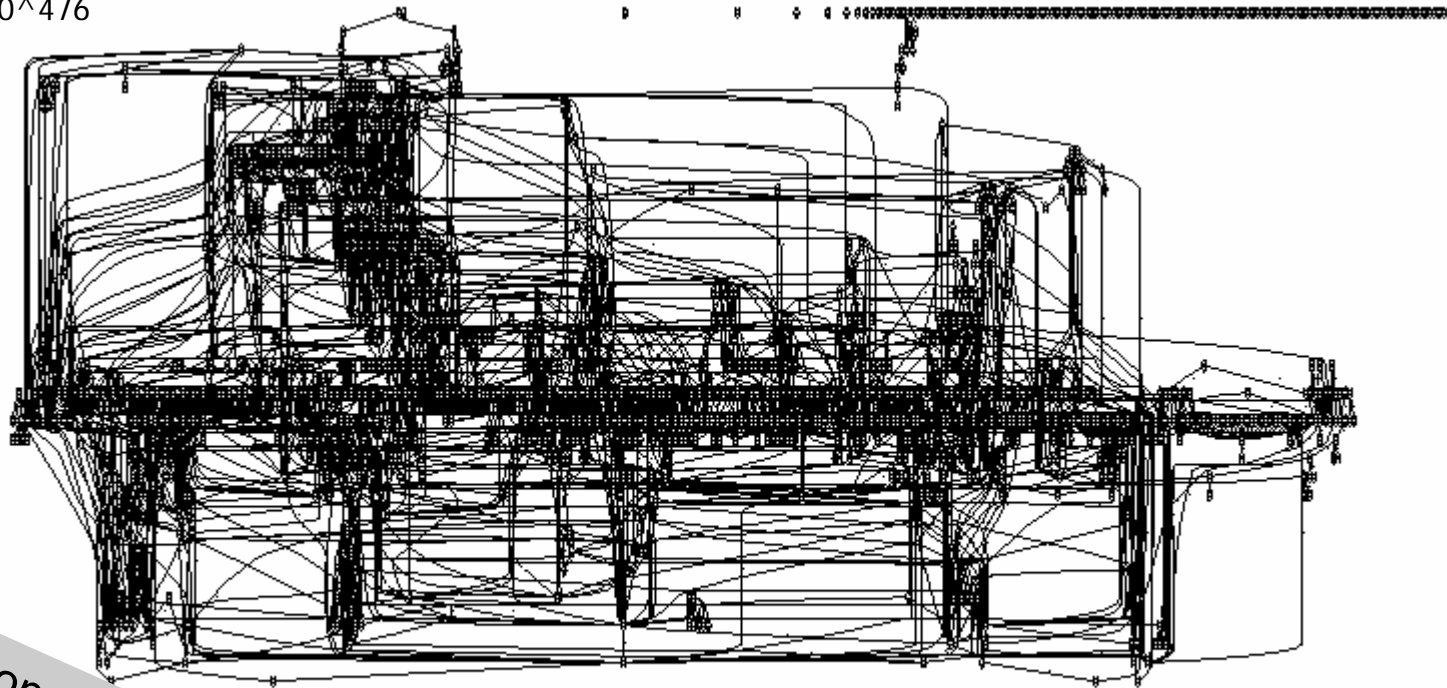
Eight Queen

Control Programs

A Train Simulator, visualSTATE (VVS)

1421 machines
11102 transitions
2981 inputs
2667 outputs
3204 local states
Declare state sp.: 10^{476}

BUGS ?



“Ideal” presentation: 1 bit/state
will clearly NOT work!

Reduced Ordered Binary Decision Diagrams [Bryant'86]

- Compact representation of *boolean functions* allowing effective manipulation (satisfiability, validity,....)

or

- Compact representation of *sets* over finite universe allowing effective manipulations.

Binary Decision Diagrams

[Randal Bryant'86]

A short review

If-Then-Else Normal Form

Definition

A boolean expression is in **If-Then-Else normal form (INF)** iff it is given by the following abstract syntax

$$t, t_1, t_2 ::= 0 \mid 1 \mid x \rightarrow t_1, t_2$$

where x ranges over boolean variables.

Example: $x_1 \rightarrow (x_2 \rightarrow 1, 0), 0$ (equivalent to $x_1 \wedge x_2$)

Boolean expressions in INF can be drawn as **decision trees**.

Shannon Expansion

Let t be a boolean expression and x a variable. We define boolean expressions

- $t[0/x]$ where every occurrence of x in t is replaced with 0, and
- $t[1/x]$ where every occurrence of x in t is replaced with 1.

Shannon's Expansion Law

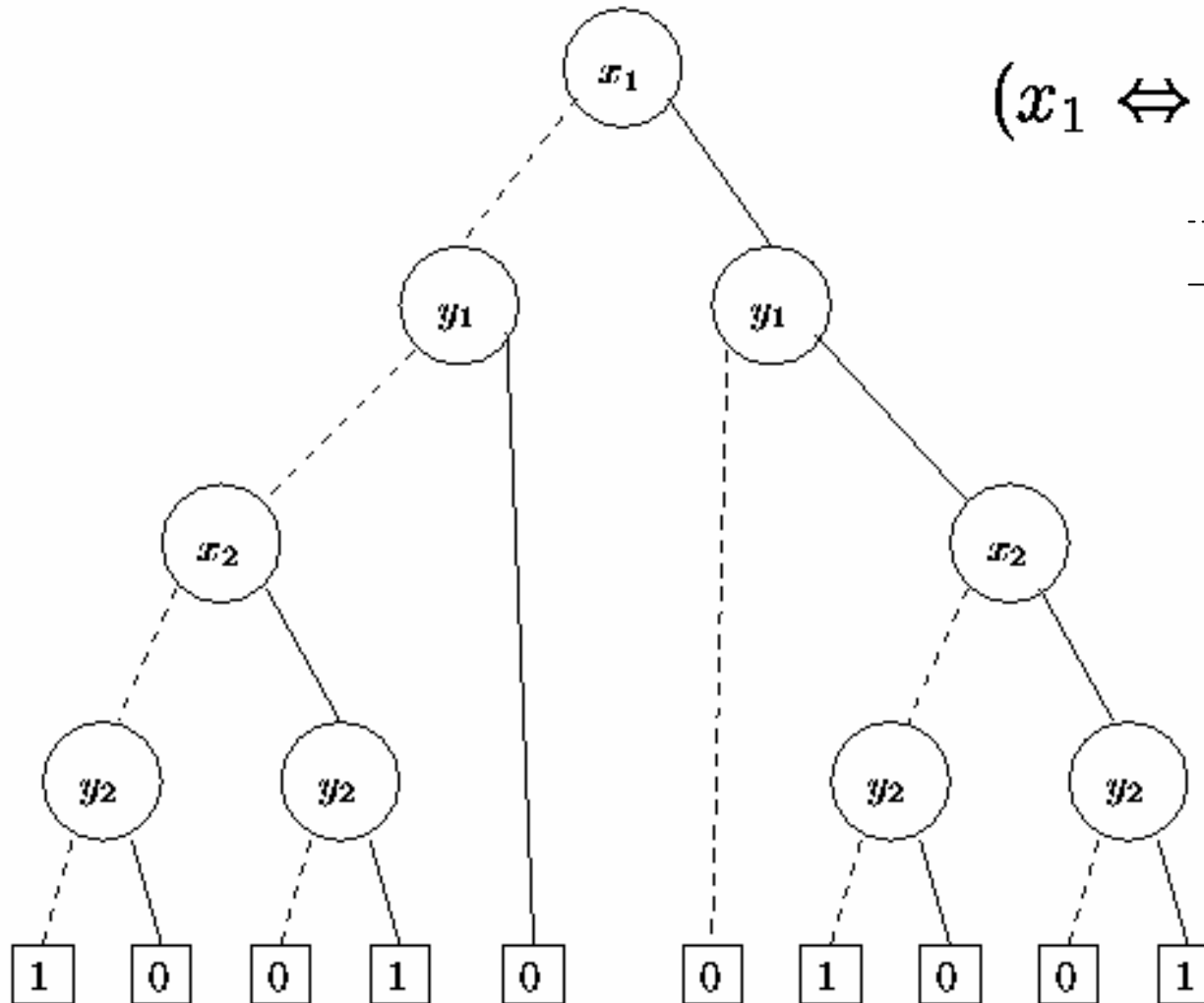
Let x be an arbitrary boolean variable. Any boolean expressions t is equivalent to

$$x \rightarrow t[1/x], t[0/x].$$

Corollary

For any boolean expression there is an equivalent one in INF.

Binary Decision Trees



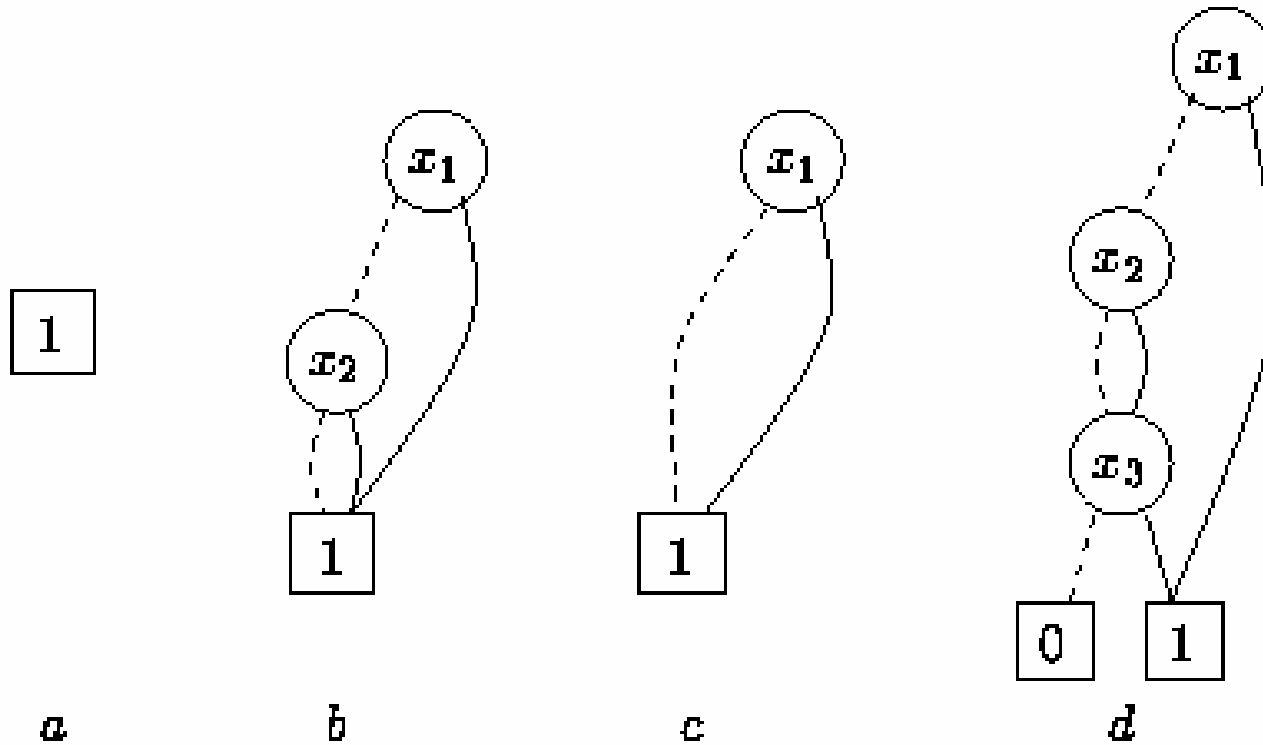
$$(x_1 \Leftrightarrow y_1) \wedge (x_2 \Leftrightarrow y_2)$$

- Variable is set to 0
- Variable is set to 1

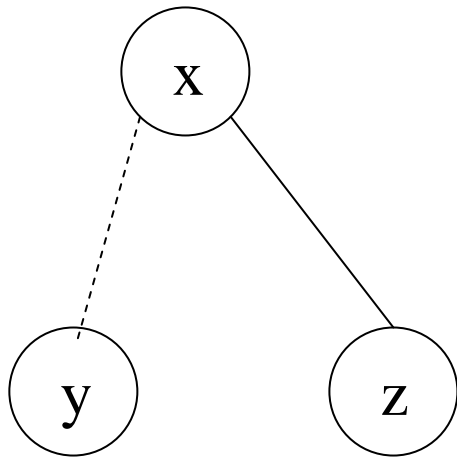
Each path determines a partial (set of) truth assignments.

Result of the boolean expression under the given assignment found in value of terminal.

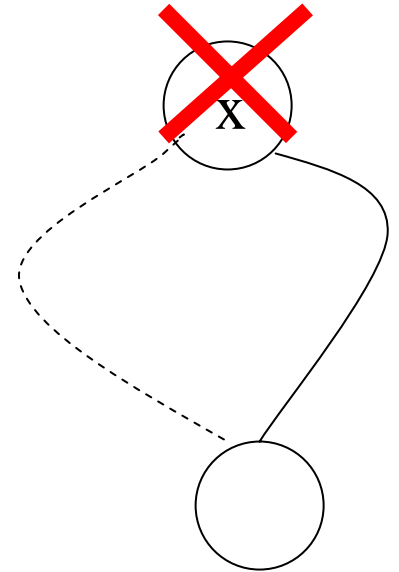
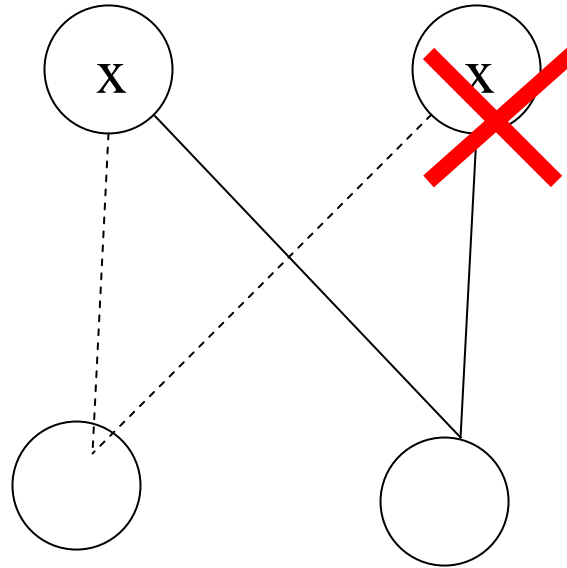
Orderedness & Redundant *TESTS*



Orderedness & Reducedness



$x < y$ $x < z$



ROBDDs formally

A *Binary Decision Diagram* is a rooted, directed, acyclic graph (V, E) . V contains (up to) two *terminal* vertices, $0, 1 \in V$. $v \in V \setminus \{0, 1\}$ are *non-terminal* and has attributes $var(v)$, and $low(v), high(v) \in V$.

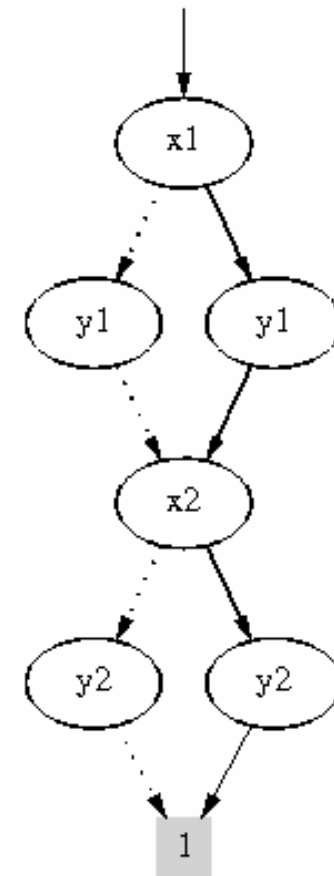
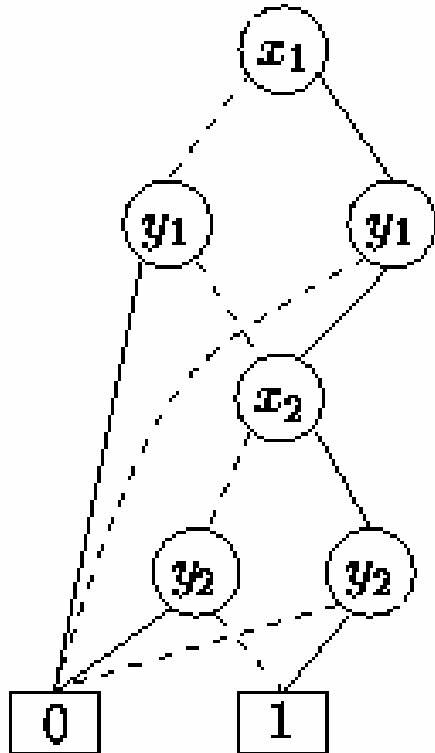
A BDD is *ordered* if on all paths from the root the variables respect a given total order.

A BDD is *reduced* if for all non-terminal vertices u, v ,

- 1) $low(u) \neq high(u)$
- 2) $low(u) = low(v), high(u) = high(v), var(u) = var(v)$
implies $u = v$

Reduced Ordered Binary Decision Diagrams

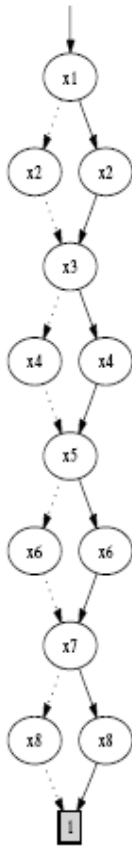
$$(x_1 \Leftrightarrow y_1) \wedge (x_2 \Leftrightarrow y_2)$$



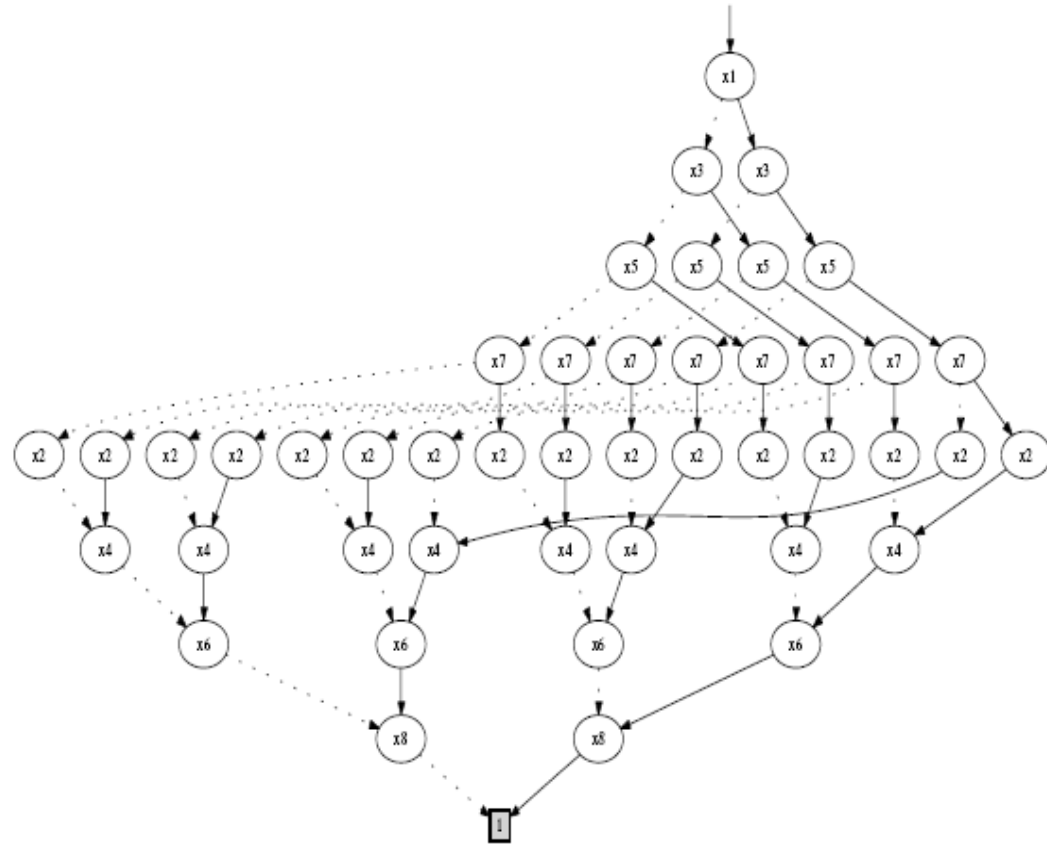
lben
Edges to 0
implicit

Ordering DOES matter

$$(x_1 \Leftrightarrow x_2) \wedge (x_3 \Leftrightarrow x_4) \wedge (x_5 \Leftrightarrow x_6) \wedge (x_7 \Leftrightarrow x_8)$$



$$x_1 < x_2 < \dots < x_8$$



$$x_1 < x_3 < x_5 < x_7 < x_2 < x_4 < x_6 < x_8$$

Canonicity of ROBDDs

$$t_0 = 0$$

$$t_1 = 1$$

$$t_u = x \rightarrow t_h, t_l, \text{ if } u \text{ is a node } (x, l, h)$$

Lemma 1 (Canonicity lemma) For any function $f: \mathbb{B}^n \rightarrow \mathbb{B}$ there is exactly one ROBDD b with variables $x_1 < x_2 < \dots < x_n$ such that

$$t_b[v_1/x_1, \dots, v_n/x_n] = f(v_1, \dots, v_n)$$

for all $(v_1, \dots, v_n) \in \mathbb{B}^n$.

Consequences: b is a tautology, if and only if, $b = \boxed{1}$
 b is satisfiable, if and only if, $b \neq \boxed{0}$

Build

Let t be a boolean expression and $x_1 < x_2 < \dots < x_n$.

Build($t, 1$) builds a corresponding ROBDD and returns its root.

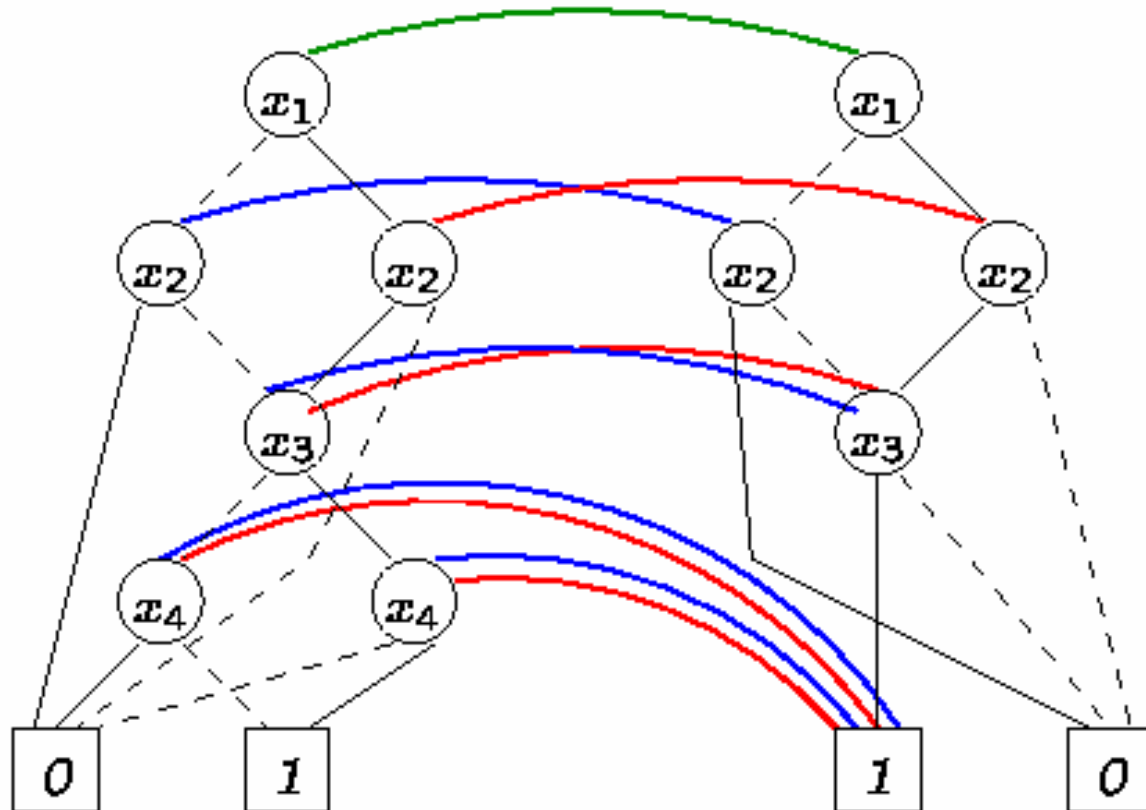
```
Build( $t, i$ ): Node =  
if  $i > n$  then  
    if  $t$  is true then return 0 else return 1  
else  
     $low := \text{Build}(t[0/x_i], i + 1)$   
     $high := \text{Build}(t[1/x_i], i + 1)$   
     $var := i$   
    return Makenode( $var, low, high$ )  
end if
```

Complexity ??

APPLY operation

```
Apply(op, b1, b2)
4:  function app(u1, u2) =
6:      if u1 ∈ {0, 1} and u2 ∈ {0, 1} then res ← op(u1, u2)
7:      else if u1 ∈ {0, 1} and u2 ≥ 2 then
8:          res ← makenode(var(u2), app(u1, low(u2)), app(u1, high(u2)))
9:      else if u1 ≥ 2 and u2 ∈ {0, 1} then
10:         res ← makenode(var(u1), app(low(u1), u2), app(high(u1), u2)))
11:      else if var(u1) = var(u2) then
12:         res ← makenode(var(u1), app(low(u1), low(u2)),
13:                               app(high(u1), high(u2)))
14:      else if var(u1) < var(u2) then
15:         res ← makenode(var(u1), app(low(u1), u2), app(high(u1), u2)))
16:      else (* var(u1) > var(u2) *)
17:         res ← makenode(var(u2), app(u1, low(u2)), app(u1, high(u2)))
18:      return res
20:
21: b.root ← app(b1.root, b2.root)
22: return b
```

APPLY example



APPLY operation with dynamic programming

```
Apply(op, b1, b2)
4:  function app(u1, u2) =
5:      if G(u1, u2) ≠ empty then return G(u1, u2)
6:      else if u1 ∈ {0, 1} and u2 ∈ {0, 1} then res ← op(u1, u2)
7:          else if u1 ∈ {0, 1} and u2 ≥ 2 then
8:              res ← makenode(var(u2), app(u1, low(u2)), app(u1, high(u2)))
9:          else if u1 ≥ 2 and u2 ∈ {0, 1} then
10:             res ← makenode(var(u1), app(low(u1), u2), app(high(u1), u2))
11:          else if var(u1) = var(u2) then
12:             res ← makenode(var(u1), app(low(u1), low(u2)),
13:                                     app(high(u1), high(u2)))
14:          else if var(u1) < var(u2) then
15:             res ← makenode(var(u1), app(low(u1), u2), app(high(u1), u2))
16:          else (* var(u1) > var(u2) *)
17:             res ← makenode(var(u2), app(u1, low(u2)), app(u1, high(u2)))
18:             G(u1, u2) ← res
19:             return res
20:
21: forall i ≤ max(b1), j ≤ max(b2) : G(i, j) ← empty
22: b.root ← app(b1.root, b2.root)
23: return b
```

Other operations

Let t be a boolean expression with its ROBDD representation.

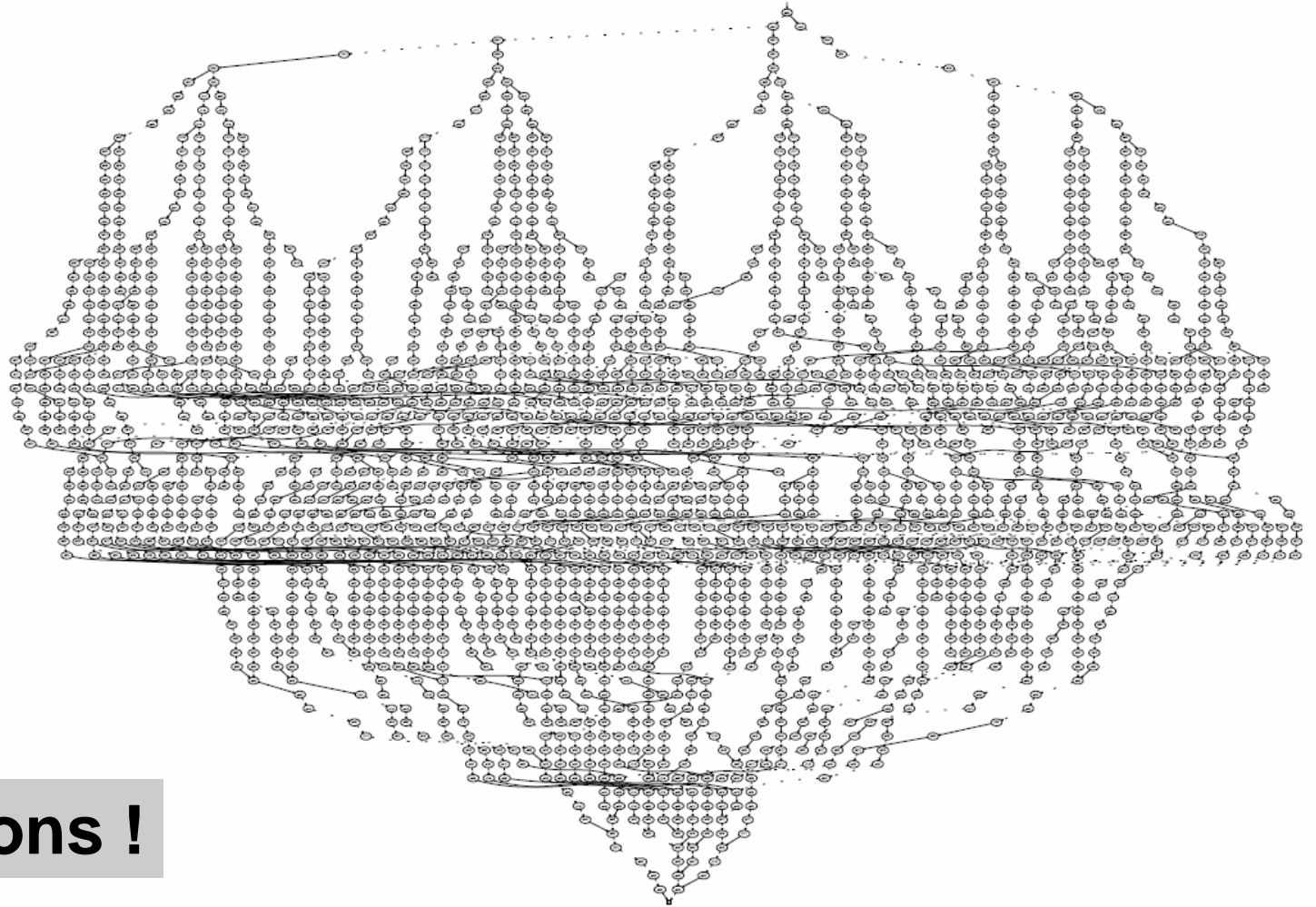
The following operations can be done efficiently:

- **Restriction** $t[0/x_i]$ ($t[1/x_i]$): restricts the variable x_i to 0 (1)
- **SatCount**(t): returns the number of satisfying assignments
- **AnySat**(t): returns some satisfying assignment
- **AllSat**(t): returns all satisfying assignments
- **Existential quantification** $\exists x_i.t$: equivalent to $t[0/x_i] \vee t[1/x_i]$
- **Composition** $t[t'/x_i]$: equivalent to $t' \rightarrow t[1/x_i], t[0/x_i]$

Constraint Solving using BDDs

1			
	2		
		3	
			4

4 x 4 Sudoku



288 solutions !

Encoding

	1	2	3	4
1	1			
2		2		
3			3	
4				4

Boolean variables $x_{i,j,k}$ for all $i, j, k \in \{1,2,3,4\}$.

Idea:

$x_{i,j,k} = 1$; if the number k is in
position (i,j) in the solution
 0 ; otherwise

$$x_{2,2,2} = 1$$

$$x_{4,4,4} = 1$$

$$x_{2,2,1} = 0$$

Constraints

	1	2	3	4
1	1			
2		2		
3			3	
4				4

Precisely one value in **each position** i, j :

$$X_{1,j,1} + X_{i,j,2} + X_{i,j,3} + X_{i,j,4} = 1 \quad \text{for each } i, j$$

Each value k appears in **each row** i exactly ones:

$$X_{i,1,k} + X_{i,2,k} + X_{i,3,k} + X_{i,4,k} = 1 \quad \text{for each } i, k$$

Each value k appears in **each column** j exactly ones:

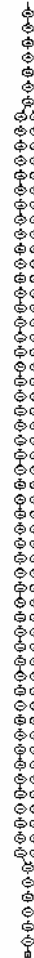
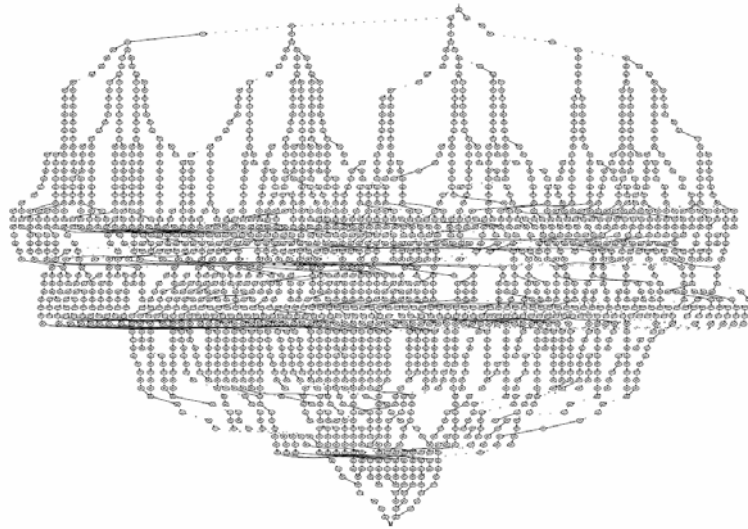
$$X_{1,j,k} + X_{2,j,k} + X_{3,j,k} + X_{4,j,k} = 1 \quad \text{for each } j, k$$

Each value k appears in **each 2x2 box** exactly ones:

$$X_{1,1,k} + X_{1,2,k} + X_{2,1,k} + X_{2,2,k} = 1 \quad (\text{e.g.})$$

Solving Sudoku

	1	2	3	4
1				
2				
3				
4				

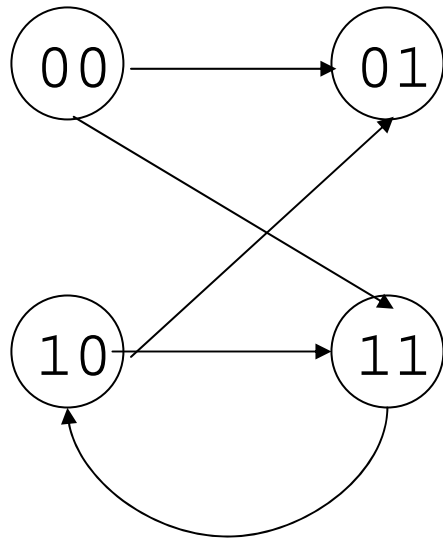


	1	2	3	4
1	1			
2		2		
3			3	
4				4

ROBDDs and Verification

[...,McMillan'90,.....,VVS'97]

ROBDD encoding of transition system

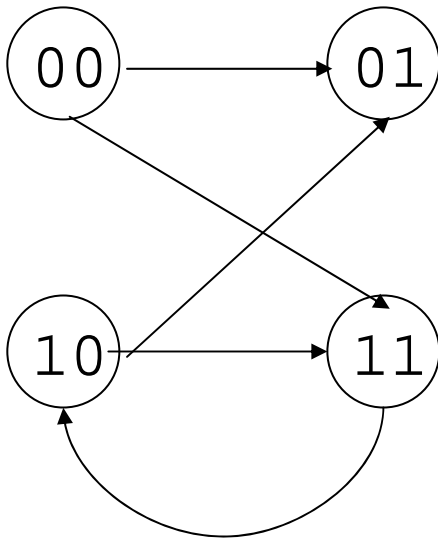


Encoding of **states** using binary variables (here x_1 and x_2).

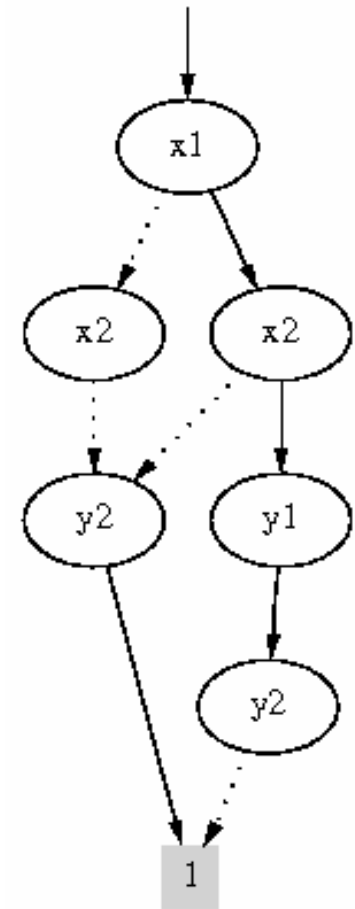
Encoding of **transition relation** using source and target variables (here x_1 , x_2 , y_1 , and y_2)

```
Trans(x1, x2, y1, y2) :=  
    !x1 & !x2 & !y1 & y2  
  + !x1 & !x2 & y1 & y2  
  + x1 & !x2 & !y1 & y2  
  + x1 & !x2 & y1 & y2  
  + x1 & x2 & y1 & !y2;
```

ROBDD representation (cont.)

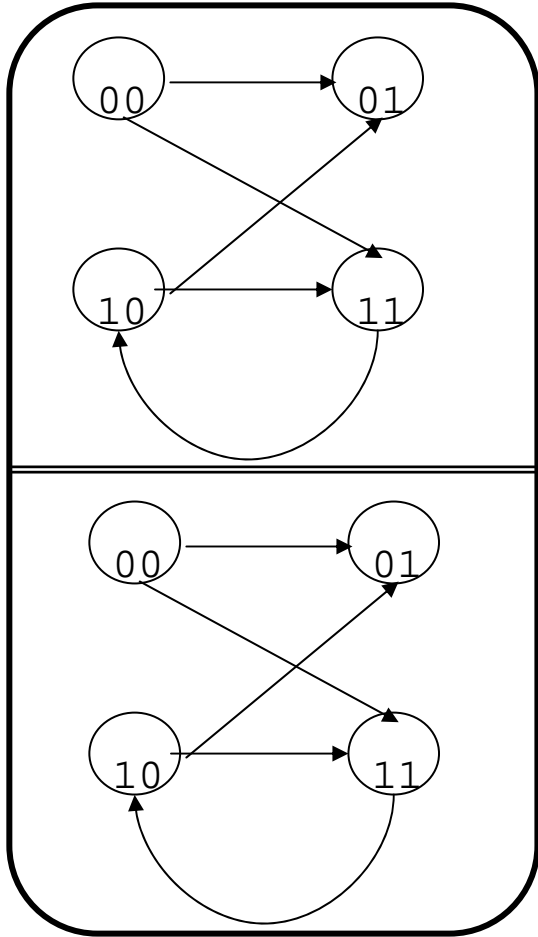


```
Trans(x1, x2, y1, y2) :=  
    !x1 & !x2 & !y1 & y2  
  + !x1 & !x2 & y1 & y2  
  + x1 & !x2 & !y1 & y2  
  + x1 & !x2 & y1 & y2  
  + x1 & x2 & y1 & !y2;
```



ROBDD for parallel composition

ATrans(\mathbf{x}, \mathbf{y})



Asynchronous composition

$$\begin{aligned} \text{Trans}(\mathbf{x}, \mathbf{y}, \mathbf{u}, \mathbf{v}) = & \\ & (\text{ATrans}(\mathbf{x}, \mathbf{y}) \ \& \ \mathbf{v}=\mathbf{u}) \\ & + (\text{BTrans}(\mathbf{u}, \mathbf{v}) \ \& \ \mathbf{y}=\mathbf{x}) \end{aligned}$$

Synchronous composition

$$\begin{aligned} \text{Trans}(\mathbf{x}, \mathbf{y}, \mathbf{u}, \mathbf{v}) = & \\ & (\text{ATrans}(\mathbf{x}, \mathbf{y}) \ \& \ \text{BTrans}(\mathbf{u}, \mathbf{v})) \end{aligned}$$

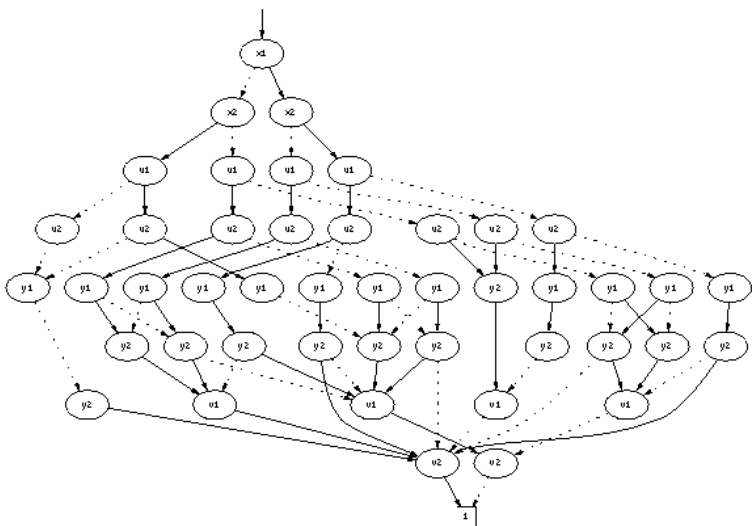
Which ordering to choose?

BTrans(\mathbf{u}, \mathbf{v})

Ordering?

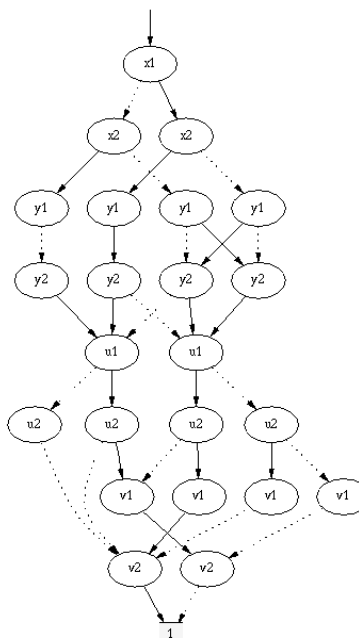
45 nodes

$x_1, x_2, u_1, u_2, y_1, y_2, v_1, v_2$



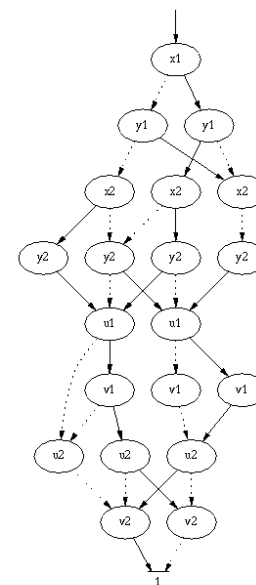
23 nodes

$x_1, x_2, y_1, y_2, u_1, u_2, v_1, v_2$



20 nodes

$x_1, y_1, x_2, y_2, u_1, v_1, u_2, v_2$



Polynomial size BDDs guaranteed
in size of argument BDDs
[Enders, Filkorn, Taubner'91]

Reachable States

```
Reach(x) := Init(x);
```

```
REPEAT
```

```
  Old(x) := Reach(x);
```

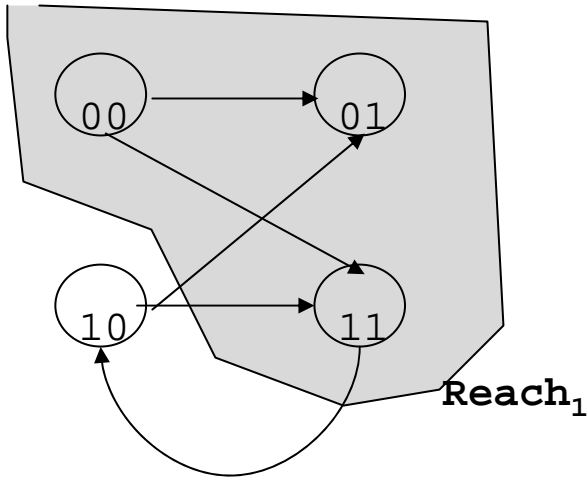
```
  New(y) := Exists x. (Reach(x) & Trans(x, y));
```

```
  Reach(x) := Old(x) + New(x)
```

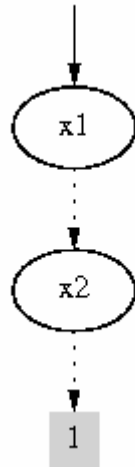
```
UNTIL Old(x) = Reach(x)
```

Relational Product:

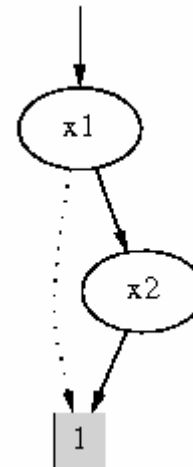
May be constructed without building intermediate (often large) &-BDD.



$Reach_0$



$Reach_1$



$Reach_2$



A MUTEX Algorithm

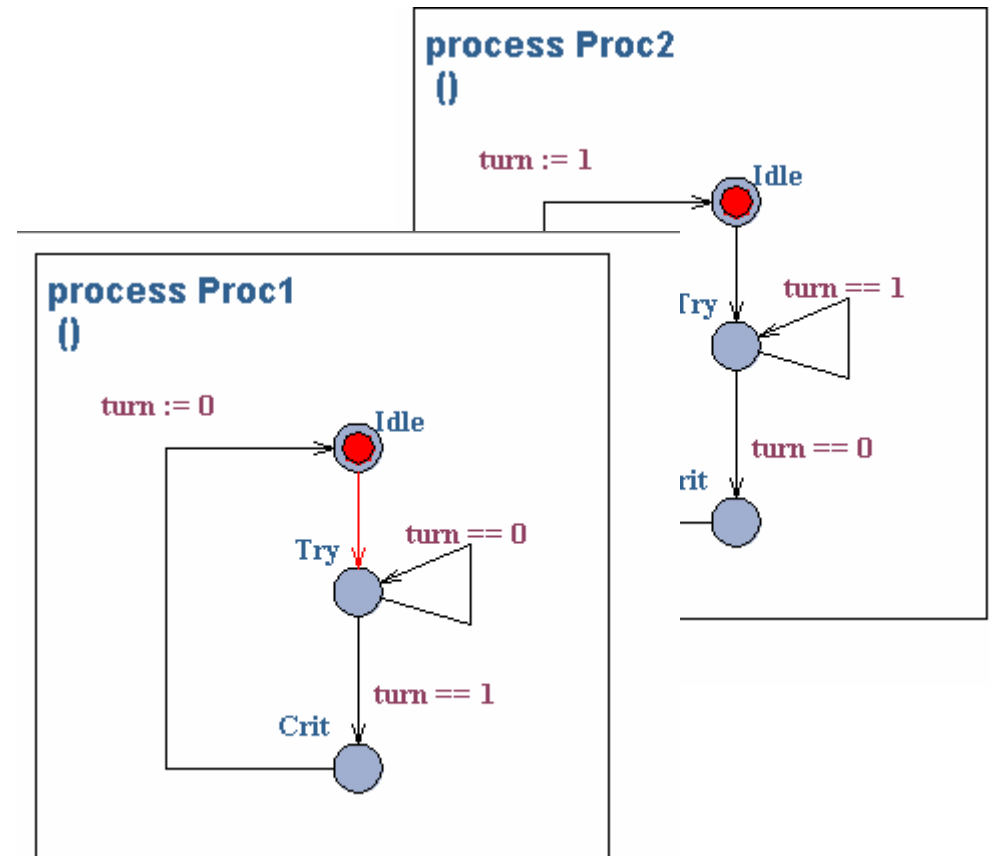
Clarke & Emerson

```
P1 :: while True do
    T1 : wait(turn=1)
    C1 : turn:=0
endwhile

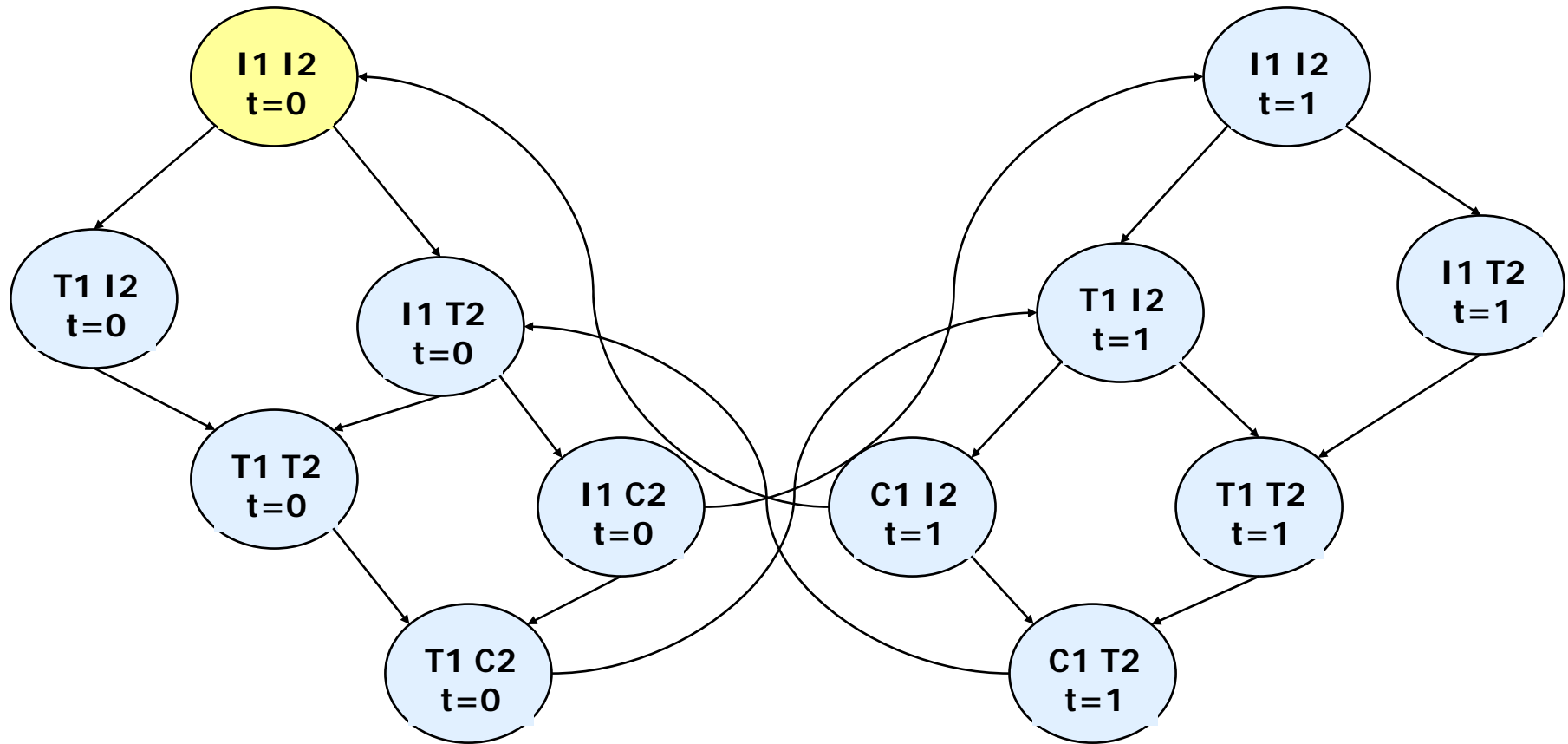
||

P2 :: while True do
    T2 : wait(turn=0)
    C2 : turn:=1
endwhile
```

Mutual Exclusion Program



Global Transition System



A MUTEX Algorithm

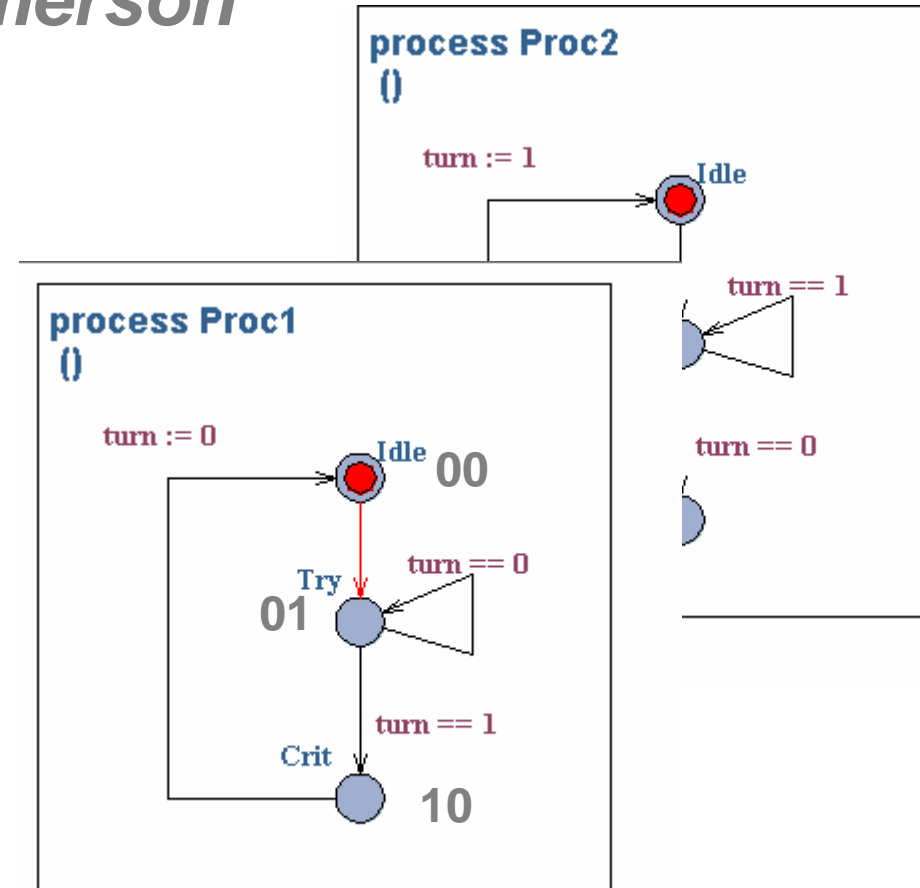
Clarke & Emerson

```
vars x1 x2;  
vars y1 y2;  
vars u1 u2;  
vars v1 v2;  
vars t s;
```

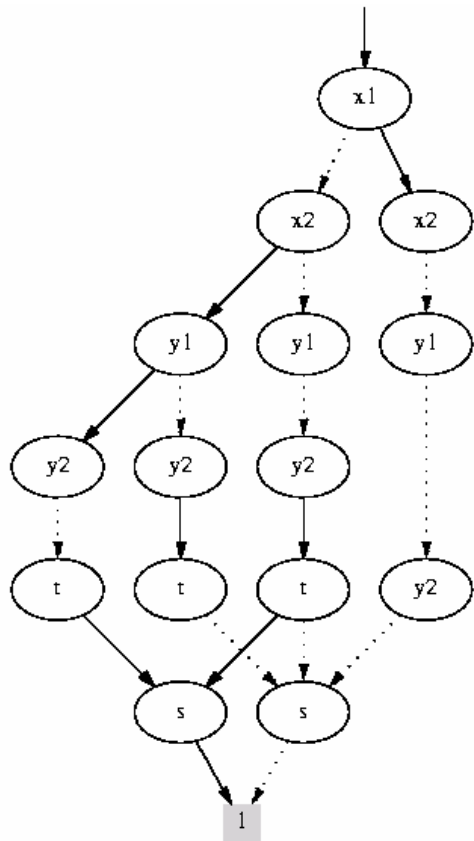
```
ATrans := (!x1 & !x2 & !y1 & y2 & (s=t))  
        + (!x1 & x2 & !y1 & y2 & !t & !s)  
        + (!x1 & x2 & y1 & !y2 & t & s)  
        + (x1 & !x2 & !y1 & !y2 & !s);
```

```
BTrans := (!u1 & !u2 & !v1 & v2 & (s=t))  
        + (!u1 & u2 & !v1 & v2 & t & s)  
        + (!u1 & u2 & v1 & !v2 & !t & !s)  
        + (u1 & !u2 & !v1 & !v2 & s);
```

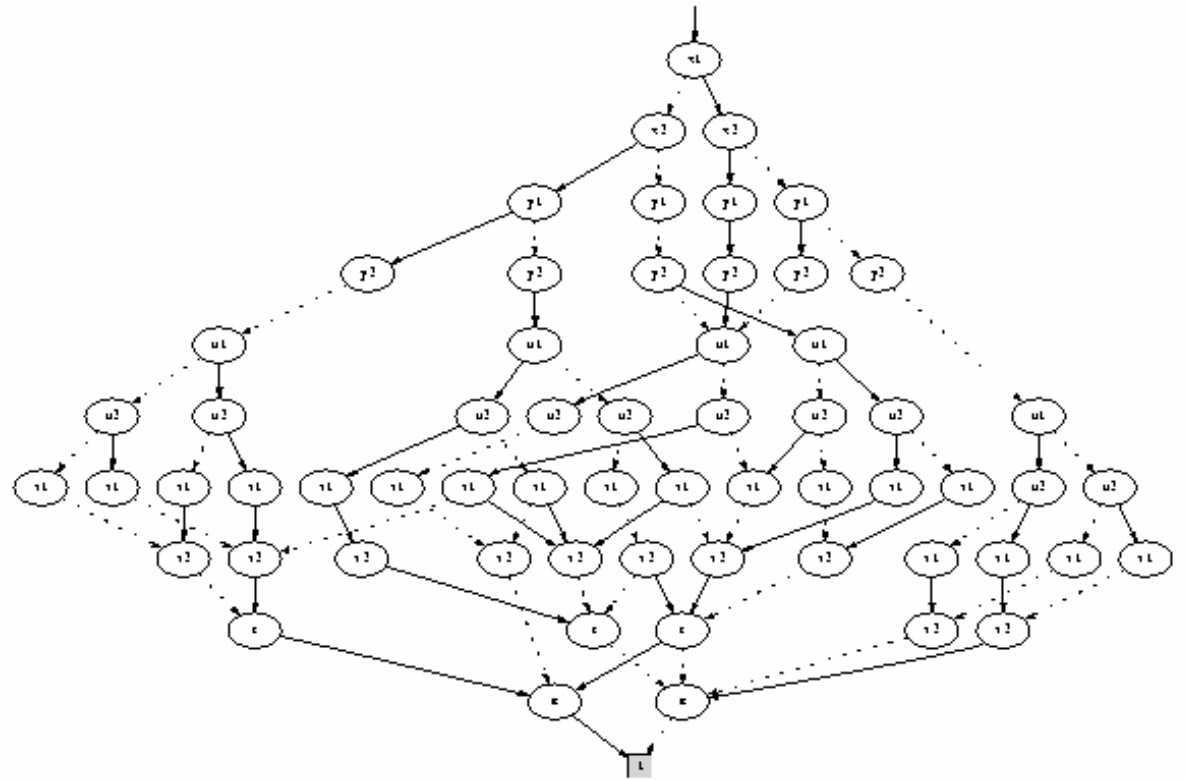
```
TT := (ATrans & (u1=v1) & (u2=v2))  
      + (BTrans & (x1=y1) & (x2=y2));
```



BDDs for Transition Relations



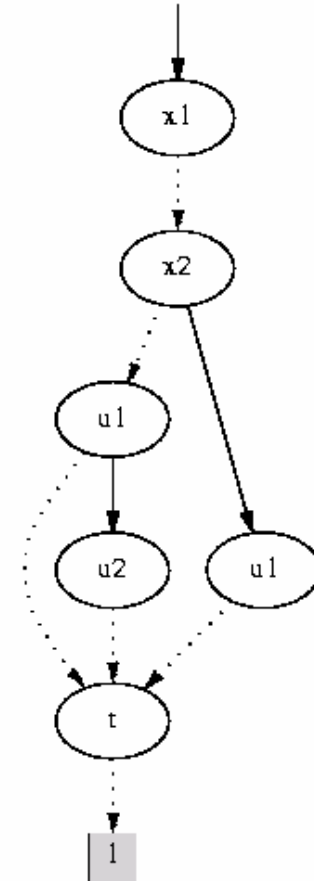
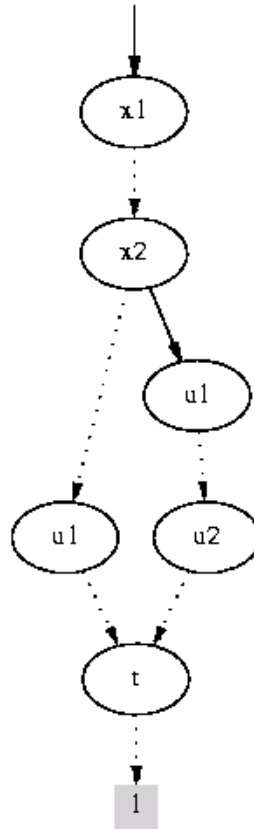
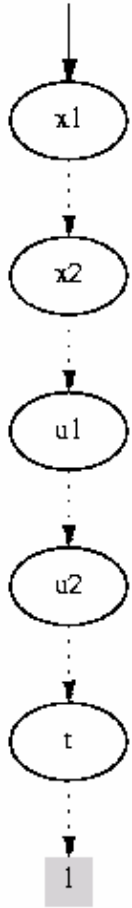
ATrans



TT

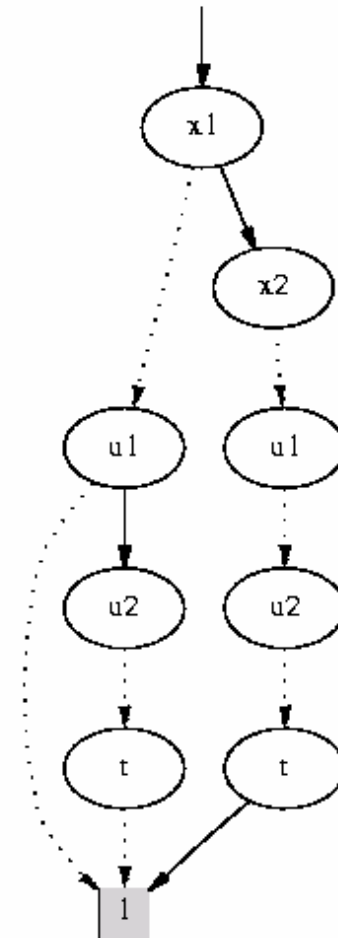
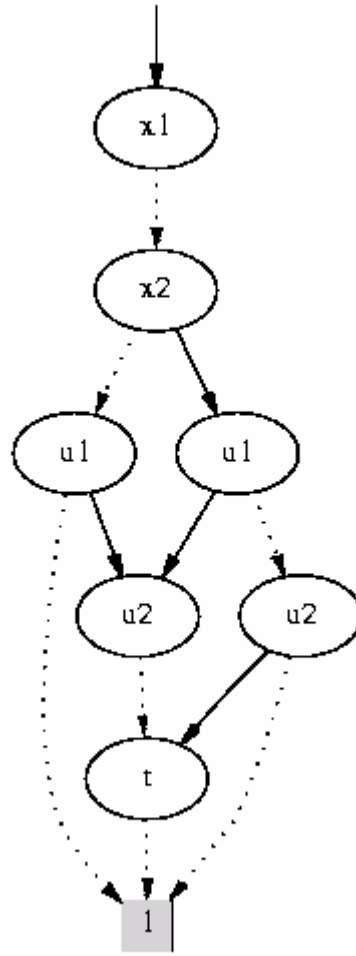
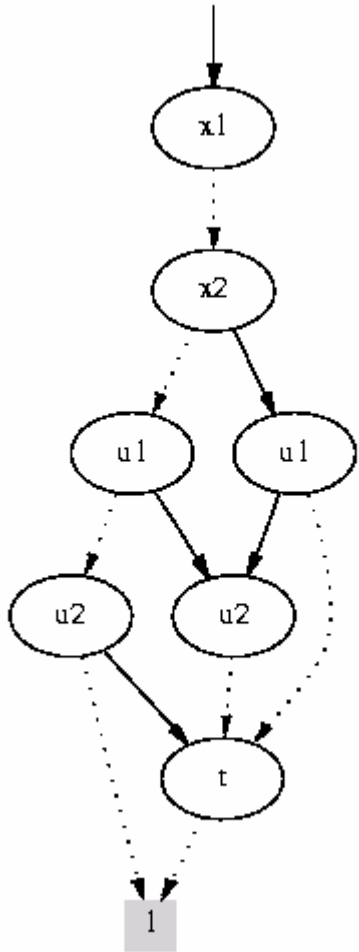
Reachable States

```
Reach( $\mathbf{x}$ ) := Init( $\mathbf{x}$ );  
REPEAT  
  Old( $\mathbf{x}$ ) := Reach( $\mathbf{x}$ );  
  New( $\mathbf{y}$ ) := Exists  $\mathbf{x}$ . (Reach( $\mathbf{x}$ ) & Trans( $\mathbf{x}, \mathbf{y}$ ));  
  Reach( $\mathbf{x}$ ) := Old( $\mathbf{x}$ ) + New( $\mathbf{x}$ )  
UNTIL Old( $\mathbf{x}$ ) = Reach( $\mathbf{x}$ )
```



Reachable States

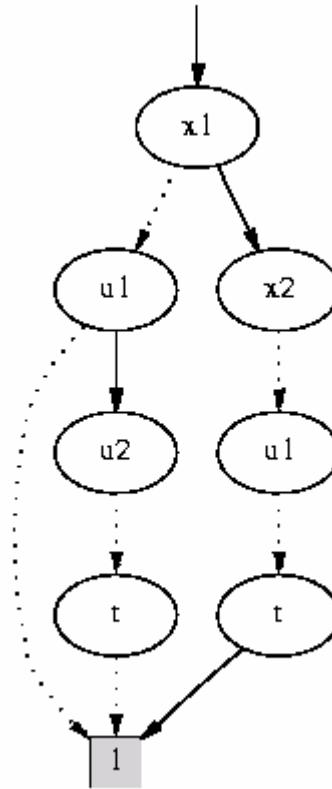
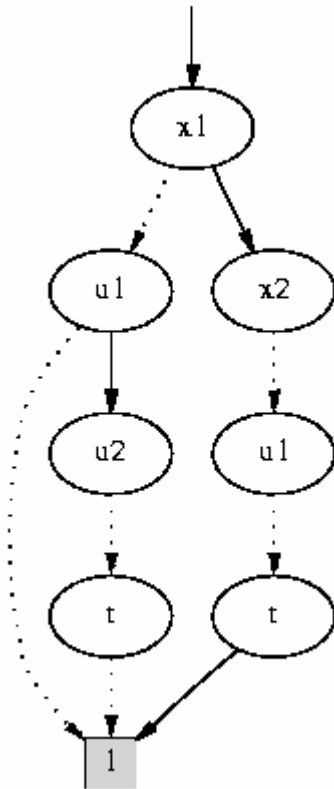
```
Reach( $\mathbf{x}$ ) := Init( $\mathbf{x}$ );  
REPEAT  
  Old( $\mathbf{x}$ ) := Reach( $\mathbf{x}$ );  
  New( $\mathbf{y}$ ) := Exists  $\mathbf{x}$ . (Reach( $\mathbf{x}$ ) & Trans( $\mathbf{x}, \mathbf{y}$ ));  
  Reach( $\mathbf{x}$ ) := Old( $\mathbf{x}$ ) + New( $\mathbf{x}$ )  
UNTIL Old( $\mathbf{x}$ ) = Reach( $\mathbf{x}$ )
```



Reachable States

```

Reach(x) := Init(x);
REPEAT
  Old(x) := Reach(x);
  New(y) := Exists x.(Reach(x) & Trans(x,y));
  Reach(x) := Old(x) + New(x)
UNTIL Old(x) = Reach(x)
  
```



Reach

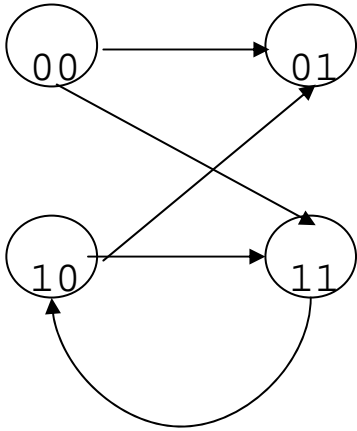
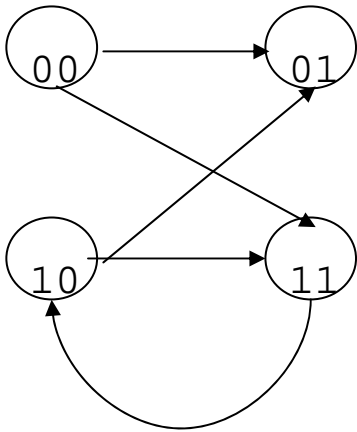
MUTEX ?

Reach &
x1 & !x2 &
u1 & !u2

1

Bisimulation

vars **x** (**y**)



```
Bis(x,u) := 1;
```

```
REPEAT
```

```
  Old(x,u) := Bis(x,u);
```

```
  Bis(x,u) :=
```

```
    Forall y. Trans(x,y) =>
```

```
      (Exists v. Trans(u,v) & Bis(y,v))
```

```
    &
```

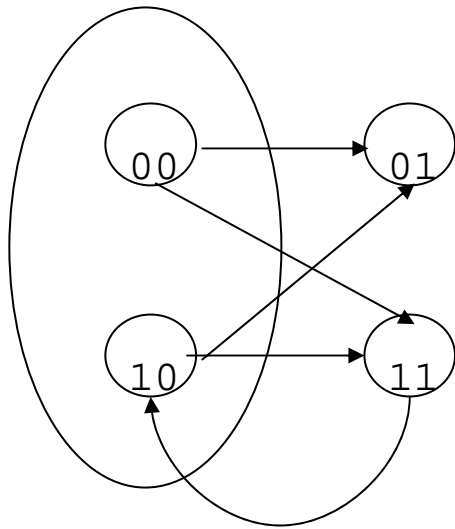
```
    Forall v. Trans(u,v) =>
```

```
      (Exists y. Trans(x,y) & Bis(y,v));
```

```
UNTIL Bis(x,u)=Old(x,u)
```

vars **u** (**v**)

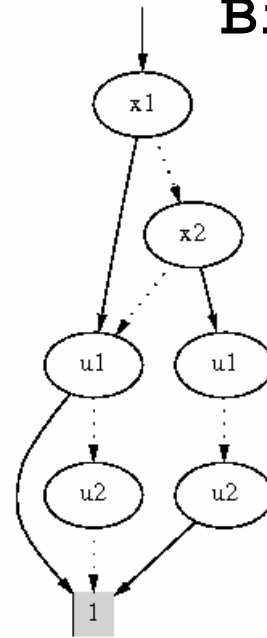
Bisimulation (cont.)



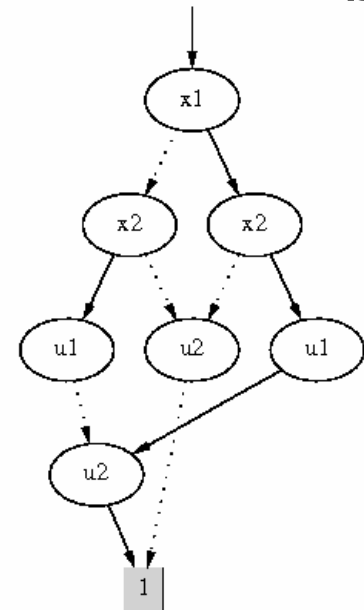
Bis_0



Bis_1

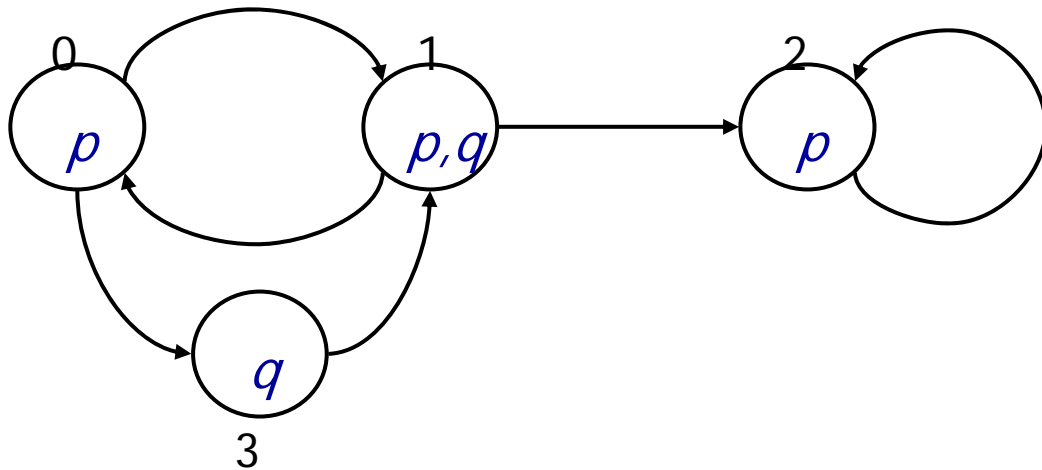


Bis_2



3 equivalence classes
= 6 pairs in final bisimulation

Model Checking



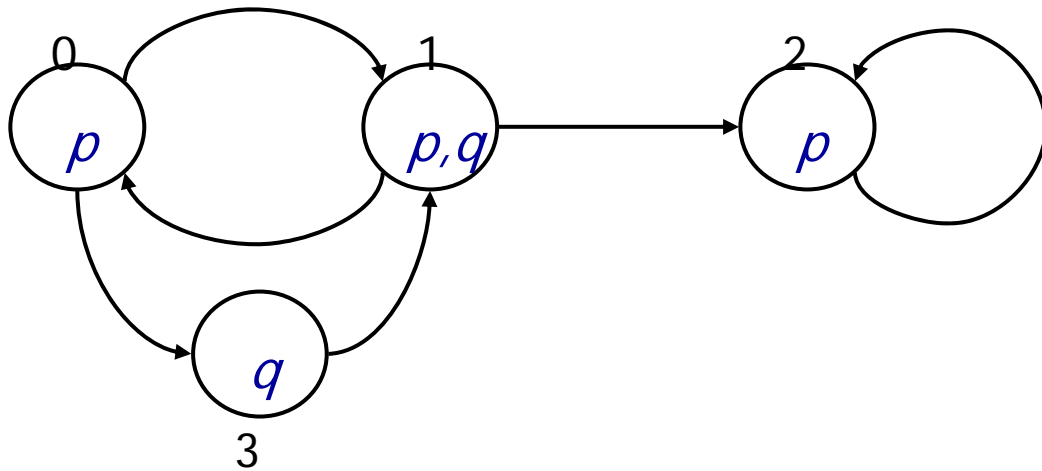
```
vars x1 x2;  
vars y1 y2;
```

```
Trans(x1,x2,y1,y2) :=  
    !x1 & !x2 & !y1 & y2  
+ !x1 & !x2 & y1 & y2  
+ ..... ;
```

```
P(x1,x2) := !x1 & !x2  
+ !x1 & x2  
+ x1 & !x2;
```

```
Q(x1,x2) := ..... ;
```

Model Checking



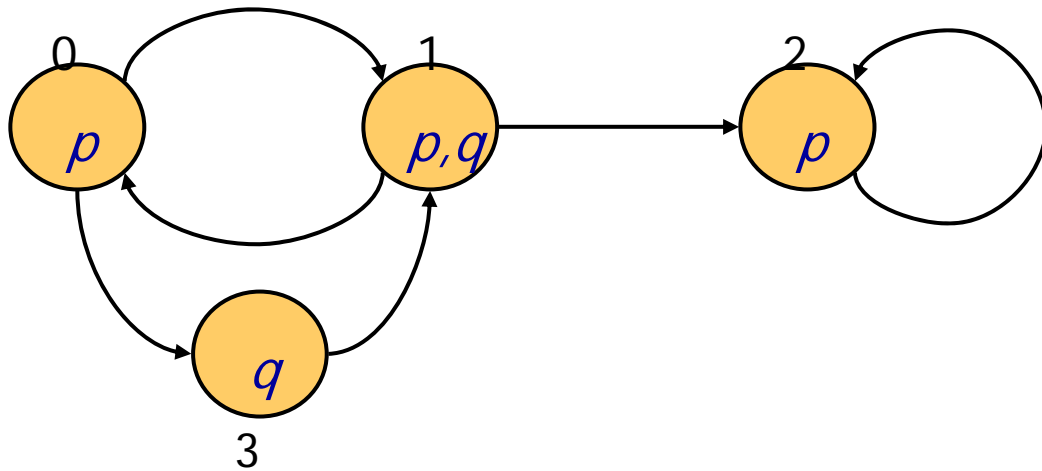
EX P

Exists $y1, y2$.

$\text{Trans}(x1, x2, y1, y2) \ \&$

$\text{P}(y1, y2);$

Model Checking



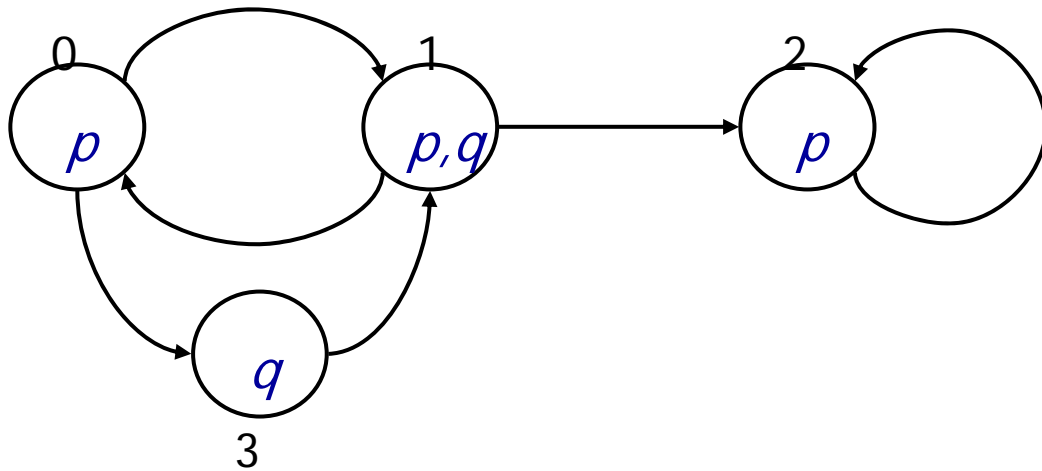
EX P

Exists $y1, y2$.

$\text{Trans}(x1, x2, y1, y2) \ \&$

$\text{P}(y1, y2);$

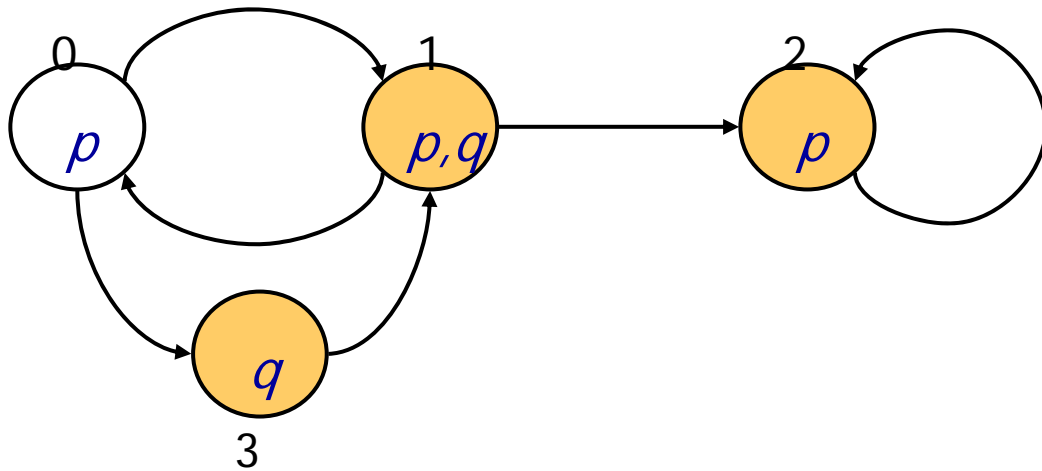
Model Checking



AX P

Forall $y1, y2$.
 $\text{Trans}(x1, x2, y1, y2) \Rightarrow$
 $P(y1, y2);$

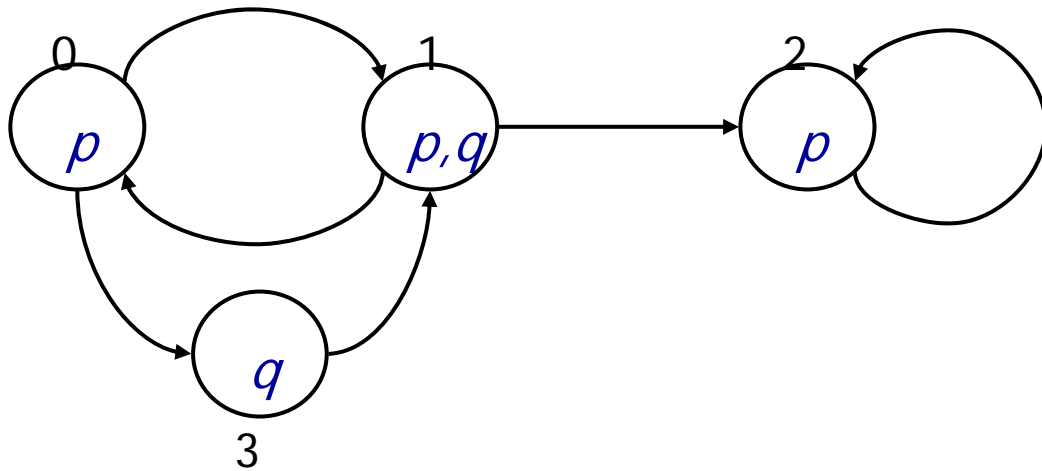
Model Checking



AX P

Forall $y1, y2$.
Trans($x1, x2, y1, y2$) \Rightarrow
P($y1, y2$);

Model Checking

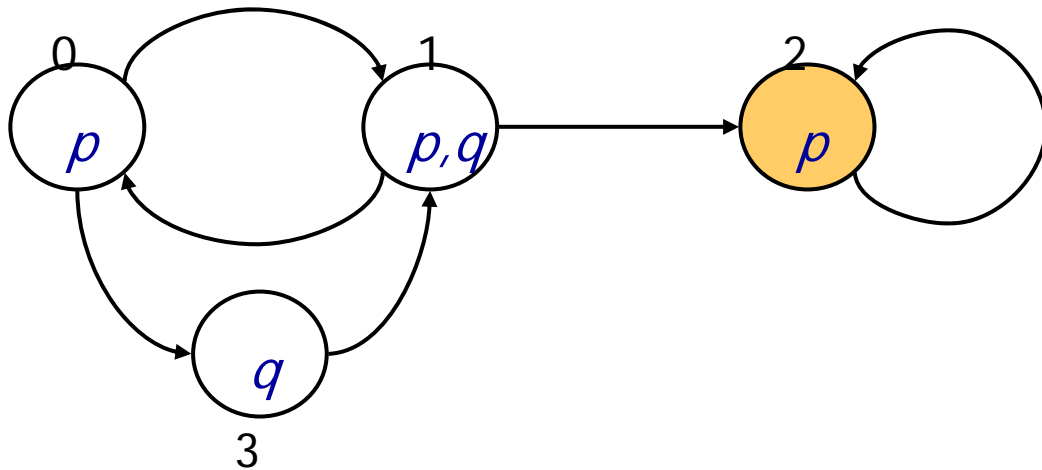


AG P

max fixpoint

```
A(x1,x2) =  
P(x1,x2) &  
Forall y1,y2.  
  Trans(x1,x2,y1,y2) =>  
  A(y1,y2);
```

Model Checking

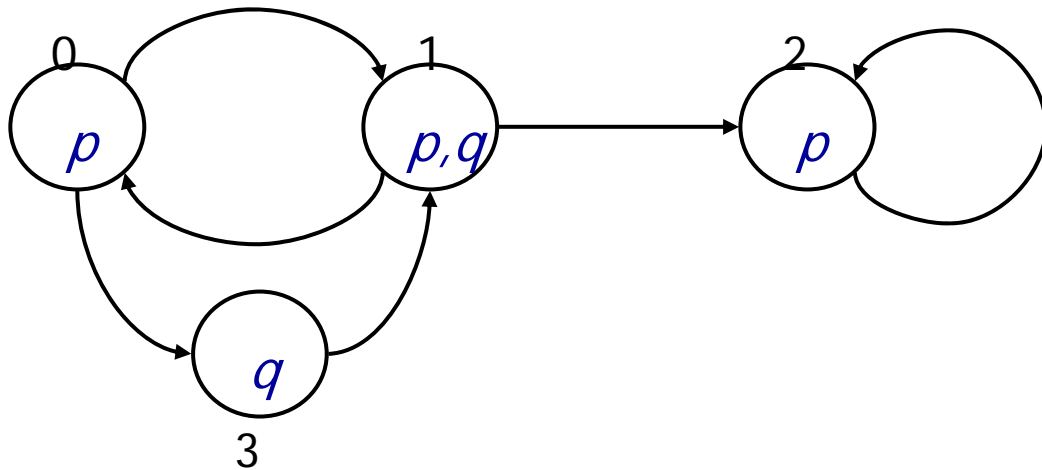


AG P

max fixpoint

```
A(x1,x2) =  
P(X1,x2) &  
Forall y1,y2.  
  Trans(x1,x2,y1,y2) =>  
  A(y1,y2);
```

Model Checking

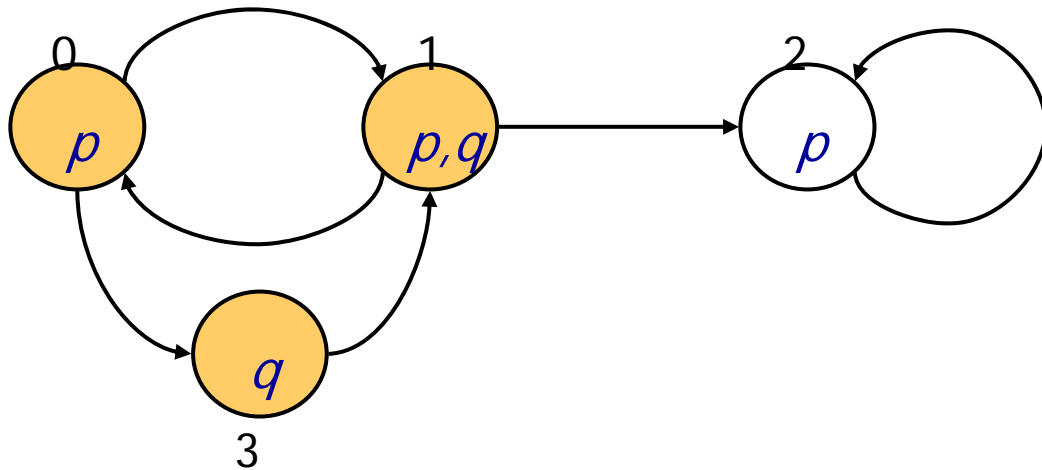


A(P UNTIL Q)

min fixpoint

$$\begin{aligned} \mathbf{U}(x1, x2) = & \\ & Q(x1, x2) + \\ & \{ P(x1, x2) \ \& \\ & \quad \text{Forall } y1, y2. \\ & \quad \text{Trans}(x1, x2, y1, y2) \Rightarrow \\ & \quad \mathbf{U}(y1, y2) \\ & \}; \end{aligned}$$

Model Checking



A(P UNTIL Q)

min fixpoint

$$\begin{aligned}
 \mathbf{U}(x1, x2) = & \\
 & Q(x1, x2) + \\
 & \{ P(x1, x2) \ \& \\
 & \quad \text{Forall } y1, y2. \\
 & \quad \text{Trans}(x1, x2, y1, y2) \Rightarrow \\
 & \quad \mathbf{U}(y1, y2) \\
 & \};
 \end{aligned}$$

Partitioned Transition Relation

Relational Product

LARGE

Exists $\mathbf{y}\mathbf{v}$. ($T(\mathbf{x}\mathbf{u}, \mathbf{y}\mathbf{v})$ & $S(\mathbf{y}\mathbf{v})$)

Asynchronous

Synchronous

$T(\mathbf{x}\mathbf{u}, \mathbf{y}\mathbf{v}) =$
 ($A\text{Trans}(\mathbf{x}, \mathbf{y})$ & $\mathbf{v}=\mathbf{u}$)
+ ($B\text{Trans}(\mathbf{u}, \mathbf{v})$ & $\mathbf{y}=\mathbf{x}$)

$T(\mathbf{x}\mathbf{u}, \mathbf{y}\mathbf{v}) =$
 $A\text{Trans}(\mathbf{x}, \mathbf{y})$
 & $B\text{Trans}(\mathbf{u}, \mathbf{v})$

Exists \mathbf{y} . $A\text{Trans}(\mathbf{x}, \mathbf{y})$ & $S(\mathbf{y}\mathbf{u})$
+
Exists \mathbf{v} . $B\text{Trans}(\mathbf{u}, \mathbf{v})$ & $S(\mathbf{x}\mathbf{v})$

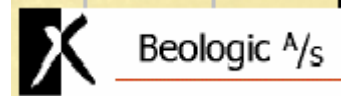
Exists $\mathbf{y}\mathbf{v}$.
 $A\text{trans}(\mathbf{x}, \mathbf{y})$
 & $B\text{trans}(\mathbf{u}, \mathbf{v})$ & $S(\mathbf{y}\mathbf{v})$

Exists \mathbf{y} .
 $A\text{trans}(\mathbf{x}, \mathbf{y})$
 & (Exists \mathbf{v} . $B\text{trans}(\mathbf{u}, \mathbf{v})$ & $S(\mathbf{y}\mathbf{v})$)

visualSTATE



CIT project VVS (w DTU)



Beologic's Products: salesPLUS visualSTATE

- 1980-95: *Independent division of B&O*
- 1995- : *Independent company*
B&O, 2M Invest,
Danish Municipal Pension Ins. Fund
- 1998: *BAAN*
- 2000: *IAR Systems A/S*

- *Embedded Systems*
- *Simple Model*
- *Verification of Std. Checks*
- *Explicit Representation*
(STATEEXPLOSION)
- *Code Generation*

Customers

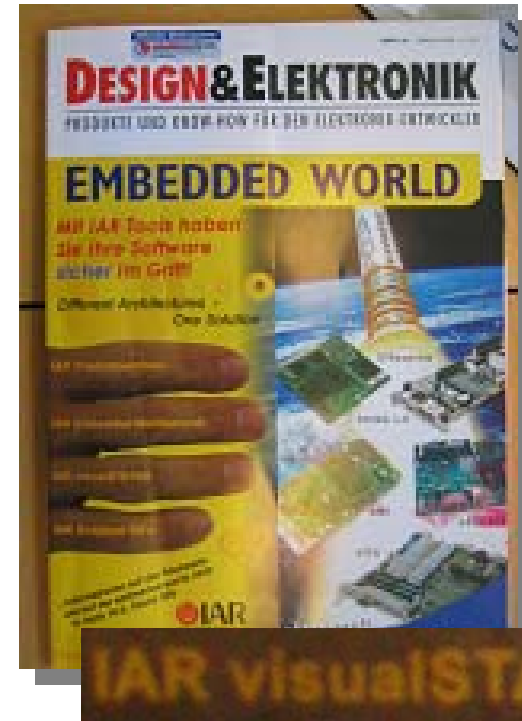
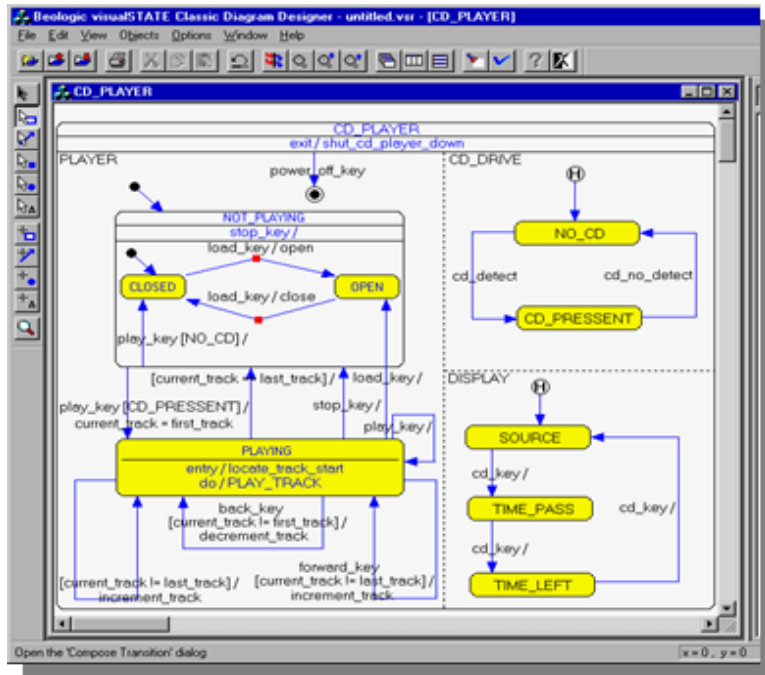
- ABB*
- B&O*
- Daimler-Benz*
- Ericson DIAX*
- ESA/ESTEC*
- FORD*
- Grundfos*
- LEGO*
- PBS*
- Siemens (approx. 200)*

Verification Problems:

- *1.400 components*
- *10⁴⁰⁰ states*

Our techniques has reduced verification by an order of magnitude (from 14 days to 6 sec)

visualSTATE



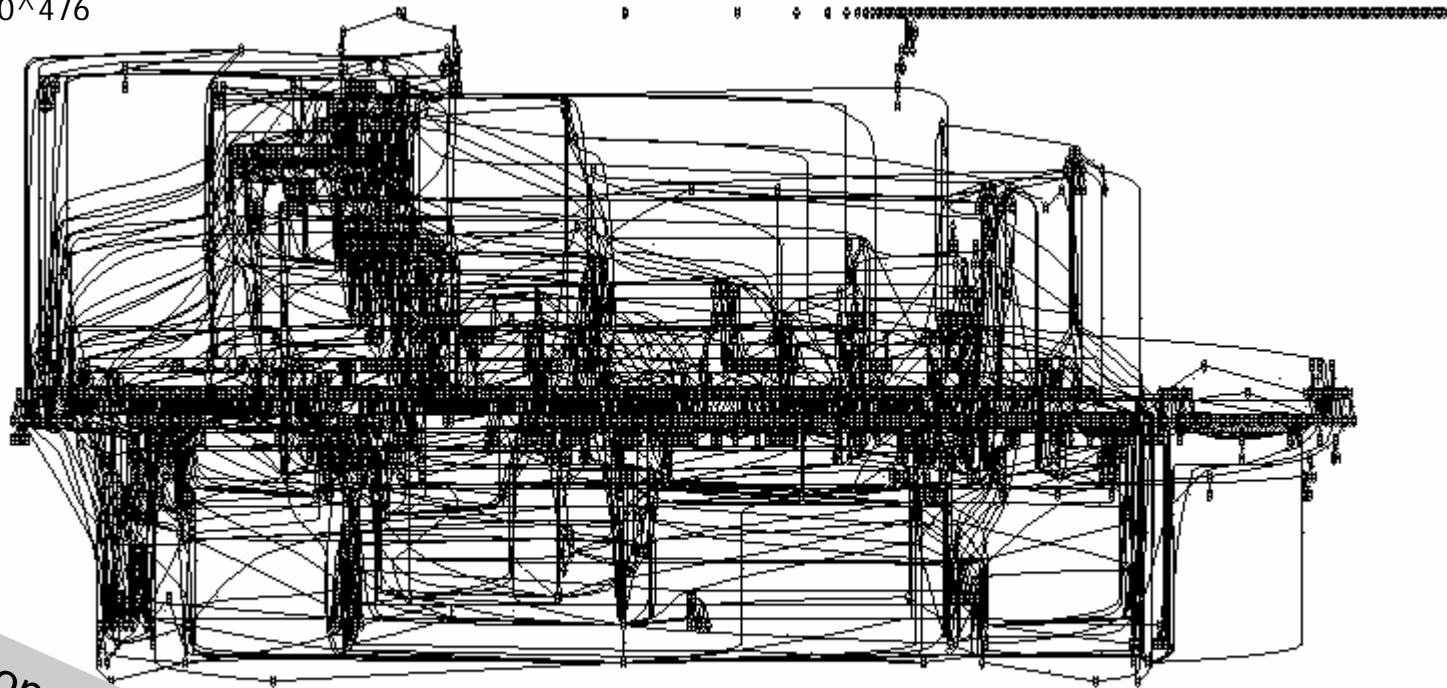
Embedded World
Nürnberg, 2005

Control Programs

A Train Simulator, visualSTATE (VVS)

1421 machines
11102 transitions
2981 inputs
2667 outputs
3204 local states
Declare state sp.: 10^{476}

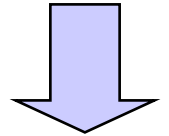
BUGS ?



“Ideal” presentation: 1 bit/state
will clearly NOT work!

Experimental Breakthroughs

Patented



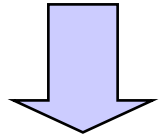
System	Mach.	State Space		Checks	Visual ST	St-of-Art		ComBack	
		Declared	Reach			Sec	MB	Sec	MB
VCR	7	10 ⁵	1279	50	<1	<1	6	<1	7
JVC	8	10 ⁴	352	22	<1	<1	6	<1	6
HI-FI	9	10 ⁷	1416384	120	1200	1.0	6	3.9	6
Motor	12	10 ⁷	34560	123	32	<1	6	2,0	
AVS	12	10 ⁷	1438416	173	3780	6.7	6	5.7	6
Video	13	10 ⁸	1219440	122	---	1.1	6	1.5	6
Car	20	10 ¹¹	9.2 10 ⁹	83	---	3.8	9	1.8	6
N6	14	10 ¹⁰	6399552	443	---	32.3	7	218	6
N5	25	10 ¹²	5.0 10 ¹⁰	269	---	56.2	7	9.1	6
N4	23	10 ¹³	3.7 10 ⁸	132	---	622	7	6.3	6
Train1	373	10¹³⁶	---	1335	---	---	---	25.9	6
Train2	1421	10⁴⁷⁶	---	4708	---	---	---	739	11

Machine: 166 MHz Pentium PC with 32 MB RAM

---: Out of memory, or did not terminate after 3 hours.

Experimental Breakthroughs

Patented



System	Mach.	State Space		Checks	Visual ST	St	ComBack		
		Declared	Reach				sec	MB	
VCR	7	10 ⁵	1279	50	---	---	1.1	7	
JVC	8	10 ⁴	352	---	---	---	---	6	
HI-FI	9	10 ⁷	1416384	---	---	---	3.9	6	
Motor	12	10 ⁷	---	---	---	---	2.0	6	
AVS	12	10 ⁷	---	---	---	---	5.7	6	
Video	13	10 ⁷	---	---	---	1.1	6	1.5	6
Car	20	---	---	---	---	3.8	9	1.8	6
N6	1	---	---	43	---	32.3	7	218	6
N5	---	---	---	269	---	56.2	7	9.1	6
N4	---	10 ⁸	---	132	---	622	7	6.3	6
Train1	---	---	---	1335	---	---	---	25.9	6
Train2	---	---	---	4708	---	---	---	739	11

Our technique have reduced verification time by several orders of magnitude (eg. From 14 days to 6 sec)

Machine: 166 MHz Pentium PC with 32 MB RAM

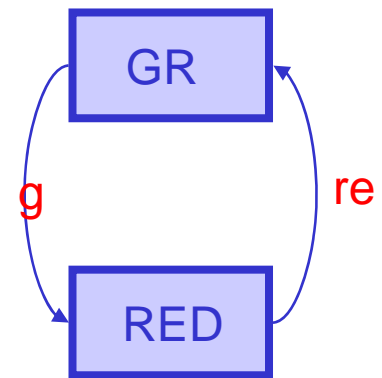
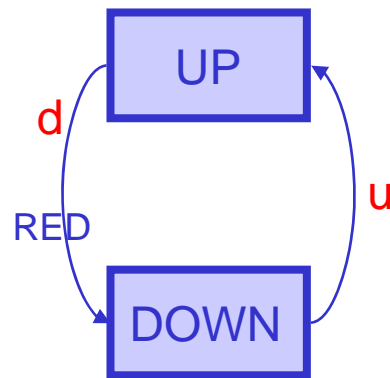
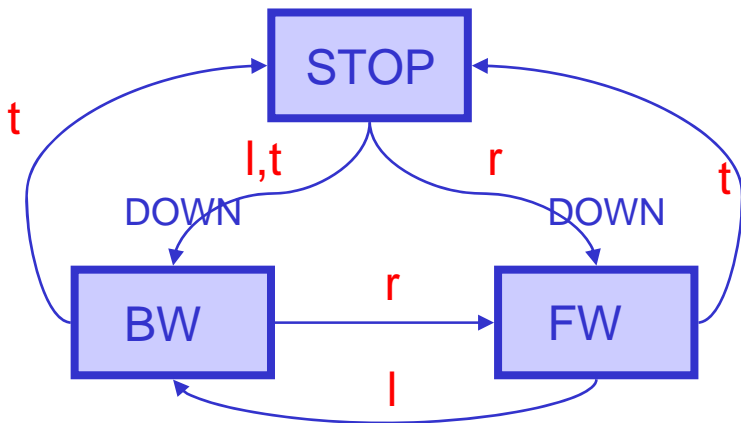
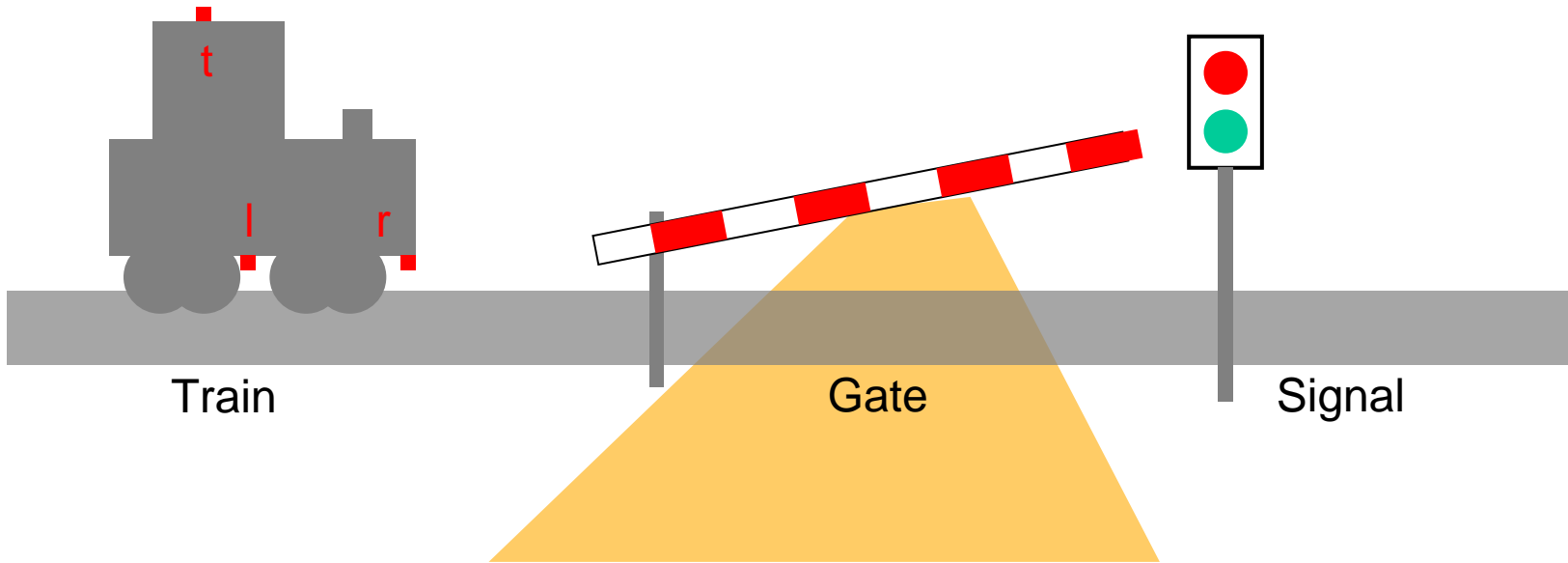
---: Out of memory, or did not terminate after 3 hours.

Compositional Backwards Reachability

[TACAS'98]

Example

The Small Train



State-Event Model

visualSTATE

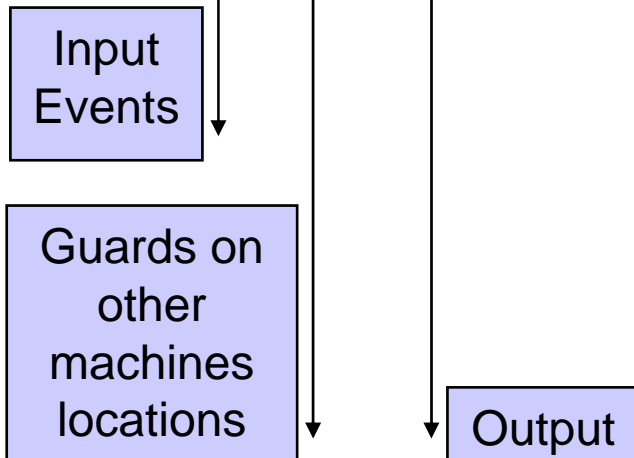
Syntax

n synchronously combined machines

$$M_i = (S_i, s_i^0, T_i)$$

where

$$T_i \subseteq S_i \times E \times G_i \times M(O) \times S_i$$



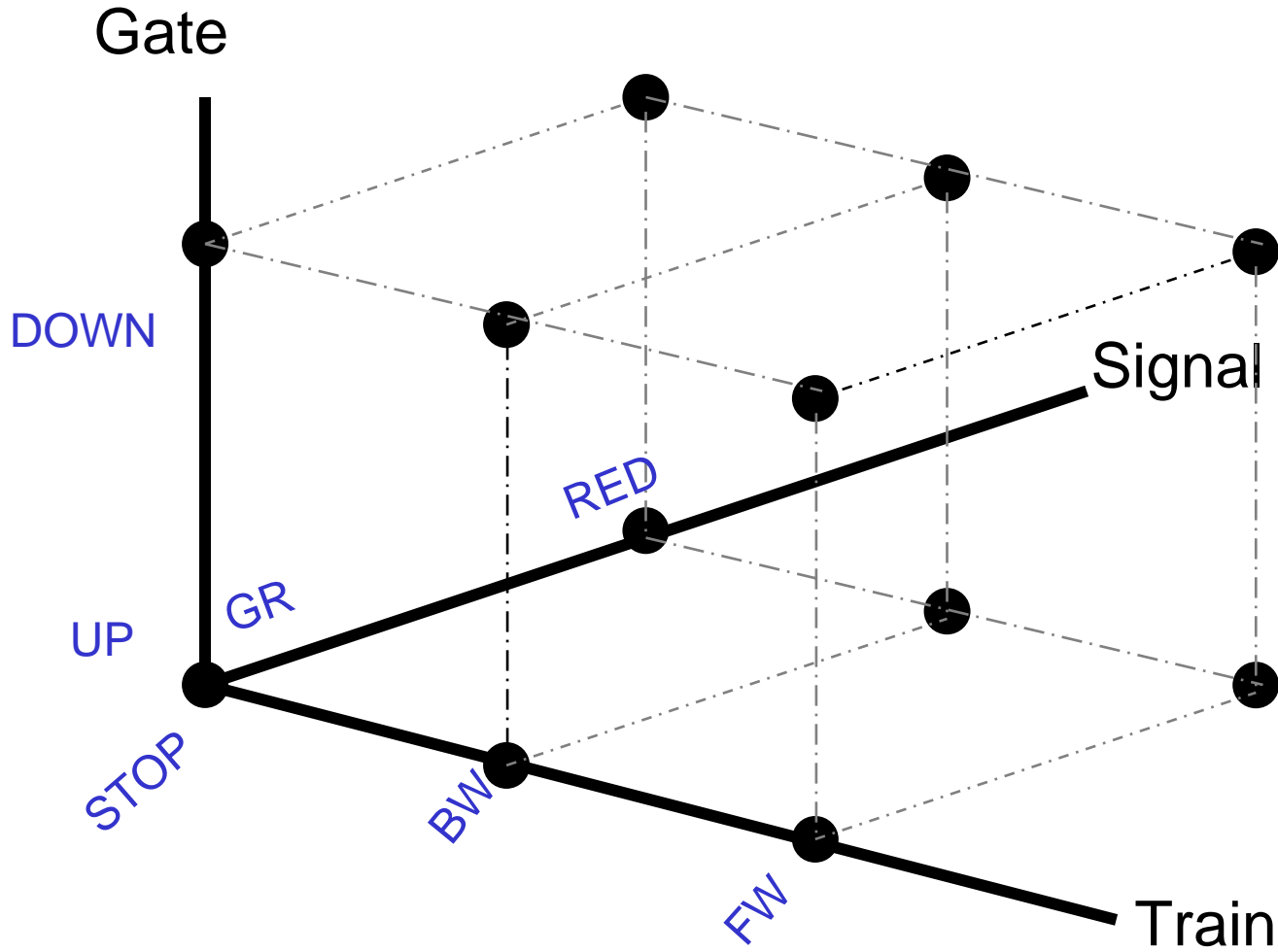
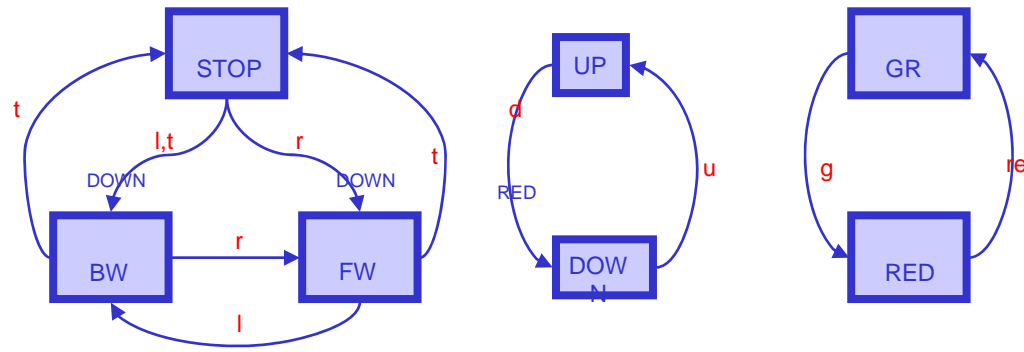
Semantics

$$(s_1, \dots, s_n) - e, \cup o_i \rightarrow (s_1', \dots, s_n')$$

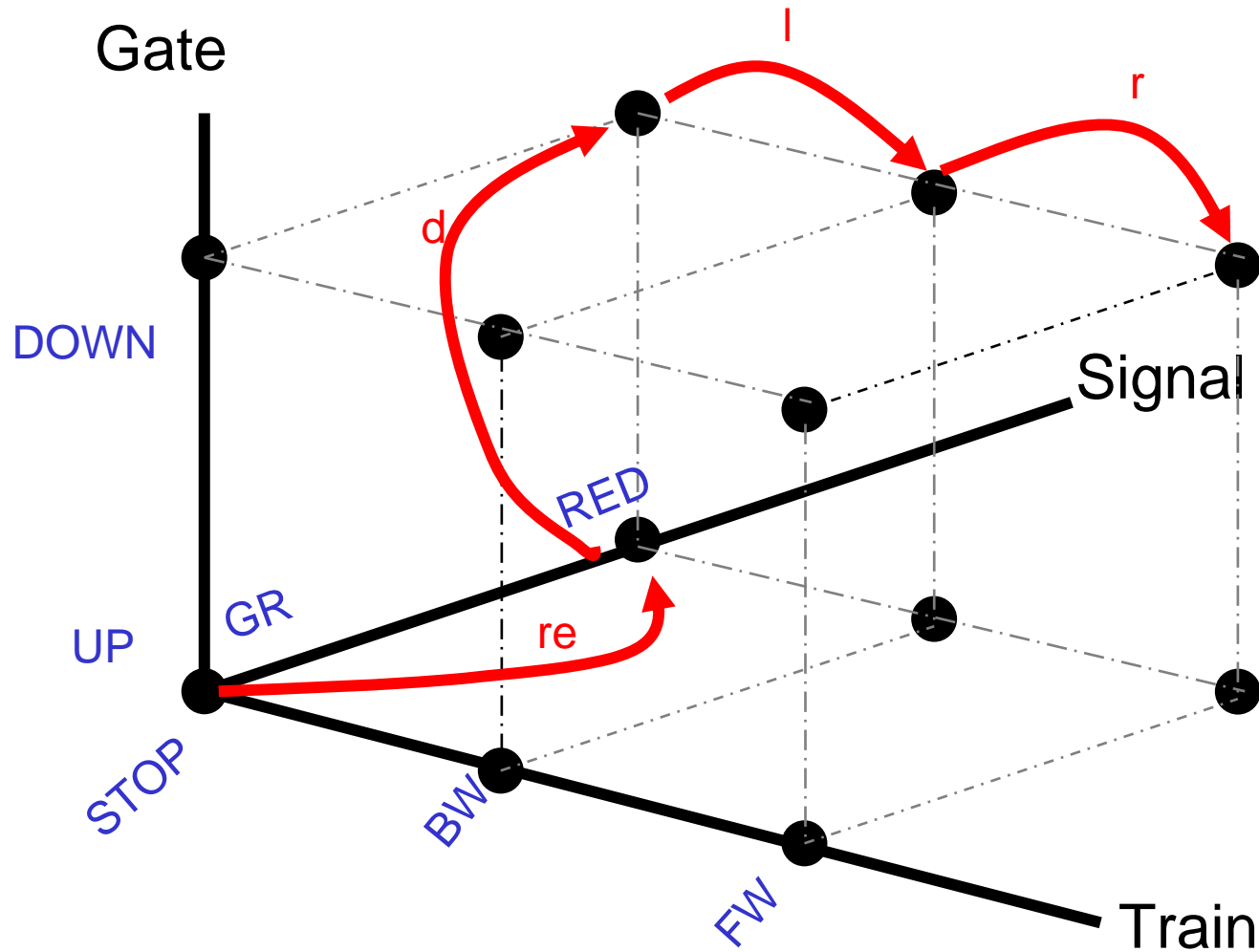
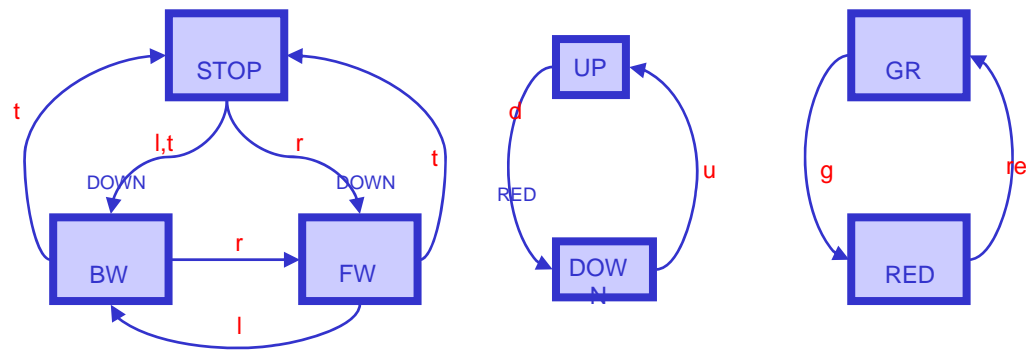
iff

$$\left\{ \begin{array}{l} s_i - e, g_i, o_i \rightarrow s_i' \text{ with} \\ \quad \quad \quad g_i(s_1, \dots, s_n) = \text{true} \\ \text{or} \\ s_i = s_i' \text{ and } o_i = \emptyset \text{ and} \\ \text{whenever } s_i - e, g_i \rightarrow \\ \text{then } \quad \quad \quad g_i(s_1, \dots, s_n) = \text{false} \end{array} \right\}_{i=1..n}$$

Small Train (cont)



Small Train (cont)

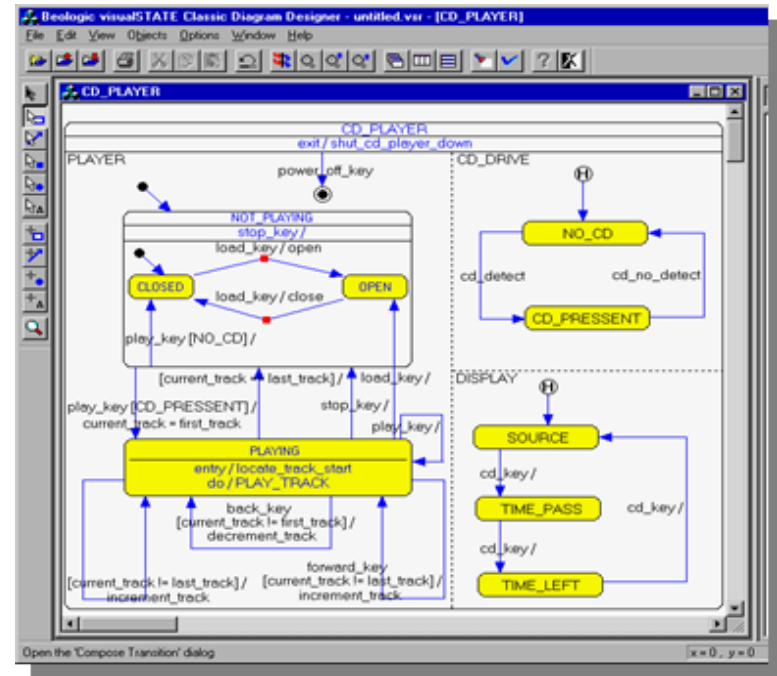


Generic Checks

visualSTATE offers checks for a number of predefined properties:

- Reachability of states
- Firing of transitions
- Input without interpretation
- Output without generation
- Conflicting Rules
- Local Deadlock
- Global Deadlock

Reachability



Not a single CHECK but several thousands!

Guard dependencies

Let

$g_1, g_2, g_3, \dots, g_N$ (N big)

be the guards we want to show reachable

If

$g_i \Rightarrow g_j$ (e.g. $g_i = \text{DOWN} \wedge \text{UP}$, $g_j = \text{DOWN}$)

then it suffices to show that g_j reachable, i.e. there is a reachable global state satisfying g_j .

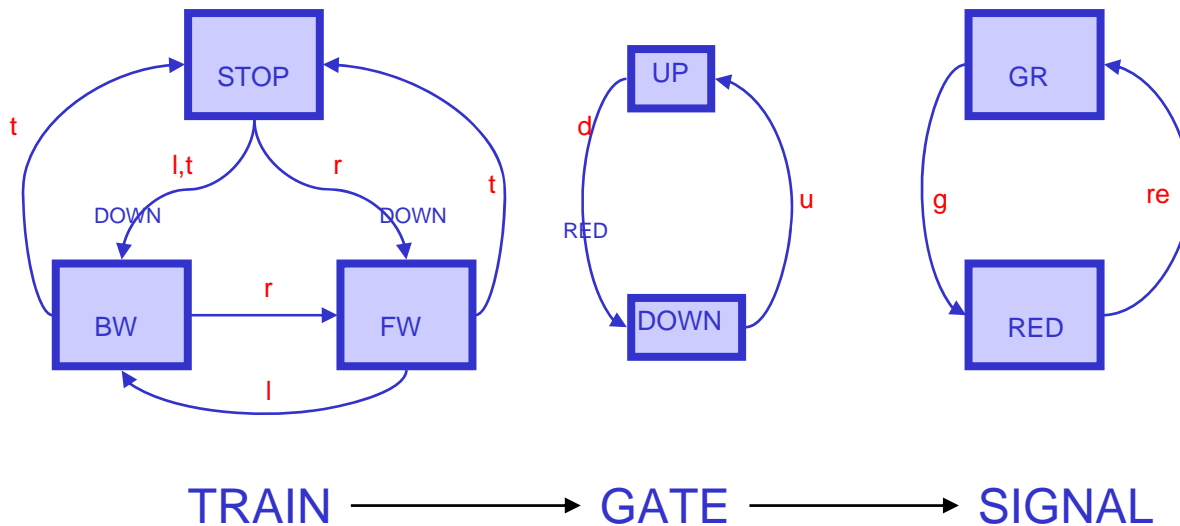
Sort $g_1, g_2, g_3, \dots, g_N$ according to size and check only if NOT implied by a previously shown reachable guard



40-70-90 % reduction.

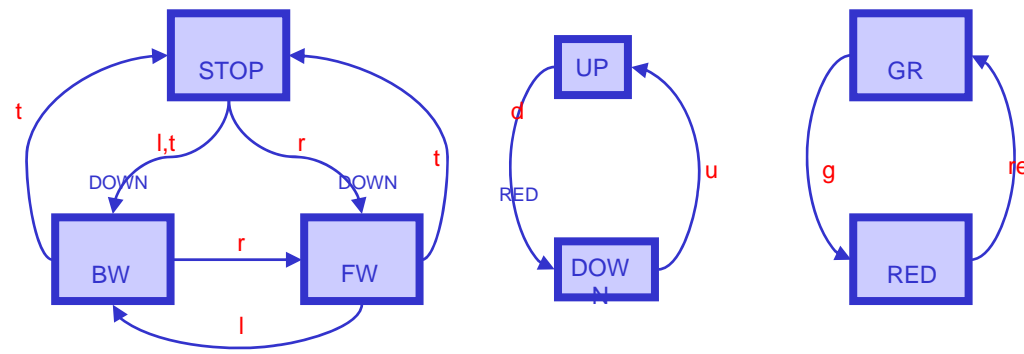
Machine Dependencies

A guard g in machine M_i that depends on/refers to a state in M_j introduces a dependency from M_i to M_j



Backwards statespace iterations can be restricted to dependency closed sets, e.g. $DC(\text{GATE}) = \{\text{GATE}, \text{SIGNAL}\}$

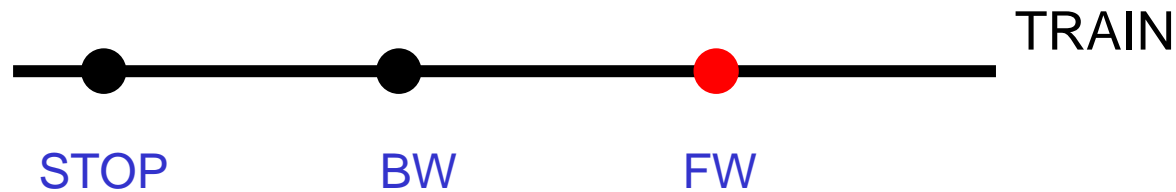
Compositional Backwards



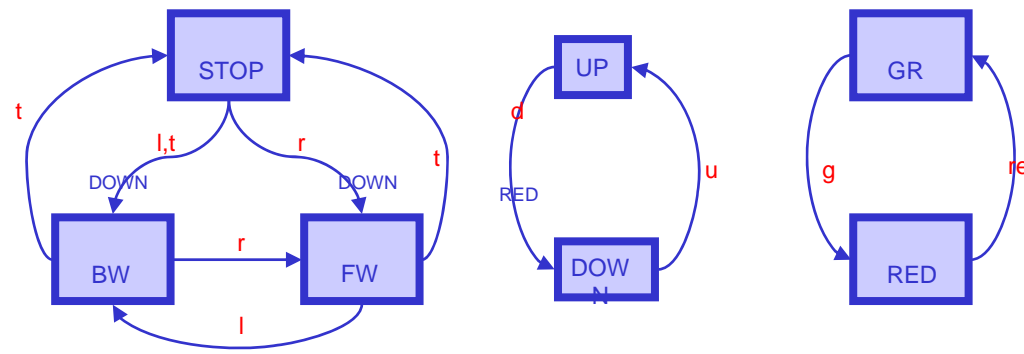
Is **FW** reachable ?

IDEA:
Compute backward reachable states as much as possible with minimal set of machines.
Increase set of considered machines when necessary!

Consider TRAIN



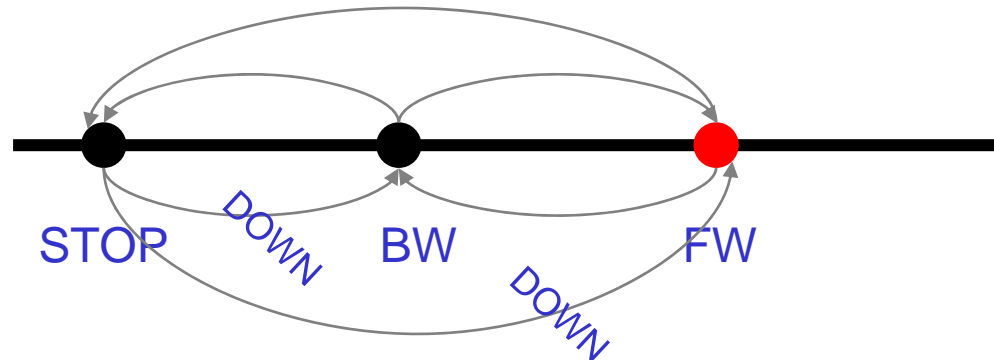
Compositional Backwards



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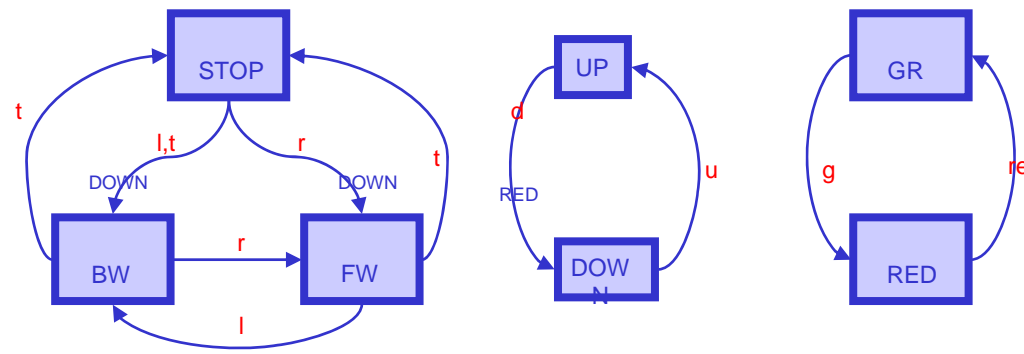
Consider TRAIN



TRAIN

Ignoring INPUT event

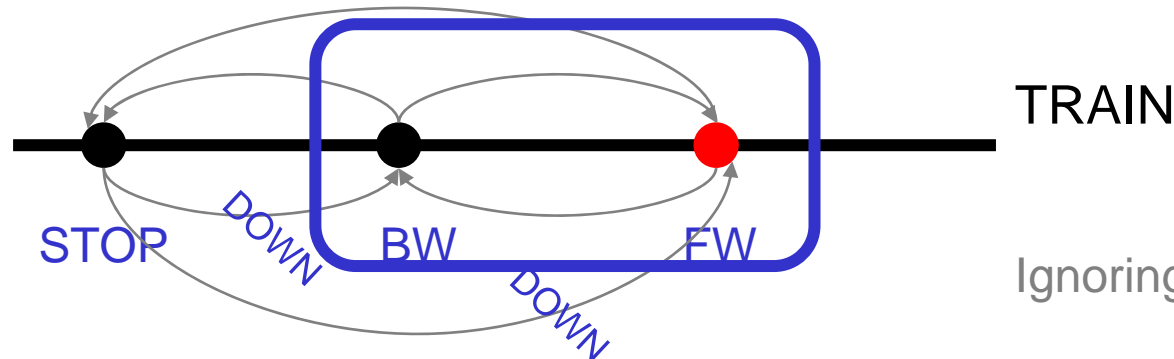
Compositional Backwards



Is **FW** reachable ?

IDEA:
 Compute backward reachable states as much as possible with minimal set of machines.
 Increase set of considered machines when necessary!

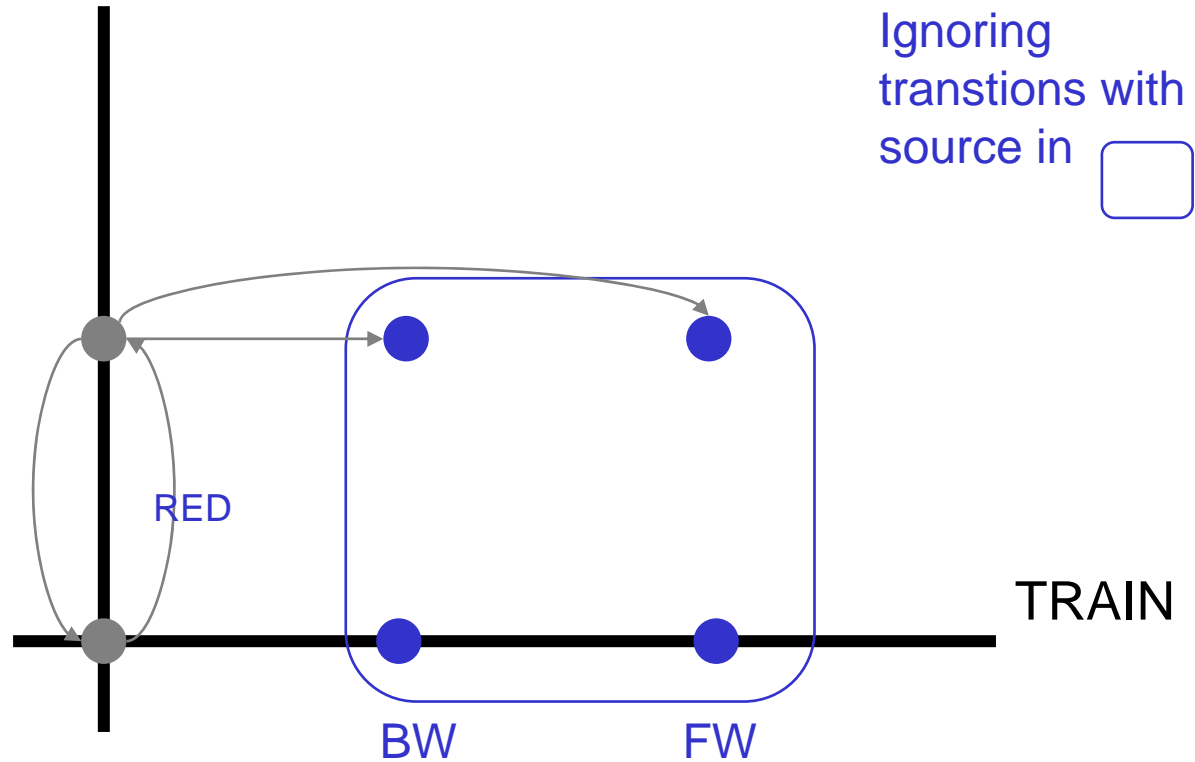
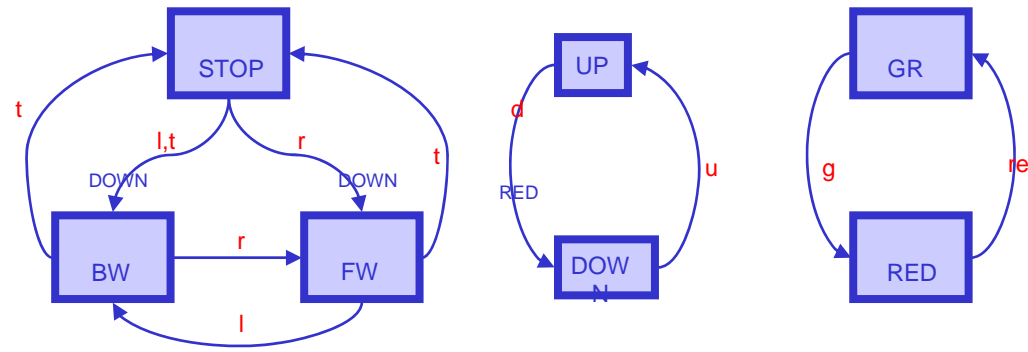
Consider TRAIN



Ignoring INPUT event

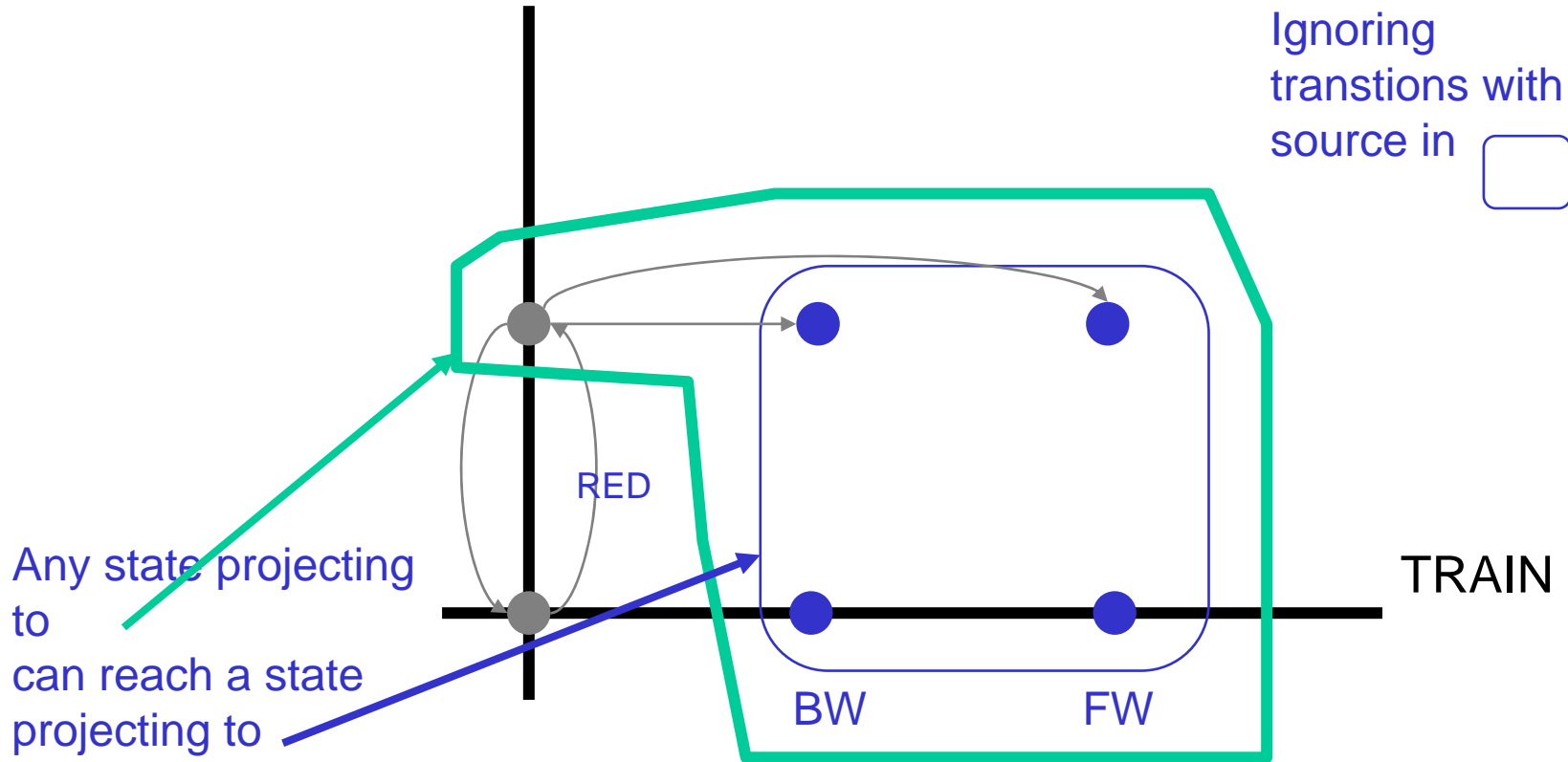
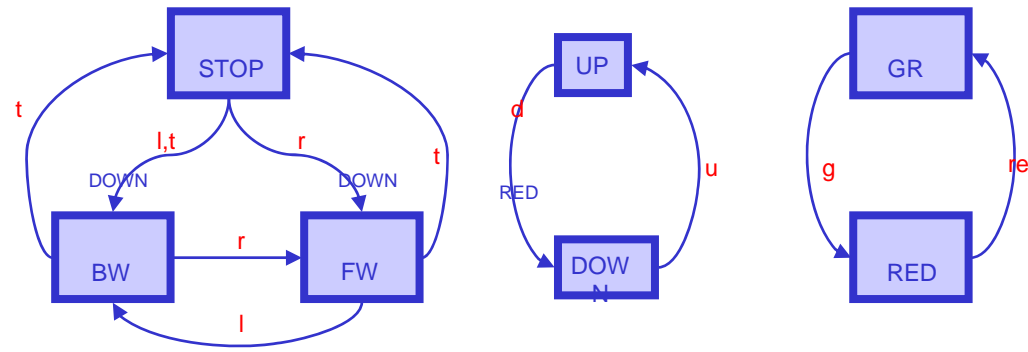
Compositional Backwards

Include GATE !

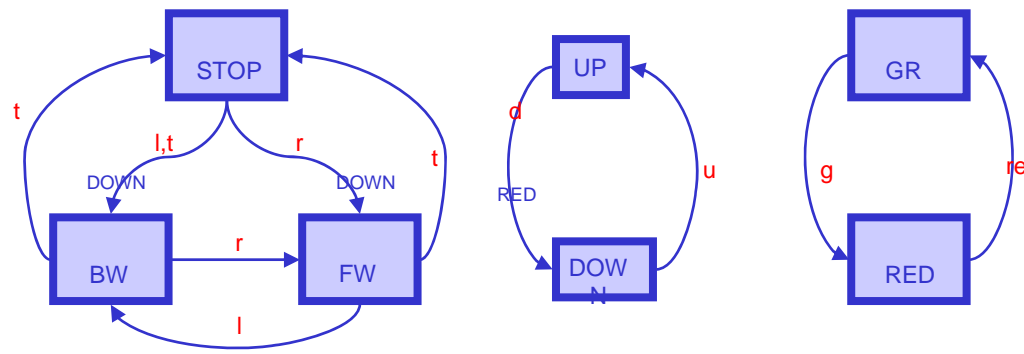


Compositional Backwards

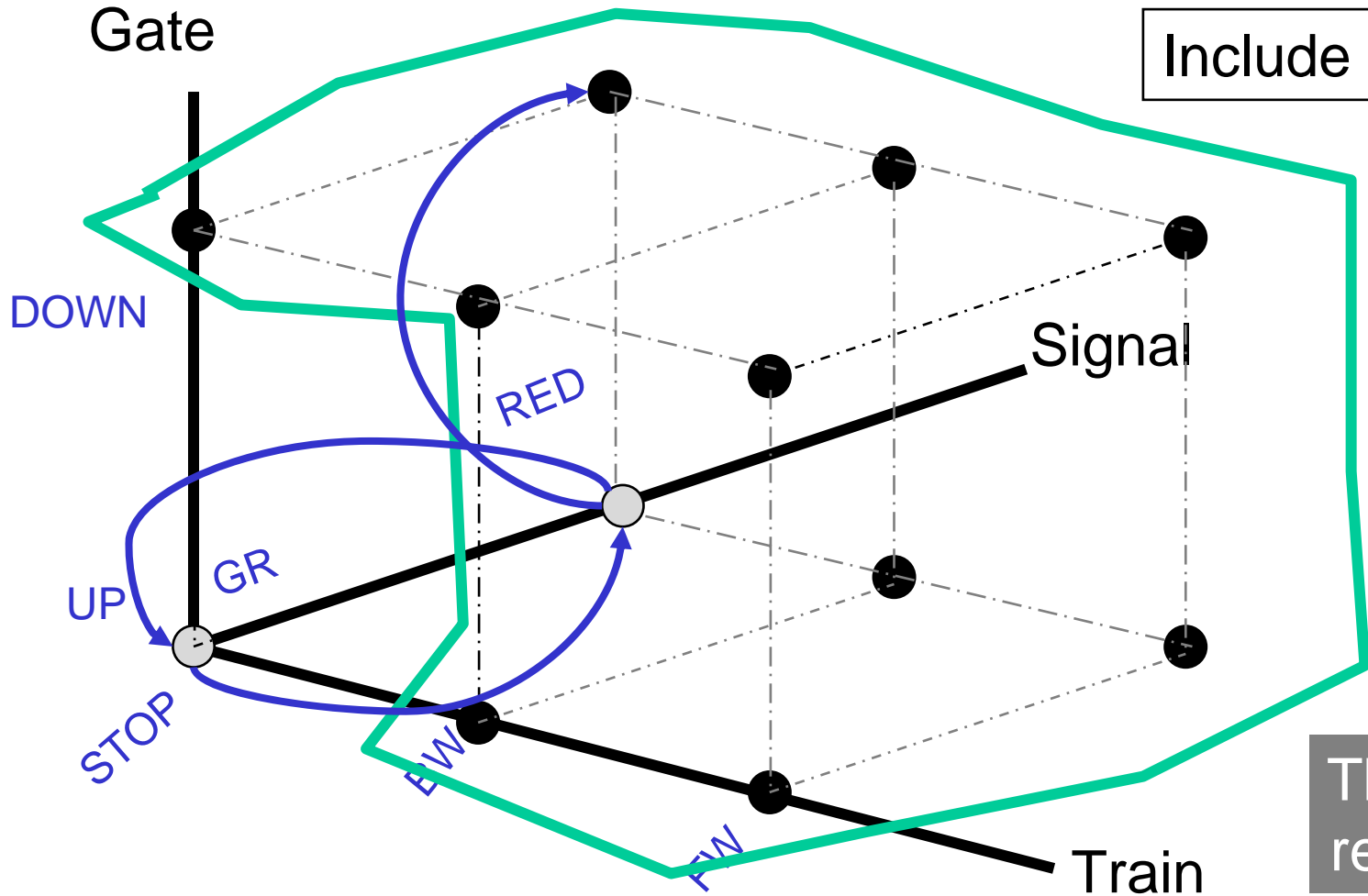
Include GATE !



Compositional Backwards



Include SIGNAL !



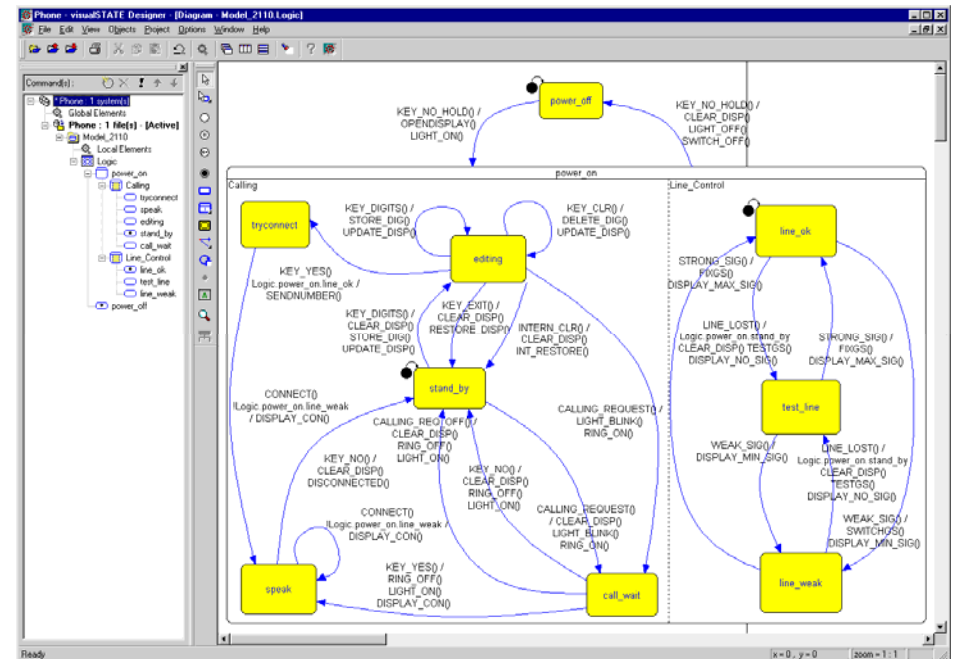
Thus FW is reachable !

Hierarchical Systems

[TACAS'99]

IDEA

Reuse already known reachability properties of superstates to conclude reachability of substates !



Experimental results

