Problem Set 8

Ph.D. Course 2012:
Nodal DG-FEM for solving partial differential equations

If you have not already done so, please download all the Matlab codes from the book from

http://www.nudg.org/

and store and unpack them in a directory you can use with Matlab.

The focus of todays exercises is the constuction and solution of DG-FEM for elliptic problems.

Let us first consider the problem

$$u_{xx}(x) = f(x), \quad u(x) = \sin(\pi x), \quad x \in [a, b]$$

- Determine $f(x)$ to have the desired exact solution.
- Formulate and implement a DG-FEM scheme for the 1D Poisson equation. Generalize the flux so you can consider both a stabilized central flux and a "flip-flop" (LDG) flux.
- Check to see if the operator is symmetric for different combinations of boundary conditions – Dirichlet/Neumann at the two boundaries.
- Solve first the problem on $x \in [0, 2]$ with $u(0) = u(2) = 0$. Solve it using both flux types and discuss whether the accuracy is as expected.
- Solve the problem with $x \in [0, 1.5]$ with $u(0) = u_x(2) = 0$. Solve it using both flux types and discuss whether the accuracy is as expected.
- Generalize the solver to also include

$$(au_x)_x = f.$$

- Choose a smooth $a(x)$ and a solution to test the accuracy of the solver.

Let us now also consider the two-dimensional case,

$$\nabla^2 u(\mathbf{x}) = f(\mathbf{x}), \quad \mathbf{x} \in [-1, 1]^2.$$

- Generate several grids with DistMesh or some other grid generator.
- Assume first that $u(x, y) = \sin(\pi x)\sin(\pi y)$ and find $f(x, y)$.

- Formulate and implement a DG-FEM scheme for the 2D Poisson equation. Generalize the flux so you can consider both a stabilized central flux and a "flip-flop" (LDG) flux.
- Solve the problem assuming Dirichlet boundary conditions and look at accuracy of the schemes.
- Use a direct solver and consider the effect of reordering algorithms on solution speed and memory usage. The goal of the reordering is to minimize fill (i.e. nonzero elements that appear during factorization).
- Consider the generalized problem

  $$\nabla \cdot (a(\mathbf{x})\nabla u) = f(\mathbf{x}), \mathbf{x} \in [-1,1]^2,$$

  and alter the code to enable the solution of this problem.

If time permits it

- Check that the problem is symmetric and positive definite for different values of N and K.
- Solve the problem using an iterative solver.
- Consider the impact of different preconditioning techniques on solution speed.
- Extend the codes to deal with inhomogeneous boundary conditions.

Enjoy!