# Introduction to General and Generalized Linear Models 

 Introduction to RAnders Nielsen, Henrik Madsen, Jan Kloppenborg

Informatics and Mathematical Modelling<br>Technical University of Denmark<br>DK-2800 Kgs. Lyngby

January 2012

## This lecture

Introduction to the R software we will be using in this course

## What is R

- Environment to carry out statistical analysis
- Based on "S" which was developed by John M. Chambers (Bell Lab)
- Received the prestigious ACM (Association for Computing Machinery) award in 1998
- Quote from the award: "For The $S$ system, which has forever altered how people analyze, visualize, and manipulate data"
- Open source and free
- Simple calculator
> $2+2$
[1] 4


## What is R

- Vector, matrix, and common linear algebra stuff
$>x<-c(1,2)$
$>A$ <- matrix $(c(1,2,3,4)$, nrow $=2)$
> solve(A, x)
[1] 10
- Simulating random numbers
> x <- runif(20, 0, 10)
$>y<-2 * x-3+\operatorname{rnorm}(20, s d=1.5)$
- Statistical modeling
$>\operatorname{coef}(\operatorname{lm}(y \sim x))$
(Intercept) X
-3.454192 2.159108


## What is R

- Graphics is excellent in R :
> par(mfrow = c(1, 2))
$>\operatorname{plot}(x, y)$
> abline(lm(y ~ x), col = "red", lwd = 3)
> plot(lm(y ~x), which = 1, lwd = 3)




## What is $R$

- R is a fairly complete programming language
- A good editor is useful for writing longer programs
- Emacs has an "Emacs Speak Statistics" mode
- On windows the editor "TINN-R" is preferred by many
- These editors allow you to paste code directly into R and have syntax highlighting
- Another acceptable way is to use "your favorite editor" and copy and paste into R
- Run all commands saved in a text file with:
> source("myfile.R")


## Extracting sub-elements in R

- Consider the vector:
$>x<-c(1: 5,-5:-1,10,20)$
$>x$
[1] $\begin{array}{llllllllllll}1 & 2 & 3 & 4 & 5 & -5 & -4 & -3 & -2 & -1 & 10 & 20\end{array}$
- Get elements number 5, 6, 7, and 8
> $x[5: 8]$
[1] 5
- Get elements below 0
$>x[x<0]$
[1] $-5 \quad-4 \quad-3-2 c c \mid$
- Get all except number 5 and 9
$>x[-c(5,9)]$
[1] $\begin{array}{lllllllllll}1 & 2 & 3 & 4 & -5 & -4 & -3 & -1 & 10 & 20\end{array}$


## Indexing also works for matrices

- The matrix:
$>A<-\operatorname{matrix}((-4): 5$, nrow $=2$, ncol = 5)
$>A$

$$
\begin{array}{rrrrrr} 
& {[, 1]} & {[, 2]} & {[, 3]} & {[, 4]} & {[, 5]} \\
{[1,]} & -4 & -2 & 0 & 2 & 4 \\
{[2,]} & -3 & -1 & 1 & 3 & 5
\end{array}
$$

- Get elements below 0
$>A[A<0]$
[1] $-4 \quad-3-2-1$
- Or assign them something else
$>A[A<0]<-0$
$>A$

|  | $[, 1]$ | $[, 2]$ | $[, 3]$ | $[, 4]$ | $[, 5]$ |
| :--- | ---: | ---: | ---: | ---: | ---: |
| $[1]$, | 0 | 0 | 0 | 2 | 4 |
| $[2]$, | 0 | 0 | 1 | 3 | 5 |

## Indexing also works for matrices

- The matrix is still:
> $A$

|  | $[, 1]$ | $[, 2]$ | $[, 3]$ | $[, 4]$ | $[, 5]$ |
| :--- | ---: | ---: | ---: | ---: | ---: |
| $[1]$, | 0 | 0 | 0 | 2 | 4 |
| $[2]$, | 0 | 0 | 1 | 3 | 5 |

- pick a row
> A[2, ]
[1] 00135
- Or two columns
$>A[, c(2,4)]$

$$
\begin{array}{lrr} 
& {[, 1]} & {[, 2]} \\
{[1,]} & 0 & 2 \\
{[2,]} & 0 & 3
\end{array}
$$

## Writing own functions

- Let's write a simple function
> weighted.ave <- function(x, w = rep(1, length(x))) \{
$+\quad$ s1 <- $\operatorname{sum}(x \quad *$ w)
$+\quad$ s2 <- $\operatorname{sum}(w)$
$+\quad$ return(s1/s2)
+ \}
> weighted.ave(c(0, 1, 2, 3, 4), c(7, 3, 10, 17, 21))
[1] 2.724138
- Notice we can supply default values


## The working space

- To get info about the current working space
> getwd()
[1] "/home/an/02424/slides"
- To set the working space to something else
> setwd("c:/my/path/to/somewhere")
- To save an image of all the current defined variables and functions use > save.image(file = "myStuff.RData")
- To load a saved image use
> load("myStuff.RData")


## Reading data from a file

- Often data is organized as below (columns separated with white space, and a line of headings).

```
sex x y
M 0.3 0.01
M 1.0 0.11
M 2.1 0.04
F 2.2 0.02
F 0.1 0.10
F 0.2 0.06
```

- Such data can be read in with:
> myData <- read.table("datafile.tab", header = TRUE)
- The resulting object myData is a so-called dataframe.
- A dataframe behaves mostly like a matrix, but not quite.
- Can contain columns of different types.
- Columns can be extracted via the \$-operator e.g. myData\$x


## Factors

- Factors are used to describe categories, and a factor in R knows how many categories is has.
> x <- rep(1:5, each = 3)
$>x$
[1] 1111222333444555
> f <- factor $(x)$
$>f$
[1] 111222333444555
Levels: 12345
> is.factor (x)
[1] FALSE
> is.factor (f)
[1] TRUE


## Fitting linear models

- General linear models lm()
- Generalized linear models glm()
- Mixed effects linear models lme() (in package nlme).


## Model formulas

- A specified model can e.g. look like
> fit <- lm ( $\mathrm{y}^{\sim} \mathrm{x}+\mathrm{f}+\mathrm{g}: \mathrm{h}+\mathrm{k}: z$ )
which would correspond to:

$$
y_{i}=\mu+\alpha x_{i}+\beta\left(f_{i}\right)+\delta\left(g_{i}, h_{i}\right)+\gamma\left(k_{i}\right) z_{i}+\varepsilon_{i}
$$

- Which are factors?
- Interactions between two factors is different from interaction between factor and covariate.
- What does interactions between two covariates mean?
- Interactions are specified with e.g. f:g
- A shorthand for specifying both main effects and interaction effects is $\mathrm{f} * \mathrm{~g}$ (same as $\mathrm{f}+\mathrm{g}+\mathrm{f}: \mathrm{g}$ )
- Adding a -1 to the formula will get rid of the common intercept $\mu$


## Quick example

- What are we doing here?
$>x<-\operatorname{rep}(1: 5$, each $=3)$
$>y<-\sin (2 * x)+\operatorname{rnorm}(15, s d=0.1)^{>}$
$>f<-$ factor $(x)$
$>\operatorname{coef}(\operatorname{lm}(y \sim x))$
$\begin{array}{rr}\text { (Intercept) } & \mathrm{x} \\ 0.3767725 & -0.1069740\end{array}$
$>\operatorname{coef}(\operatorname{lm}(y \sim f-1))$

| $f 1$ | $f 2$ | $f 3$ | $f 4$ | $f 5$ |
| ---: | ---: | ---: | ---: | ---: |
| 0.8841206 | -0.8044325 | -0.2608534 | 1.0267678 | -0.5663493 |

- Are any of these models useful?


## Build in distributions

- R has quite a few build in distributions. The naming convention is: $\mathrm{d}<$ name> (x) Density function $\mathrm{p}<$ name $>$ ( x ) Cumulated density function (probability $\leq x$ ) $\mathrm{q}<$ name $>(\mathrm{p})$ Quantile (the point $x$ where the probability $\leq x$ is $p$ ) $r<n a m e>(n)$ Simulate random numbers from the distribution
- A basic R installation has: beta, binom, cauchy, chisq, exp, f, gamma, geom, hyper, logis, multinom, nbinom, norm, pois, signrank, t, tukey, unif, weibull, wilcox
- Is anything missing?
- The multivariate normal, but that is in an extension package
> install.packages("mvtnorm")
> library(mvtnorm)
- Now we also have: dmvnorm, pmvnorm, qmvnorm, rmvnorm,


## Basic control-flow

- The matrix
> A <- matrix((-4):5, nrow = 2, ncol = 5)
- Sum all positive elements (in a sub-optimal way)
> $S$ <- 0
> for (i in 1:nrow(A)) \{
$+\quad$ for (jin 1:ncol(A)) \{
$+\quad i f(A[i, j]>0)$ \{
$+\quad S<-S+A[i, j]$
$+\quad\}$
$+\quad\}$
+ \}
> S
[1] 15
- Would be better to use
$>\operatorname{sum}(A[A>0])$
[1] 15


## Loop avoidance

- Generally speaking loops are slow, and can mostly be avoided
- Use build in functions (sum(), min(), max(), which(), which.min(), rowMeans(), colMeans(), rowSums(), colSums(),...)
- Use linear algebra
- Use the build in apply functions. A few examples are:
apply () Use a function over one index (or more) of a matrix (array) tapply () Use a function within a number of groups
lapply () Use a function for each element in a list
> apply(A, 2, sd)
[1] 0.70710680 .70710680 .70710680 .70710680 .7071068
- If none of the above does the trick, then it is possible to implement the computer intensive part in a small piece of C code and call that from within $R$ (see e.g. help for . C()).


## Getting help

- From within R simply type "?" followed by the function name e.g:
> ?dnorm
> ?"for"
- Many manuals and reference cards at http://www.r-project.org
- http://stackoverflow.com/questions/tagged/r
- http://www.google.com
- Very helpful community on mailing list.

