
Problem Set 6

Ph.D. Course 2012:

Nodal DG-FEM for solving partial differential equations

If you have not already done so, please download all the Matlab codes from the book from

<http://www.nudg.org/>

and store and unpack them in a directory you can use with Matlab.

In this problem set we will be concerned about how to generate meshes for our simple needs. Furthermore, once we have the meshes we need, how do we define and setup specialized meshes for the specific problem? We shall see that once we have obtained a mesh from our favorite mesh generator, we can take advantage of distance functions for creating such simple maps.

- Download the DistMesh matlab package from <http://www-math.mit.edu/~persson/mesh/> or choose your own favorite mesh generator. Then make it possible to load or create a mesh from within Matlab. Having generated a mesh you should have at least the three mesh tables EToV, VX and VY which completely defines the geometry of the problem you want to solve. The DistMesh package comes with predefined signed distance functions.
- Make sure that all elements in your mesh are ordered in a counter-clockwise fashion. Why is this important?
- Download the Mesh2D v23 Matlab function from <http://www.mathworks.com/matlabcentral/fileexchange/10307>. In the following you can use the MeshQuality.m script to assess the quality of your 2D mesh. If the quality is poor, then see if you can improve your mesh using Laplacian smoothing using the commands

```
% smoothmesh from Mesh2D v23
>> [p,EToV] = smoothmesh([VX(:) VY(:)],EToV,maxit,tol);
>> VX = p(:,1); VY = p(:,2);
```

- Setup up a script to define a square mesh $\mathbf{x} \in [-1,1]^2$ using your favorite mesh generator. Then make specialized maps vmapX and mapX for each X=E,W,N,S corresponding to West, East, North and South boundary edges using distance functions. To validate your setup use these new maps instead of mapB to impose boundary conditions in the Advect2Dxxx solver for the Rotating Hill Problem set 5.

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- Solve the Rotating Hill problem on an L-shaped domain created by your favorite mesh generator, $\mathbf{x} \in [-1, 1]^2 \setminus [0, 1]^2$.
 - Discuss what is required to make periodic maps in 2D? Then see if you can create periodic maps to solve problems in your Advect2Dxxx solver on a periodic domain.
 - Create a triangulation for the square mesh and make a simple conversion to a quad mesh using CatmullClarkSubdivision.m. Download the Quad extension DGFEM package from the course website and use the StartUp2Dquad.m file from this package to setup basic operators. What changes are required in your solver scripts to solve your problem on a quad mesh instead of a triangular mesh? Test it out, e.g. by running the Advect2Dxxx or rotating hill (cf. problem 5) solvers.

If time permit it

- Familiarize yourself with some of the basic operations in 2D, e.g.
 - How to differentiate a discrete function $f(x, y)$.
 - How to interpolate from a coarse to a fine grid and possibly back again.
 - How to determine the modal damping profiles of a given filter function in 1D/2D.
 - See if you can construct a nodal filter which preserves the edge values of the element after filtering.

Enjoy!