Feedback on Exercise 3

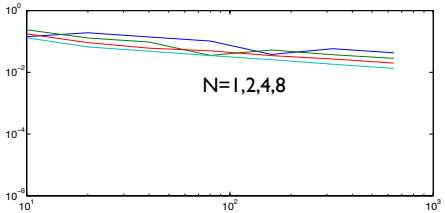
August 9, 2012

Consider

$$\partial_t u + a(x)\partial_x u = g(x,t), \quad x \in [-2,2].$$

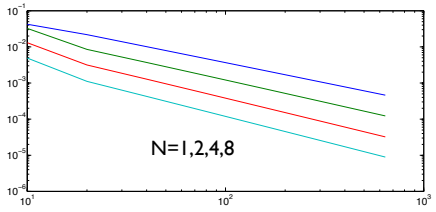
$$a(x) = 1 \text{ and } g(x,t) = 0.$$

$u(x,0) = -(\text{sign}(x) - 1)/2.$



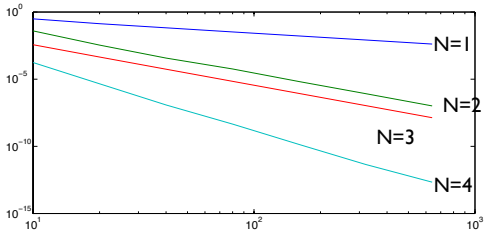
$h^{1/2}$

$u(x,0) = |x|.$



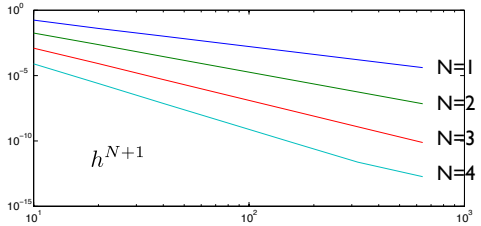
$h^{3/2}$

$$a(x) = \begin{cases} 1.5 & |x| \le 0.5 \\ 1 & \text{otherwise} \end{cases} \qquad u(x,t) = \sin(\pi(x - a(x)t)).$$



Central flux

Upwind flux



Consider

$$u_t + M(x)u_x = -p_x$$
$$p_t + M(x)p_x = -u_x$$

Expressed as

$$\mathbf{q}_t + \mathcal{A}(q)_x = 0, \qquad \mathbf{q} = \begin{bmatrix} u \\ p \end{bmatrix}, \quad \mathcal{A} = \begin{bmatrix} M & 1 \\ 1 & M \end{bmatrix}.$$

Eigenvalues of A

$$\lambda(A) = M \pm 1$$

|M| > 1 - both eigenvalues have same sign
|M| < 1 - different signs